Kinematics of Mechanisms and Machines
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Lecture – 37
Gear Kinematics

In this lecture, I am going to discuss on Gear Kinematics and in the subsequent lecture we are going to use the kinematics relations that we obtain here in understanding gear trains.

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So, I will first introduce gears and gear kinematics, then I will discuss the Universal law of gearing action. Then, I will show you how to calculate the Transmission ratio both the transmission ratio for motion as well as the transmission ratio for torques.

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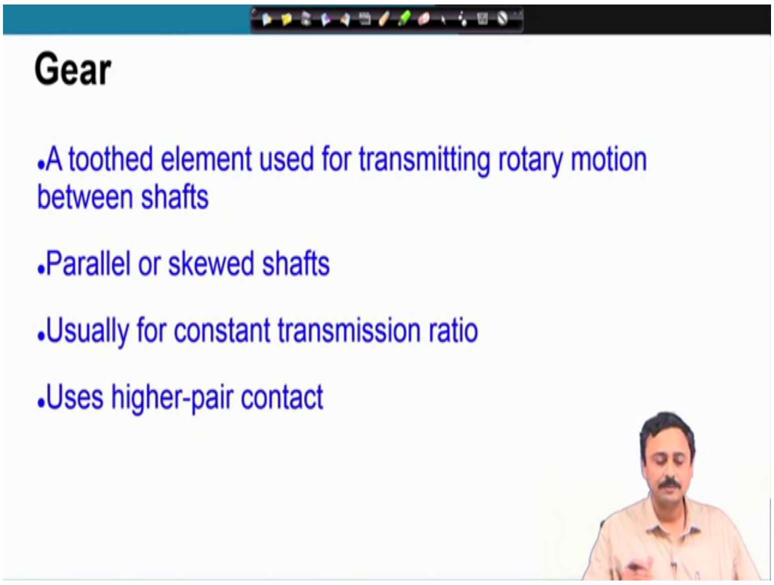

So, what is a gear? A gear is a toothed element used for transmitting rotary motion between shafts. The shafts can be parallel or skewed. Usually, we use gears for constant transmission ratio.

But there are very special applications which we are of course not going to discuss, where you can have periodically varying transmission ratios. A gear uses higher pair contact which means that these are either line contacts or point contacts.

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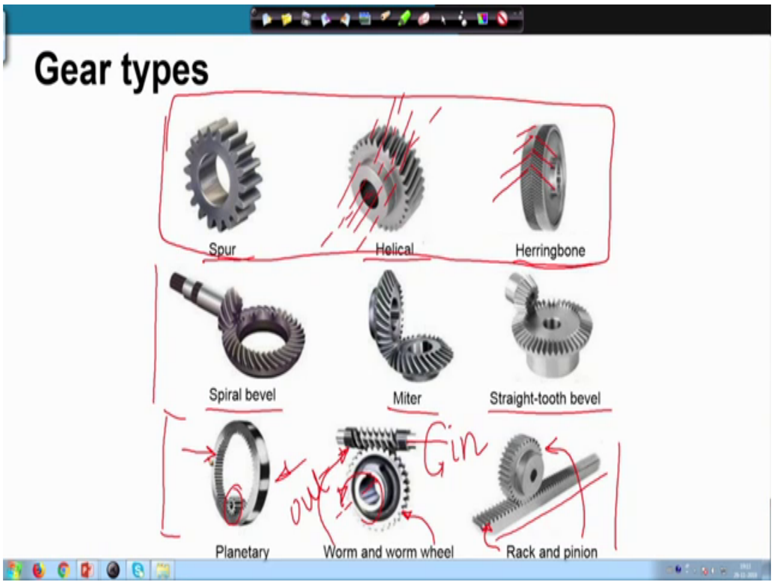

Here, I show you some pictures of typical gears. This is called the straight tooth Spur gear. This is the Helical tooth gear. If this is the gear axis, then the teeth are on helixes centred around the centre line of the shaft. These Helical gears. A double helical gear is a Herringbone gear, you can see that these have these are helical gears; two helical gears put together. These are Herringbone gears. Now, this set of gears are used to connect two parallel shafts. In the next set we have the Spiral bevel, the Miter and the Straight-tooth bevel gears. These are used to connect two non parallel intersecting and sometimes non intersecting shafts.

Here in these examples they are all intersecting shafts and they are at 90 degree, but it can have shafts intersecting at other angles as well. In the third row, I have some special kinds of gears. The Planetary gear which we are going to discuss in detail in a subsequent lecture. We have this Worm and worm wheel; this is the worm and this is the worm wheel. Now worm and worm wheel, this is non reversible which means that you can only rotate the worm and hence, produce a rotation on the worm wheel. So, this is the input and the rotation of the wheel is the output. You cannot rotate the worm wheel and have a rotation of the worm.

So, this is not back drivable. And finally, you have this Rack and pinion. The rack can be considered to be a gear whose radius of curvature is infinity and you have the pinion. So, this the rack and the mating gear is the pinion. In the planetary gear you have a pinion and this shows an internal gear, but you can have planetary gears with two external gears as well.

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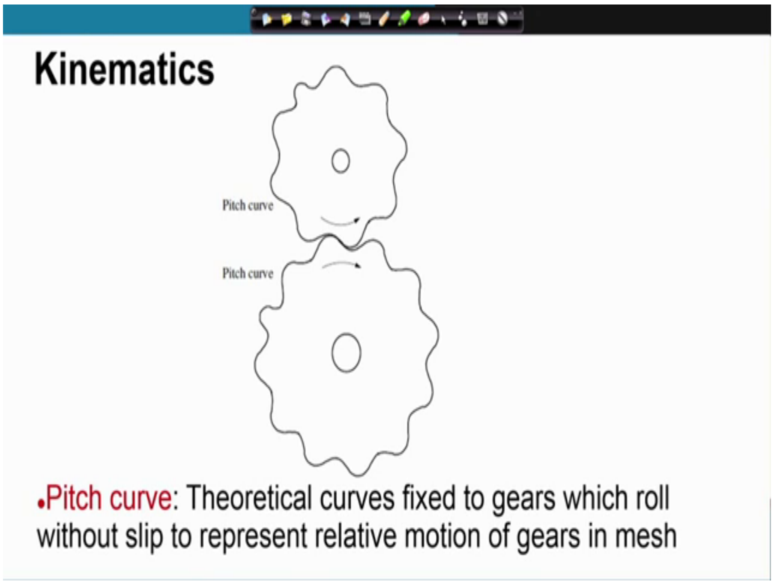

Let us know come to the Kinematics of gearing action. The our discussion on kinematics will hinge on this very important concept of pitch curve. So, what is the Pitch curve? Pitch curve is a theoretical curve which is fixed to a gear. Here, there are two machine gears. Therefore, there are two pitch curves. These curves are such that the relative motion of the gears are represented can be thought of as rolling of one pitch curve over the other. So, there is relative rolling without slipping of the two pitch curves. That represents the relative motion or relative rotation of the gears that are in mesh. So, it is very important notice that pitch curve is a curve that undergoes pure rolling motion. So, the two pitch curves are in pure rolling motion with respect to one another and it represents the rotational motion of the gears in mesh.

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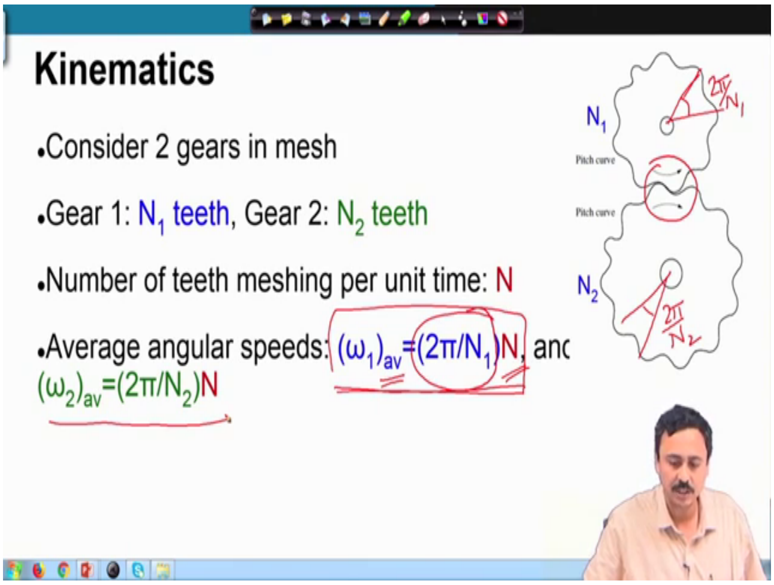

Let us consider a 2 gears in mesh and try to look at the transmission ratio. Let us try to understand transmission ratio, motion transmission ratio of 2 gears. Let us say that this Gear 1 has N 1 teeth and this Gear 2 has N 2 teeth which I have shown here. Let us consider that in unit time there are N teeth which come in contact in the contacting zone. So, in our figure here; so, this is our contacting zone. So, if I consider unit length of time, it could be second; it could be minute. Let me count the number of teeth that are passing in through that contacting zone. Now the because the teeth are in mesh for both gears. So, equal number of teeth will pass through this contacting zone.

So, let that be N. So, if you have this then the average angular speed of Gear 1 can be expressed in this form. You can recognise that this part of the expression is nothing but the average this is the angle. So, if there are N 1 teeth suppose if I consider this as the teeth, then this angle is 2 pi over N 1 and similarly, this angle is 2 pi over N 2. Now, since N teeth pass per unit time. Therefore, this expression gives us the average angular speed of Gear 1 and this expression similarly gives us the average angular speed of Gear 2.

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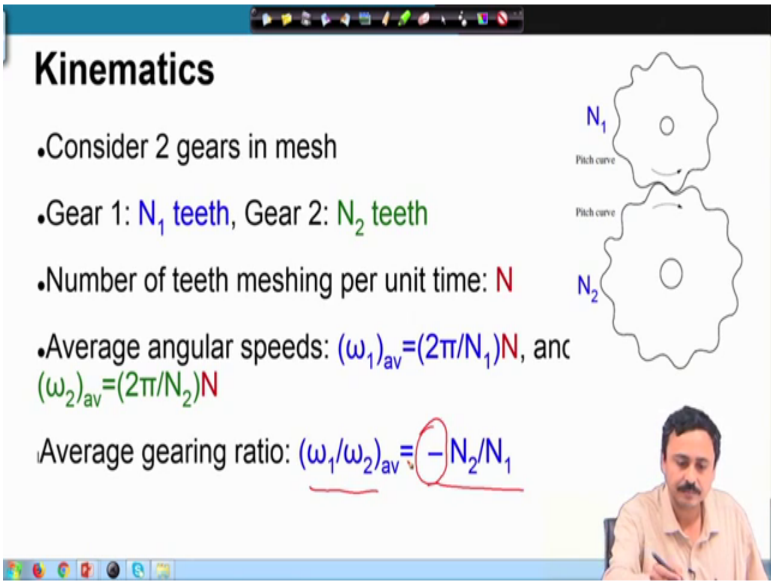

Therefore, the ratio of the average angular speeds is which is given by omega 1 by omega 2 average is minus of N 2 by N 1. Here, you will notice that I have put in a negative sign to stress upon the fact that the rotations are opposite. This sign is very important when we when we consider multiple gears as you will see.

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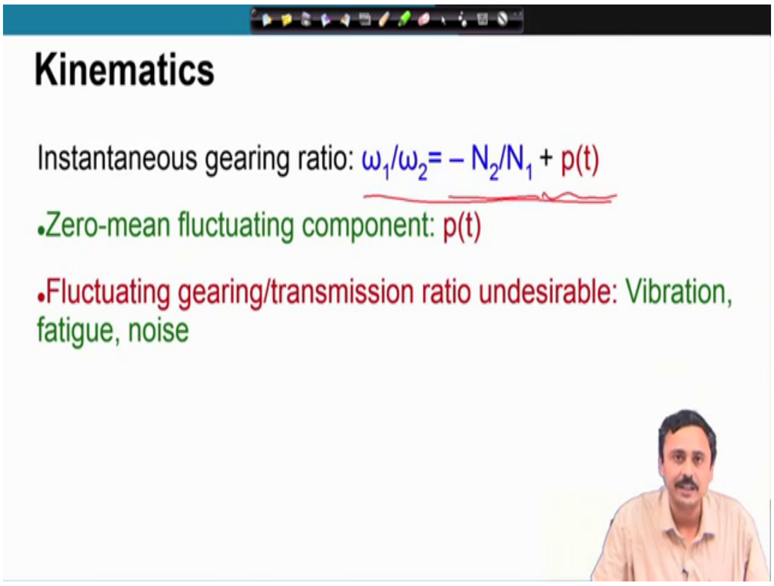

However, this expression that we have just now derived is for the average angular speed, but the instantaneous angular speed ratio can fluctuate. So, we can have in general a term like this. So, omega 1 by omega 2 instantaneous is of course, minus N 2 by N 1 this is the average value plus there is a fluctuation which is denoted by this function p of t. So, this is a time varying fluctuating function with Zero-mean. So, a fluctuating component p t. Now, of course as I have mentioned before that we would like to have constant transmission ratio. So, why do we need constant transmission ratio?

This is because if you have fluctuating transmission ratio, then you will have vibrations; you will have whenever you have vibrations which means force also will fluctuate. If there are fluctuating forces or stresses will be fluctuating and therefore, the gear teeth etcetera and the shafts everything will be under fatigue loading. So, because of fluctuating loads being transmitted, the system will be able will have fatigue. So, the thing might fail by fatigue either the tooth or the shaft and further more it will introduce noise in the system. All these things are undesirable for a gearing system.

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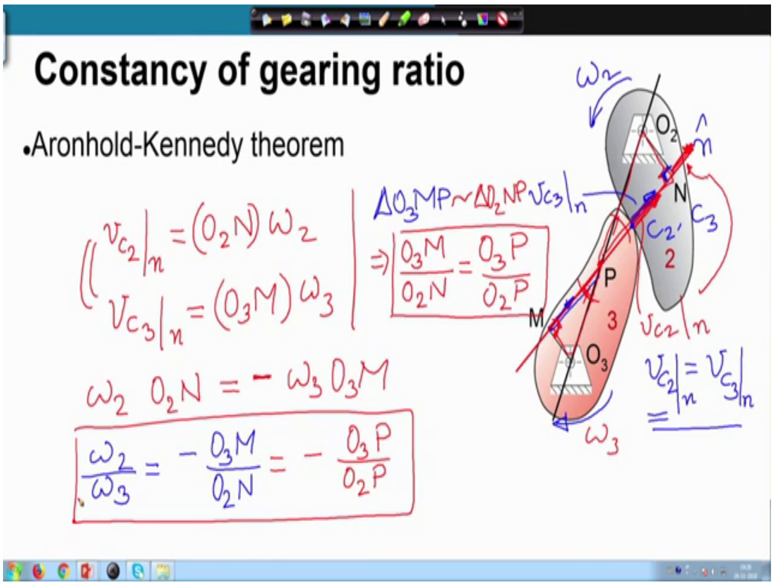

Therefore, we would like to have a constant transmission ratio. So, we have to understand how or under what conditions, we can get constant transmission ratio. So, for that let us go back to our discussions on Aronhold-Kennedy theorem which we have discussed before. Here, I have a figure of 2 laminas which are hinged at O 2 and O 3. So, lamina 2 is hinged at O 2 and lamina 3 is hinged at O 3 and they are in contact and we considered that this is some kind of a transmission between 2 and 3. Therefore, we expect that this contact will be maintained; this contact is to be maintained.

Now, what is the condition that this contact to be maintained? The condition is the relative normal velocity between the two bodies at C must vanish. So, they must have the same normal velocity. Let us look at how the Aronhold-Kennedy theorem helps us in getting at the relation or getting at the condition that must be satisfied in order that that the transmission ratio is constant. Now, let us recall what is Aronhold-Kennedy theorem. It is states that if 3 bodies are in relative motion, then the relative IC's the relative Instantaneous Centres of rotation of the 3 bodies, they are collinear; they lie on a single straight line.

In this figure, this is the IC between the ground. I have number ground as 1. So, O 2 represents the relative IC between 1 and 2. O 3 represents the relative IC between 1 and 3 and then, from the Aronhold-Kennedy theorem, we must have the relative IC between 23 I 23 on the line joining O 2 and O 3; this is what is a statement of Aronhold-Kennedy theorem. Let us locate the line of the centres which are drawn here in black. Now, how to locate the relative IC I 23? Let us look at the common normal at the point of contact.

So, this red line is the common normal at the point of contact. As I have mentioned that this common normal. So, this is the point of contact that is a C. Now, I can imagine C has a point belonging to 2 as well as C belonging to 3, body 3. So, these are coincidence points. These are coincidence points at this configuration. If the contact is to be maintained, then velocity of C 2 in the normal direction; if this the normal direction and velocity of C 2 in the normal direction must be equal to velocity of C 3 in the normal direction. This is the condition of maintenance of contact between bodies 2 and 3 ok. Now, how to find out these velocities? To do that, let us draw normals from O 2 on to the common normal.

So, this angle is 90 degree. Similarly, I will drop another normal from O 3 onto N. So, I will drop another normal from O 3 on to N. So, I will drop another normal onto the common normal N. If my angular speed of body 2, if I call it omega 2 is in this direction which means it is counter clockwise; then, you realise that in order to maintain contact, body C 3 must rotate in the clockwise direction. Now let us look at the velocities. This is what we want to find out velocity of C 2 along the normal and velocity of C 3 along the common normal. If you look at these constructions, then velocity of C 2 along the normal let me draw it in blue is same as the velocity of point M in this direction.

We have this we have discussed that because of because this body 3 is rigid. Therefore, of velocity of M along the common along the common normal which is along the common normal N is same as the velocity of C 3. So, this is the velocity of C 3 along the normal. So, let me write that. So, this is velocity of C 3 along the normal. Similarly velocity of C 2 along the common normal is this. So, this is velocity of C 2 along the common normal and this you will find is same as this velocity of N velocity of N along the normal. So, velocity of N is exactly the velocity of N because I have dropped N as a perpendicular from O 2.

Then, I can write velocity of C 2 in direction N is nothing O 2 N times omega 2 and velocity of C 3 along N must be O 3 M times omega 3 and these two must be equal. Therefore, I can write omega 2 O 2 N as omega 3 O 3 M. Now, I will introduce that sign as we have discussed because as you can see omega 2 and omega 3 are in a position. This is omega 3 which is clockwise and omega 2 is counter clockwise. So, to introduce that direction I have introduced the sign. Therefore, omega 2 by omega 3 is nothing but minus of O 3 M by O 2 N at this instant. So, whatever is the configuration the ratio omega 2 by omega 3 is minus of O 3 M by O 2 N.

Now, if you observe these two triangles; the triangle O 3 MP and triangle O 2 NP. So, if you observe these two triangles and this angle is same as this angle and the other angle is 90 degree. Therefore, these two triangles are similar. So, triangle O 3 MP is similar to triangle O 2 NP. Therefore, I can write O 3 M by O 2 N equal to equal to O 3 P by O 2 P. So, from similarity, I can write this which means that in our ratio of omega 2 by omega 3, we can write this as minus of O 3 P by O 2 P. Now, what this tells us is that if omega 2 by omega 3 is to be a constant, then this ratio O 3 by O 2 P must be a constant and that can happen only when P is a fixed point on the line of centres O. So, omega 2 by omega 3 is a constant, if P is a fixed point on the line of centres. This is because the distance O 2 O 3 is fixed. So, if P is a fixed point on the line of centres, then the ratio omega 2 by omega 3 is a constant.

This is very important conclusion that we have drawn and this thing this statement is known as the Fundamental law of gearing action. So, let me bring in that statement now.

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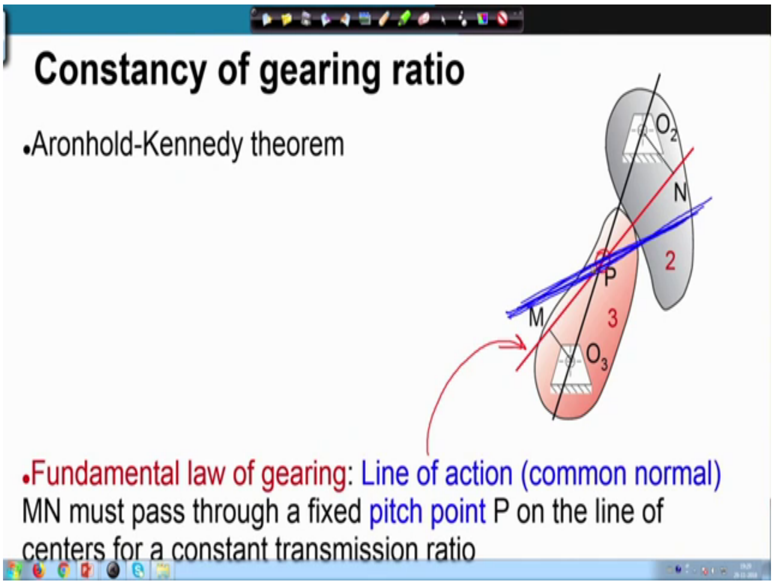

So, Fundamental law of gearing states that the line of action. So, this common normal MN is called the line of action. So, Fundamental law of gearing states that the line of action MN must pass through a fixed point P which is known as the pitch point on the line of centres for a constant transmission ratio. This is the Fundamental law of gearing action. So, P must be a fixed point for constant transmission ratio. Note that it says that P should be a fixed point; it faces no constraint on the orientation of this line.

As this these laminas move the line possibly can change, but it should pass through a single point; then the transmission ratio is a constant. So, that is guaranteed, but this rotating line of action as it is known as if it rotates, then there are other problems that we are going to discuss. Now, why this is called the Line of action? You see this line of action is the common normal at the point of contact. So, this is the point of contact.

So, Line of action is the common normal through the point of contact. If I neglect friction, then the force then the force that is being transmitted is along this line. The force is transmitted along the common normal; if friction is negligible then this is the major force contribution. So, that is why it is called the line of action and P the point through which the line of action intersects passes through on the line of centres, this point P is known as the pitch point.

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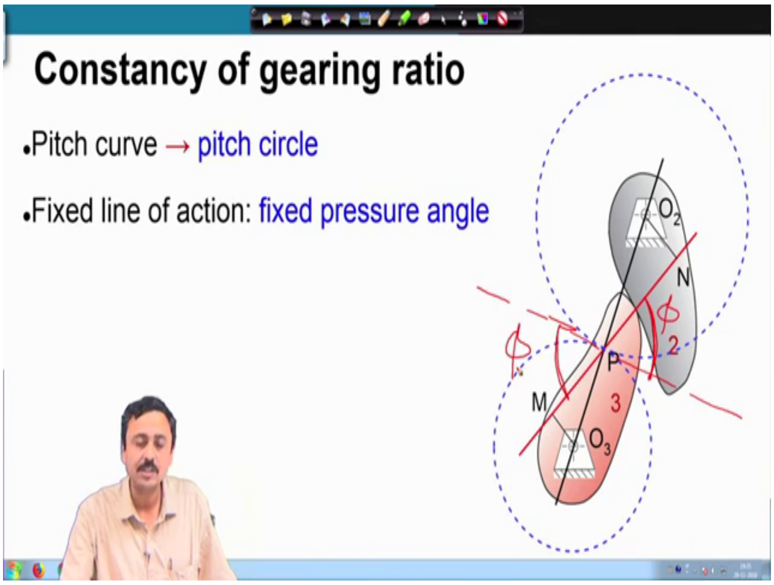

Now, let us get back to the concept of Pitch curve and see what is the pitch curve. As I have mentioned that the pitch curve is the curve which is fixed to a gear and the motion of the gear is kind of relative rolling of one pitch curve on the other. Now if you then consider ask this question what is than the pitch curve. So, pitch curve is the locus of this point P; locus of this point P on either body 2 or on body 3. If you consider the locus of P as seen from an observe as seen by an observer on body 2, then you get the pitch curve of body 2 and if you look at the locus of P sitting on body 3, then you get the pitch curve corresponding to body 3.

And because the pitch point P, at the at the pitch point P, there is a instantaneous centre of rotation of body 2 and 3 which means it is as if body 2 is rolling about this point with respect to the body 3 and body 3 is also rolling with respect to body 2 with respect to with respect to body 2 the at this point. So, this is the point of rolling of the two. Therefore, these are the curves which are rolling at with a contact at P. So, these are the pitch curve. So, locus of P as seeing by an observer fix to body 2 is the pitch curve of Gear 2 or body 2 and similarly pitch curve of a body 3 is the locus of P as seeing by an observer fix to body 3. Now what happens if P is a fixed point on the line of action? There the locus is a circle why because the distance of P from O 2 remains fixed.

So, the locus of P is a circle with centre at O 2. Similarly the locus of P as seen on body 3 is a circle. So, this is the pitch curve which is now called the Pitch circle for body 2. This is the pitch circle of body 3 and it is the relative rolling of these two pitch circles. Rolling I the this rolling is of course, I mean this pure rolling. So, which means that there is no slip, so this is the pure rolling between the pitch circles that represents the motion of the two mashing gears. Now let us look at the Line of action. As I was mentioning this line of action is the line along with the force is getting transmitted. We define something called a pressure angle is the angle made by this line of action with the common tangent at the through the pitch point.

So, this angle usually denoted by phi is known as the pressure angle. Now, why this pressure angle? Because the force is acting along this line and the angle made by this line, the line of action with the common tangent this is the pressure angle. Therefore, denotes the inclination of the direction of force with the common tangent.

Now, if this line rotates it may passing through a single point, but if it rotates which means that the force direction is changing. If force direction changes once again we have all those problems stresses will be fluctuating. Again, we will have vibrations, we will have fatigue failure. So, it is ideally we would like that not only it passes the line of action passes through a single point the pitch point, but it also maintains a constant angle constant pressure angle.

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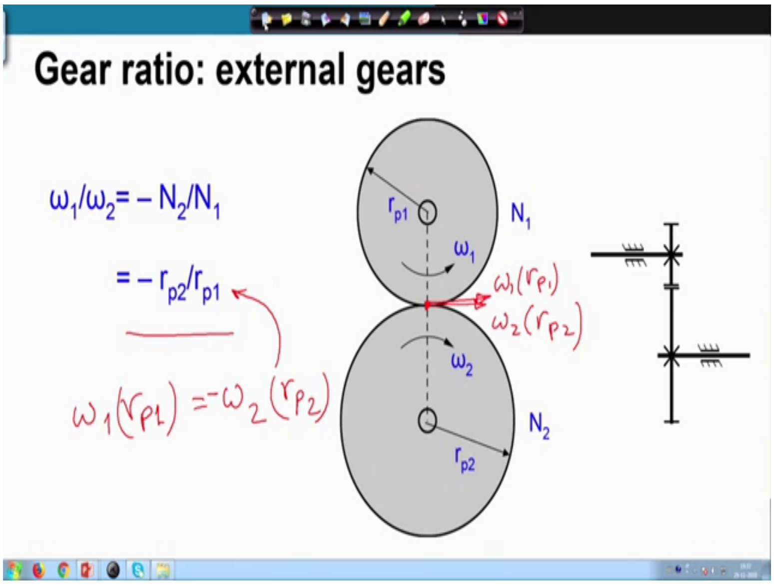

Now, let us look at some examples with the these are the pitch circles. This is notation that we will be consistently using. If you look at these mashing gears, this is this represents a gear; if this is the shaft and if I have a cross here, it means that the gear is fixed to the shaft. On the other hand, if I have a gear in a shaft and I indicate like this which means that it is freely rolling. It is not fixed to the shaft; it can rotate with respect to the shaft. With this nomenclature, let us look at the transmission ratio now because the pitch curves are circles. So, omega 1 omega 2 is minus N 2 by N 1 which we have derived already. There is no fluctuating component. The transmission ratio is perfectly constant is minus N 2 by N 1.

And from velocity matching at the point at the pitch point, this is the point of contact of the 2 pitch circles the pitch point it is, you can also write it as minus of r p 2 by r p 1. Essentially this is nothing but the velocity matching omega 1 times r p 1 is nothing but the velocity. This velocity and that must be equal to omega 2 times r p 2 will bring in that negative sign to indicate the direction. So, these two must be equal. So, from here I have this relation.

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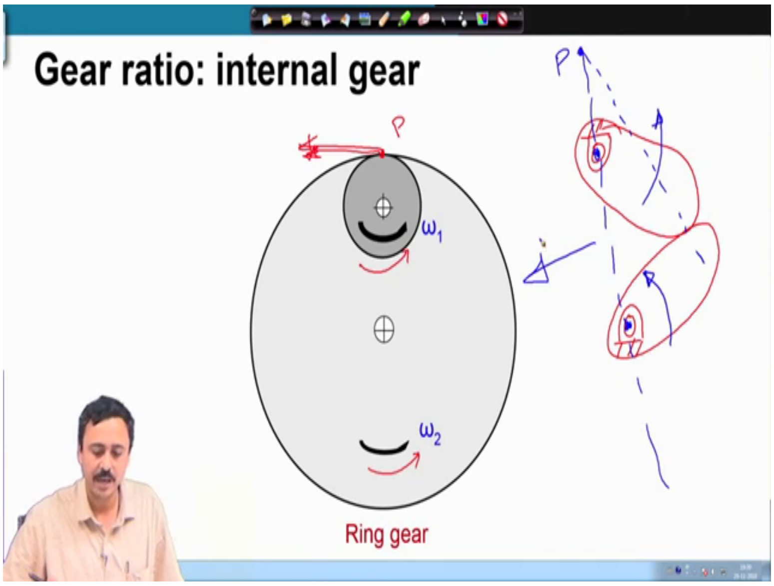

Now, we can also have machine gear machine with an internal gear. In that case this gear is known as a ring gear. Here you see that the directions of rotation omega 1 and omega 2, they have to be the same. Because the velocity for both the external gear and the ring gear, this must match at this point of contact. Now this is the pitch point. Now how do we understand this in terms of the example that we have considered before, if you consider to lamina? This is the line of centres and if you look at the common normal, it intersects outside the two instantaneous centres; these two instantaneous centre these centres of these two these two gears.

Now, once this goes outside. So, this is the pitch point. Then you will notice that if this is rotating in a counter clockwise sense, this also has to rotate in a counter clockwise sense to maintain contact; where is the pitch point is not within the two centres of the gears, then we have this situation where we have a ring gear with an external gear.

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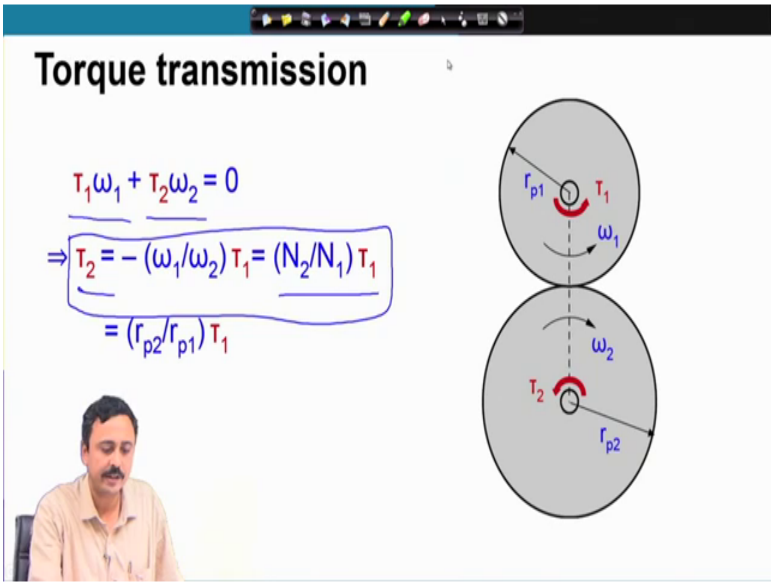

So, this is our notations for indicating a ring gear fixed to a shaft. Now we have omega 1 by omega 2 as N 2 by N 1. So, we have positive sign here because omega 1 and omega 2 are in the same direction and that can also be related to the pitch circle radii r p 2 by r p 1.

Let us look at the torque transmission from the principle of virtual work and from power balance. Here, we are use the power balance. You can write the rate of work done at gear 1 plus rate of work done at gear 2 considering this as the system that must vanish which means that tau 2. This is tau 2 is the torque that is acting on gear 2. So, that can be related to torque on gear 1 from this relation about and since omega 1 omega 2 is minus of N 2 by N 1. So, finally, we have this as the torque transmission and that can also be related in terms of the pitch circle radii i.

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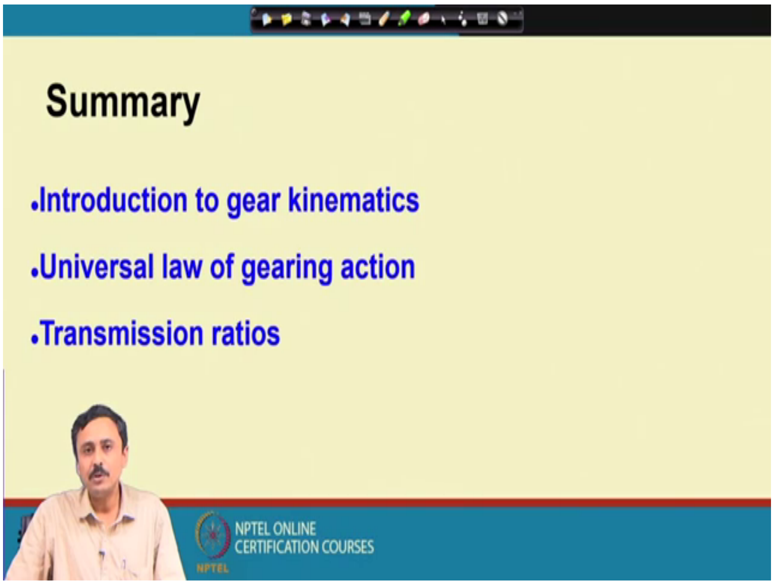

So, let me summarise what we have discussed. I have introduced gear kinematics in terms of the Pitch curve which we have found how to determine pitch curve and if we want constant transmission ratio, then we need the universal law of gearing action; fundamental of gearing action and we have discussed the transmission ratios both for motion transmission and torque transmission. So, with that I will close this lecture.