Kinematics of Mechanisms and Machines
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Lecture – 36
Force Analysis Examples

In this lecture, I am going to work out some problems on Force Analysis. We have already discussed force analysis using the principle of virtual work.

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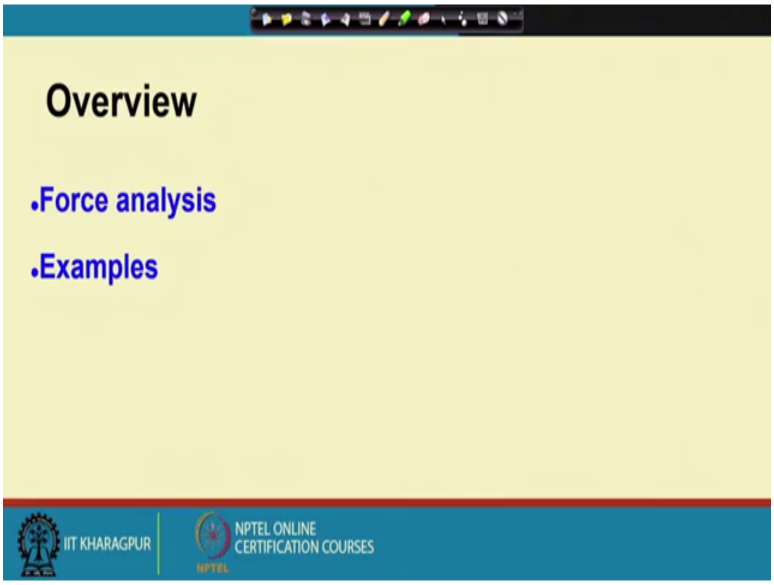

Here, I am going to take some examples and illustrate the application of principle of virtual work force analysis.

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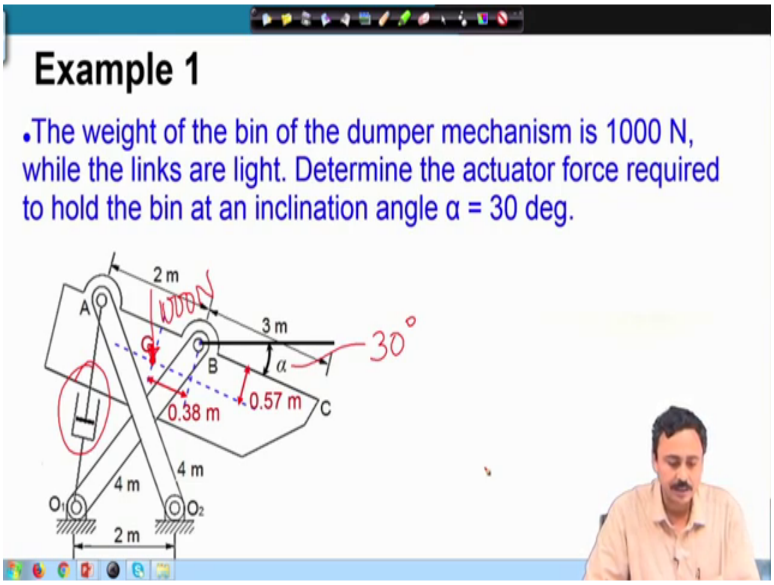

Here is our first example it states, the weight of the bin of the dumper mechanism is 1000 Newton, while the links are light. Determine the actuator force required to hold the bin at an inclination angle alpha equal to 30 degree.

Let us first see what this says. This bin has it is centre of mass at G and the weight of this bin is 1000 Newton. The question is, what should be the force applied by the actuator to keep the bin at an angle alpha of 30 degree. So, it requires to hold the bin at this angle, when the weight of the bin is 1000 Newton. The first thing that we must accomplish is the displacement analysis. We must perform the displacement analysis to find out all angles; so, that we can relate the kinematic input output relation.

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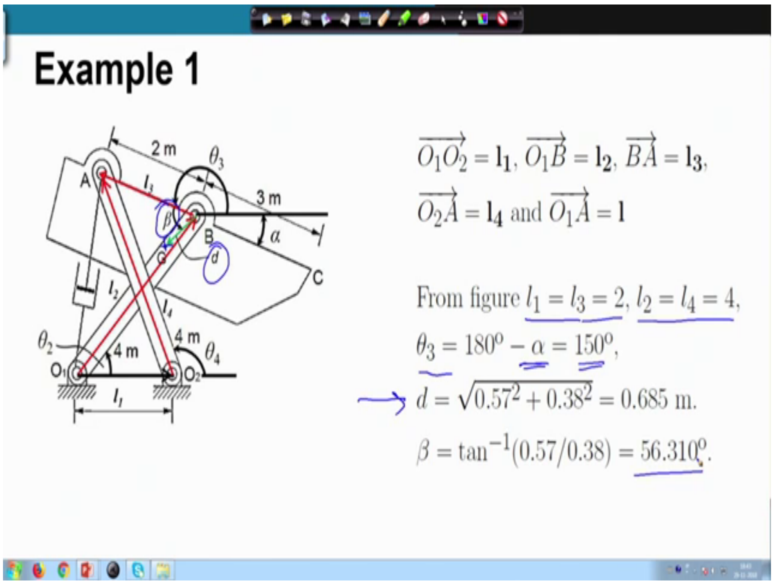

We have discussed this problem under displacement analysis previously, but now, the input to the this problem they are different, what we are given is this angle alpha here which means we are given theta 3 and from this data we are going to first solve for theta 4, theta 2, theta and l. As before let us define this vectors for example, this l1 vector is O1 O2, l2 vector is O1B. So, this is l2 BA is l3, so, this is the coupler this is the bin. O2A is l4, so is this link and O1A is this vector which we indicate by l.

Now, if you write the various data that are given, l1 l3 these link lines are given as 2 meter, 12 and l4 given are 4 meter. Theta 3 can be calculated because alpha is specified this turns out to be 150 degree. And what we have done is, we have determined the location of G in polar coordinates. So, we find out d which is 0.685 meter and we find out this angle beta which turns out to be 56.31 degree.

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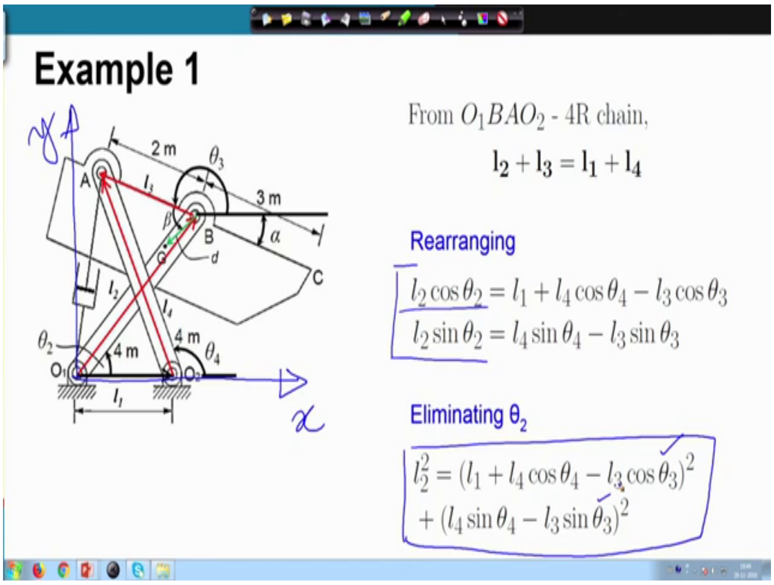

Now, with this we write out the loop closer equation for the 4 r chain which we have done before. And we rearrange this to express l2 here I have written out the scalar equations, l2 cosine theta 2 and l2 sine theta 2, our coordinate system is like this as we have used before. Therefore, in the component form, we have l2 cosine theta 2 and l2 sine theta 2 expressed like this. And if I eliminate theta 2, if I eliminate theta 2 then I obtain this relation.

You can see here, in this relation this involves two joint angles, theta 3 and theta 4 of which theta 3 is known therefore, we can solve this to find out theta 4. So, this is what we are going to do next.

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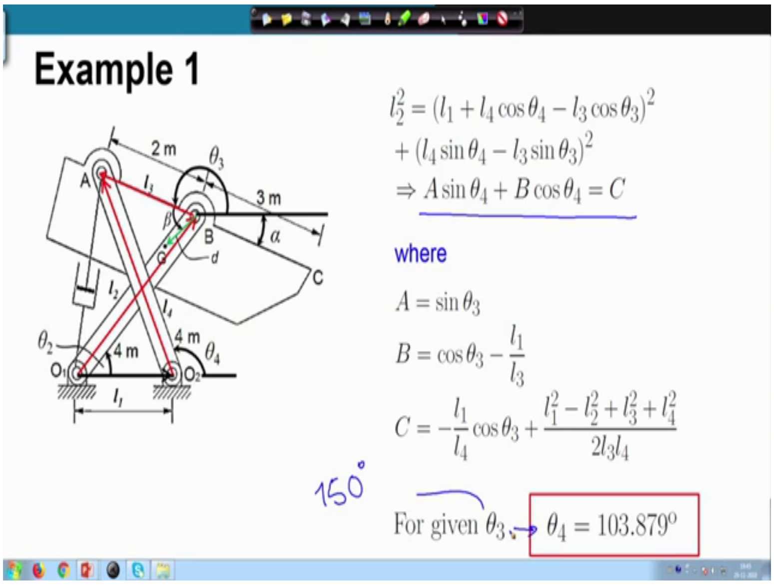

We can express theta 4 in terms of this relation and if you look at the expressions of A, B and C, they involve theta 3 which is known to us. We have solved this equation before and for given value of theta 3 which is 150 degree, theta 3 was 150 degree, we obtain theta 4 as 103.879 degree.

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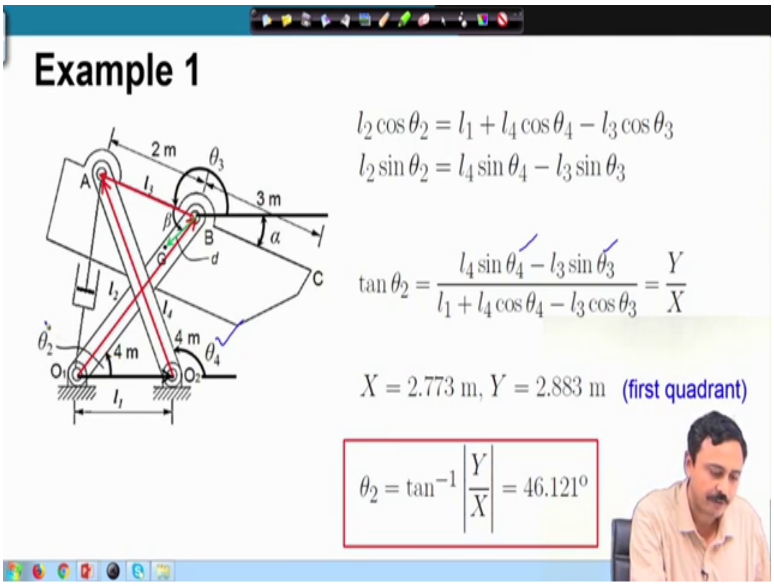

Now, with this accomplished we look at tangent of theta 2 by just dividing these two component equations. Since, I already know theta 3 and theta 4; so, I know the right hand side completely, and hence I can find out theta 2. X and Y, this capital X and capital

Y they turn out to be these values, for the values of theta 3 and theta 4 we have. And since, both are positive the solution is in the first quadrant, this is our relevant solution and this turns out to be 46.121 degree. So, up till now, we have found theta 4 and we have found theta 2.

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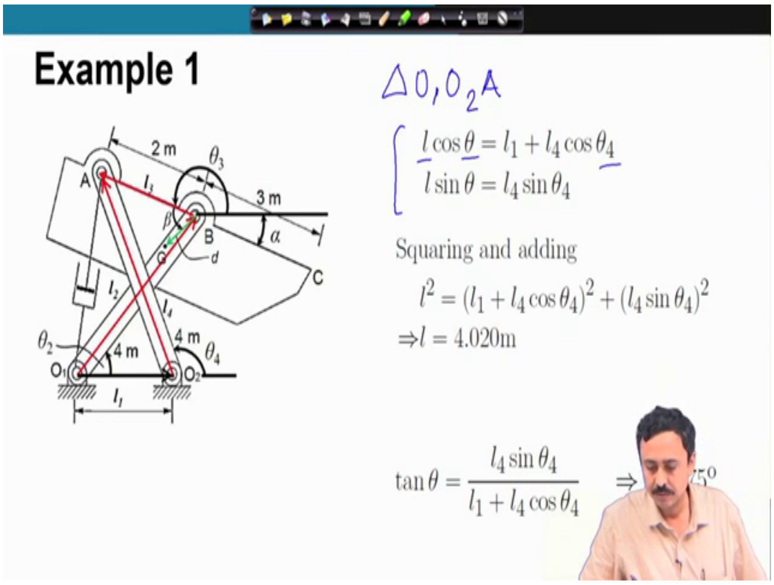

Next, we determine theta and l. If you look at the triangle O1, O2, A; if you write the vector loop equation for this triangle, then from that you can find out these component equations. Here, the relations involved l, theta and theta 4; of which theta 4 is known we have to find out l and theta.

If you square and add that eliminates theta and you directly solve for l. And if you divide then you get tangent of theta and for the known value of theta 4, we get theta as 75 degree. Therefore, we now finally, have theta as 75 degree that may indicate theta; so, this is theta, theta 2, theta 3, theta 4 and l; l is this length that is l. So, basically it is the throw of the prismatic actuator.

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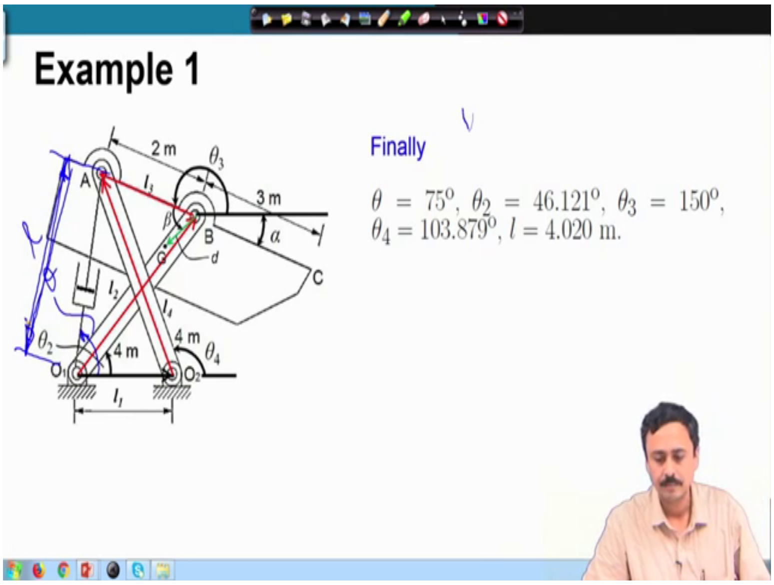

Now, we are ready to write out the virtual work expression or our relation. Here, F is the force that is required at the actuator. So, F is the force that is required at the actuator. So, F times delta l is the virtual work done at the input, which is the prismatic actuator plus the work done because of the weight, because of this infinitesimal shift in the centre of mass G which is indicated by delta r G. So, that is the virtual displacement of the centre of mass and W this matrix, this W is the force, the force vector.

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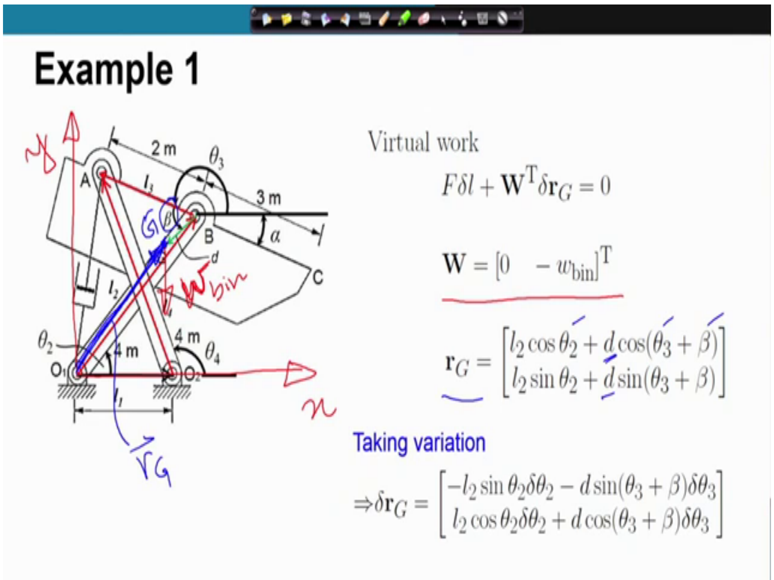

So, let me show you the expressions in the coordinate system, that we have chosen this is x and this is y, W the expression of W as a vector is like this. So, along the X there is no component because weight X vertically downwards. So, this is W bin. And it is in the negative Y direction therefore, the weight vector is 0 and minus W bin.

Next, we can express r G which is the position vector of the centre of mass in this coordinate system; so, it is this vector. So, if this is the point G, the centre of mass of the bin, this vector is r G.

As you can see this involves theta 2, theta 3 and beta which we have found out before angle beta. And it also involves d, this length d which we have also found out from geometry. So, we know this vector r G, then we can take the variation of this vector which turns out in this form as you know beta and d are constants, it is only theta 2 and theta three. So, if you give a virtual displacement to the mechanism, this is how the centre of mass will shift. So, this is the virtual displacement of the centre of mass. Now, related in terms of the virtual displacements of delta theta 2 and delta theta three.

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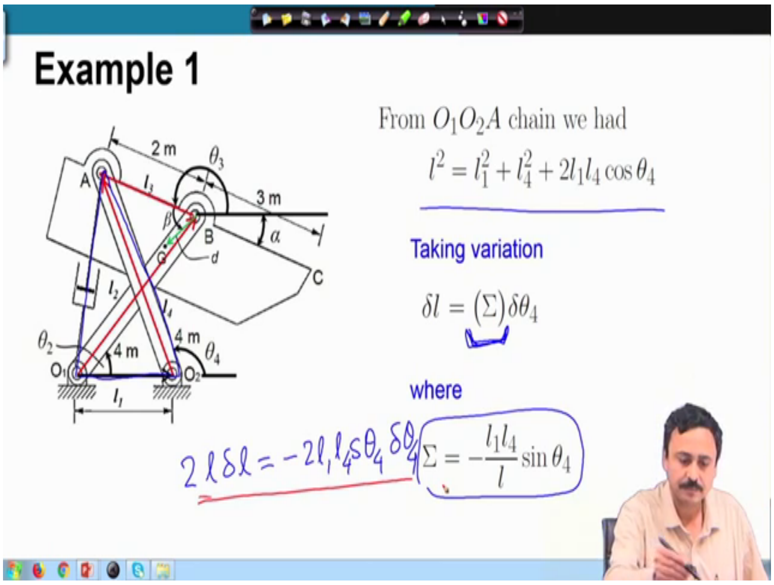

Now, let us look again, at this chain O1 O2 A which is this chain, from here we had earlier written l square in terms of theta 4; now, if you take variation of this, then you can easily see that the variation of this will give 2 l delta l will be equal to minus 2 l 1 l 4 sine theta 4 delta theta 4. And this sigma, capital sigma turns out to be this expression which you can read out here from this variational equation you can read out sigma.

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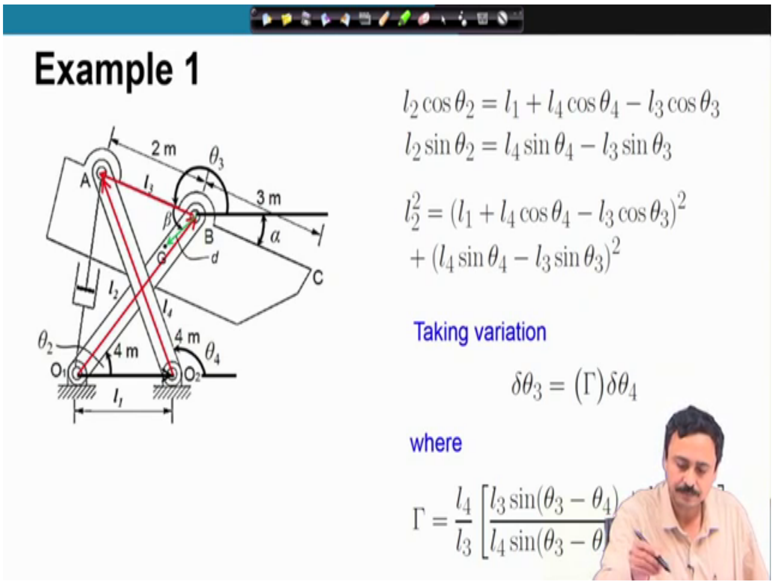

Then, we also use this kinematic relation, if you once again square and add then this relates theta 4 and theta 3 which we have used before; if you again take variation of this, this will relate delta theta 3 with delta theta 4 through this function gamma, this capital gamma is in terms of theta 3 theta 4 which are known from our displacement analysis therefore, we completely know gamma.

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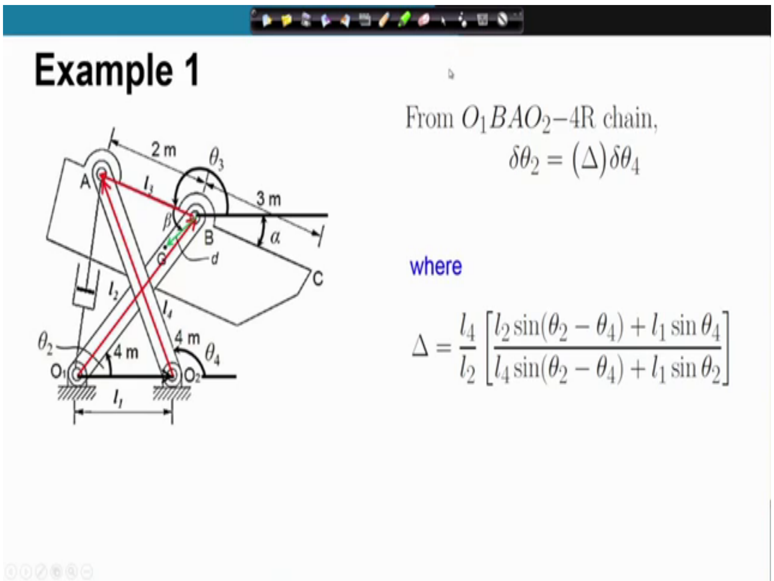

Next, we will look at this 4R chain and we can find out the relation between delta theta 2 and delta theta 4 and they turns out to be in terms of this capital delta and the expression of delta is given here. So, we can find out using the displacement relations we can find out these relations between delta theta 2 and delta theta 4, delta theta 3, delta theta 4 and delta theta and delta theta 4.

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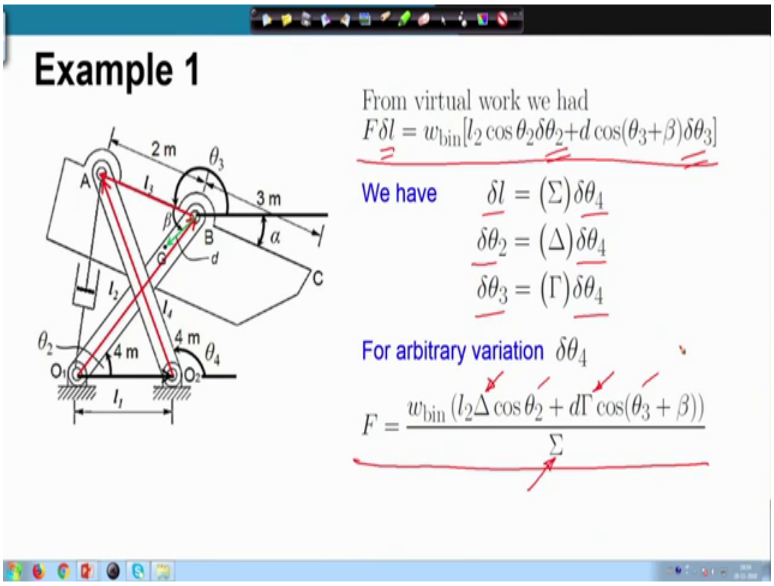

So, we have found out all these relations, we have related delta l in terms of delta theta 4, delta theta 2 in terms of delta theta 4, delta theta 3 in terms of delta theta 4. If you look at the statement of the principle of virtual work, it involves delta l, delta theta 2 and delta theta 3. If you substitute these expressions in the statement of virtual work then, you can eliminate delta theta 4 for because it can be arbitrary.

For arbitrary delta theta 4, if the statement is to be true; then the coefficient of delta theta 4 must vanish. And from there you can solve for the force to be exerted at the actuator at the prismatic actuator this involves delta, it involves gamma, it involves capital sigma and the joint angles theta 2, theta 3 and other parameters of the mechanism. If you substitute all these values which we have, then you can find out the force F.

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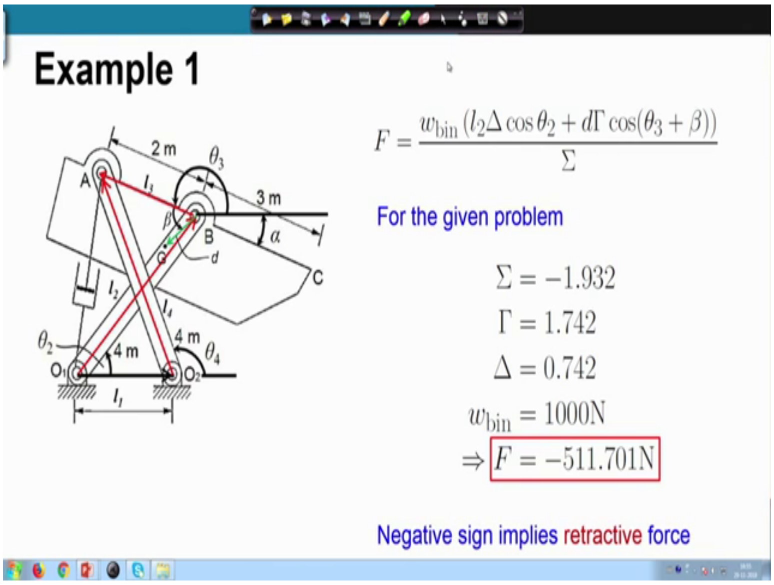

So, let us do that from the displacement analysis we have all the angles and hence we can solve for sigma, gamma, delta as shown here and weight of the bin is 1000 Newton; if you substitute this, these values in the expression of force you will obtain force as minus 511.7 Newton. Here you will get a negative sign which indicates that the force is a retractive; that means, it is pulling it is not pushing, it is not expanding, but it is trying to contract. So, the force is retractive force. So, that completes the solution for this example.

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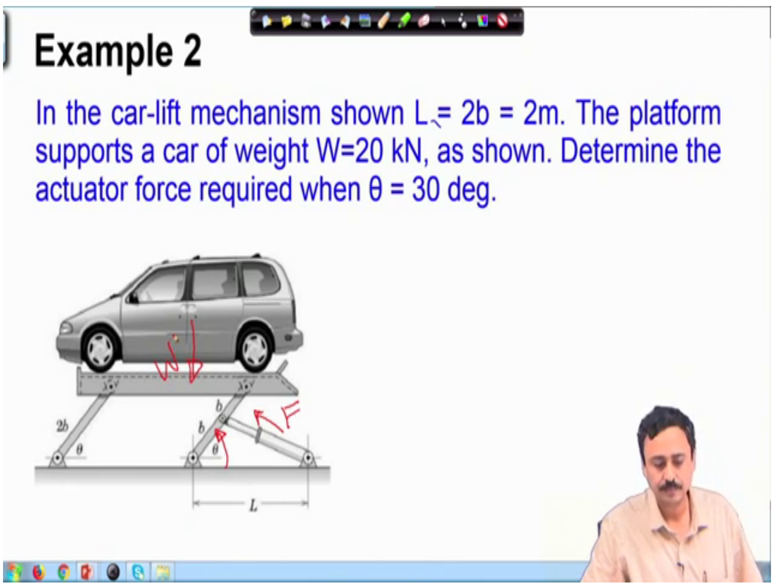

Then let us move to our second example, in this we have the car lift mechanism which we have discussed before; this in this mechanism the dimensions l and B are related and l is equal to 2 B, equal to 2 meter; the platform supports a car of weight 20 kilo Newton, determine the actuator force required when theta is 30 degree. This is theta; so, when theta reaches a value of 30 degree, we have to find out the force in the actuator to support the weight of the car.

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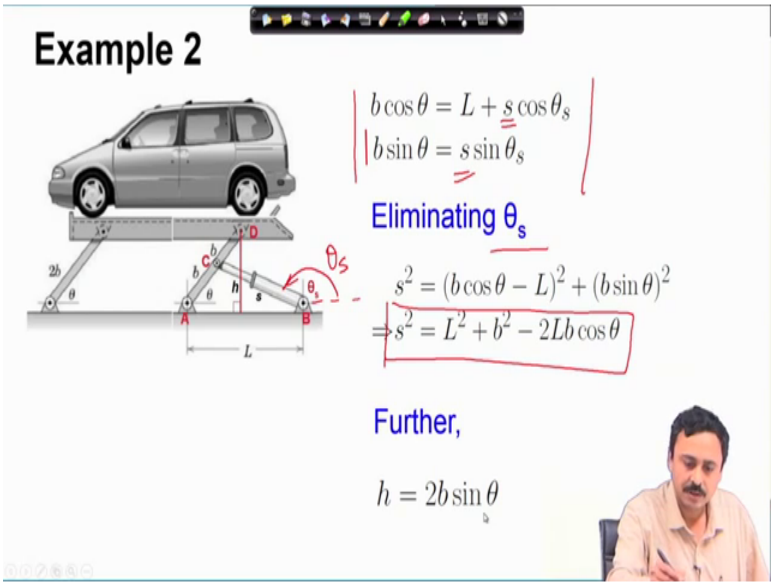

We had these relations from our displacement analysis, you can look at the vector loops and write out these relations which we had done before, during displacement analysis. This theta s is this angle, this angle is theta s therefore, these relations relate theta s and theta. And of course, you also have s, s is the throw of the prismatic actuator. You also have from here, these from these component equations you can eliminate theta s and obtain this relation between s and theta.

Also, previously we had obtained the expression of the height of the platform on which the car is supported, in terms of b and theta.

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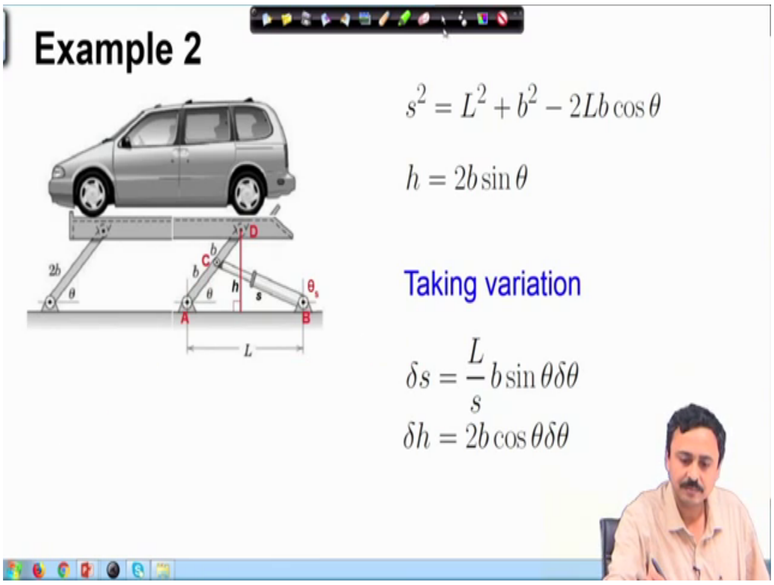

So, finally, what we have is the expressions of relating s and theta and h and theta. If you take variations of this, then you can easily obtain these variational relations between delta s and delta theta and delta h and delta theta.

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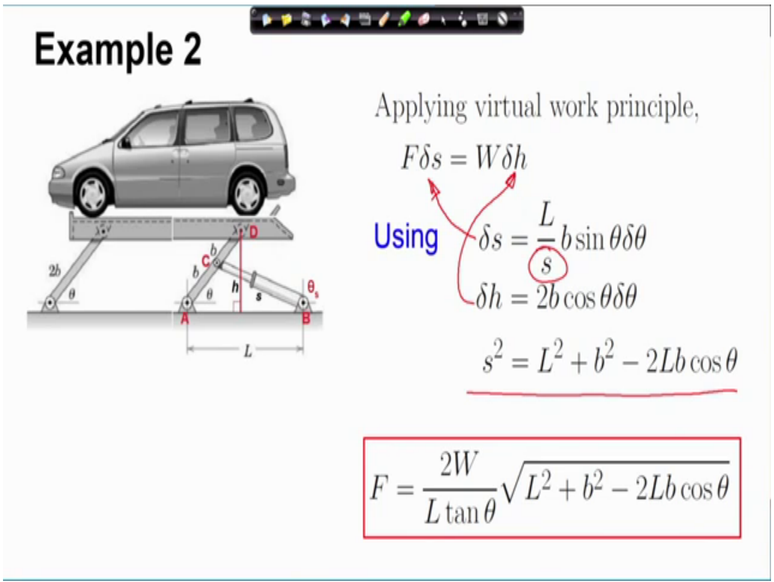

Once we have these, we can go to the principle of virtual work; now, F delta s is the virtual work done at the actuator and W delta h is the work done because of the gravity. If you substitute those expressions of delta s and delta h, which are like this in terms of delta theta; if you substitute these expressions here. Now, you will notice that this expression of delta s also involves s and we already know s in terms of theta.

So, using all this finally, you can find out the expression of the force required at the actuator using the consideration that delta theta will be arbitrary; the variation in theta will be arbitrary therefore, the force must be given by this expression, this is in terms of theta. And we are already given that, theta s 30 degree, weight is 20 kilo Newton and the lengths l and b are specified and if you substitute you will get force as 42.93 kilo Newton; now, here it is positive so which means that, this is in the expansion mode.

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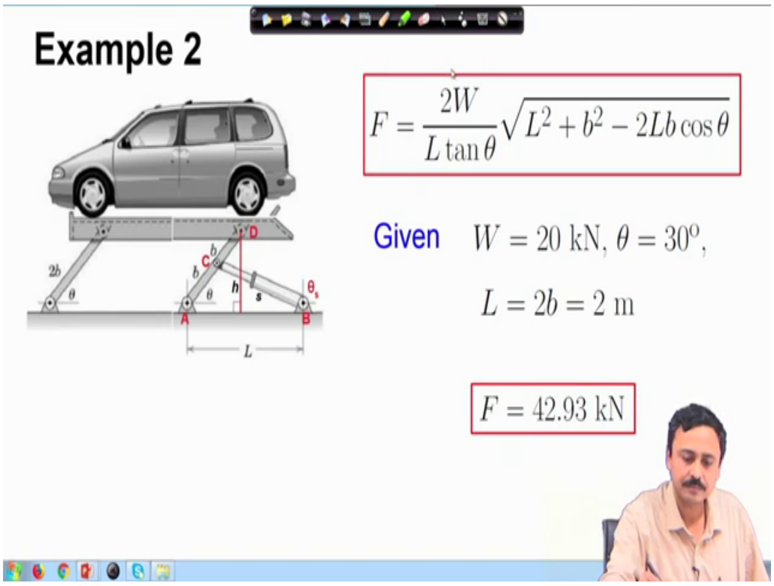

So, the force on the produced by the by the actuator of prismatic actuator is expensive. So, that completes example 2.

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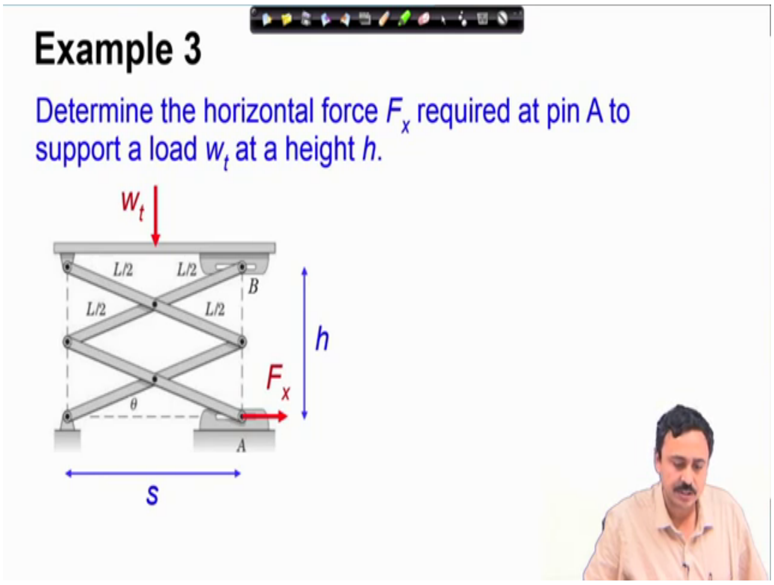

Let us now look at the (Refer Time: 21:29) mechanism, which we have discussed previously also. Determine the horizontal force effects required at the pin A to support a load W t at a height h.

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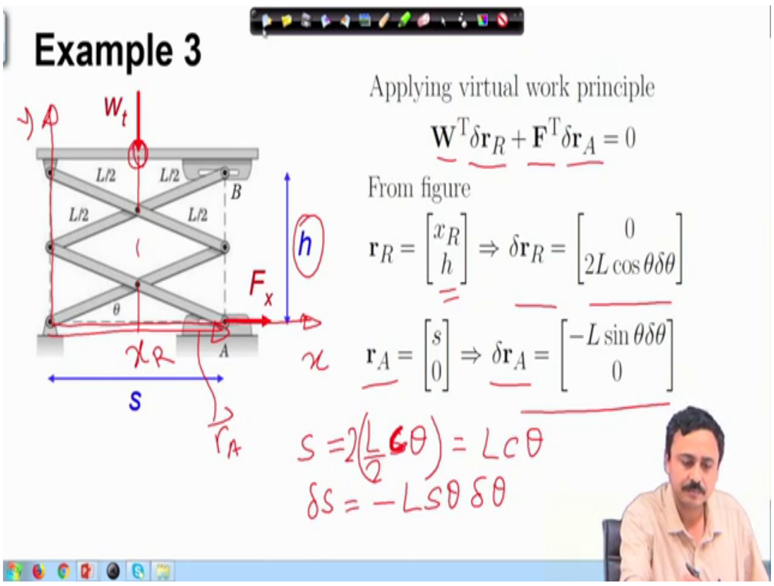

Now, directly we can write out the principle of virtual work, in terms of the load that we have the load vector on the platform; the virtual displacement of the platform the force vector applied at pin A and the virtual displacement vector at pin A. Using the coordinate system that we have used before, we can express the location of the application of point of force, as x R, this is x R and h, the Y coordinate is h. Therefore, the variation is given here we have the expression of h and x R is a fixed quantity. So, therefore, the variation is given in terms of delta theta. So, this is the variation of the point of application of the load.

Also, we have the location of pin A; the vector location of pin A. So, this is given by r A, this one is r A which is S 0 and if you use the expression of S and take variation. So, expression of S was L by 2 sine theta and 2 times of that; so, that was L cosine theta; 2 times l by 2 cosine theta, so that gives us L cosine theta. So, delta S is minus L sine theta delta theta. So, that is what we have as the variation delta r A.

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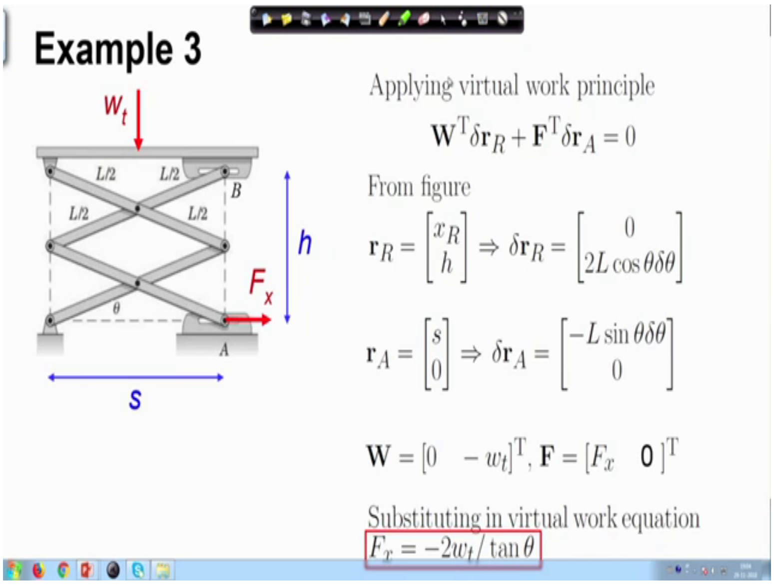

Now, if you substitute all of this in the expression of virtual work with the weight vector as 0 minus w t, this is in the coordinate system we are using. And the force that is being applied at pin A, this is applied force therefore, we are considering only F x and 0. So, if you use all this in the principle of virtual work, we get F x as minus 2 times w t by tan theta. Now, we have to express tan theta in terms of h. So, that we are going to do next.

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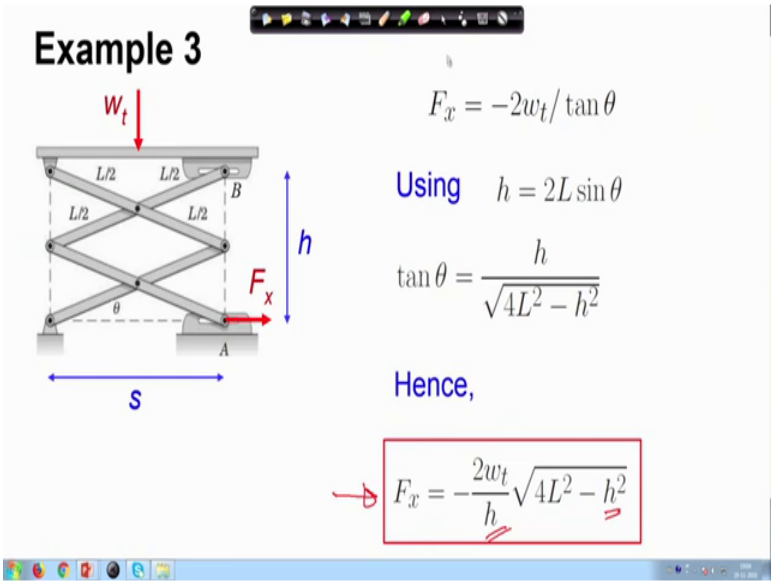

So, F x turns out as an expression in terms of tan theta. We already know that h is 2 L sine theta therefore, I can express tangent of theta. And once I have tangent of theta, I substitute in the expression of F x, this expression now is in terms of h, the height of the platform as we required; so, that concludes example 3.

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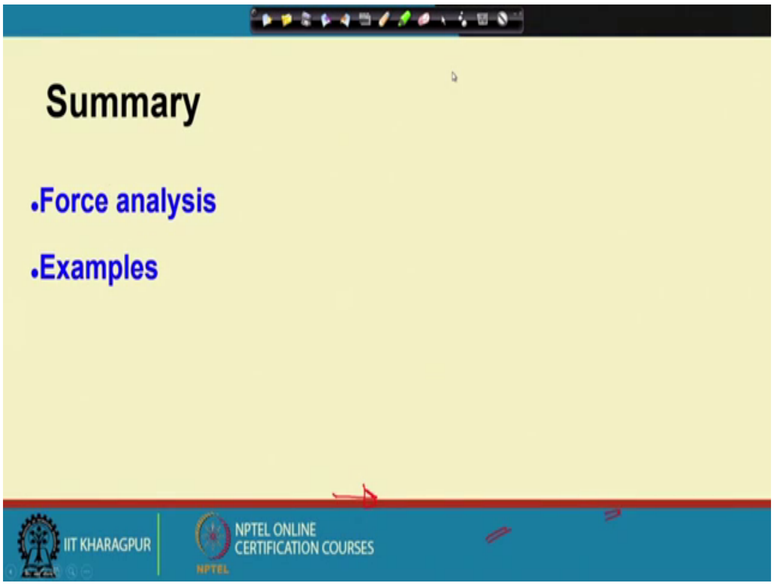

So, we have considered some examples of force analysis; we considered 3 examples and what we find is first step is to solve the displacement problem, then write down the expression of the virtual work and then finally, use the variations and use in the expression of the virtual work and solve for the required quantities. So, with that I will close this lecture.