Kinematics of Mechanisms and Machines
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Lecture – 35
Force Analysis – II

We have been discussing Force Analysis of Mechanisms. So, as I mentioned in the previous lecture that this Force Analysis is not about dynamic forces, but equilibrating forces. And, we have use the principle of virtual work to analyze force transmission by mechanisms.

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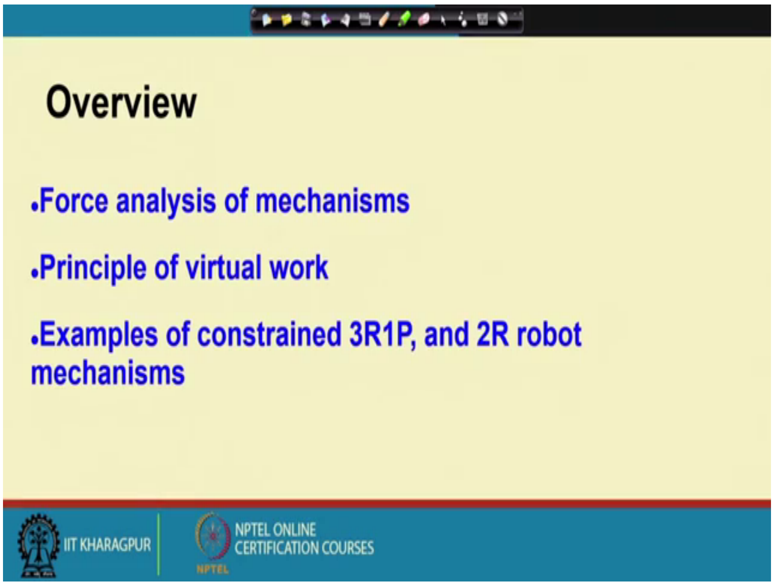

So, in today's lecture the overview of this lecture is as follows, we will look into the problem of force analysis of mechanisms using the principle of virtual work. And, consider examples of the constrained 3 R 1 P chains, and 2 R planar robot mechanisms.

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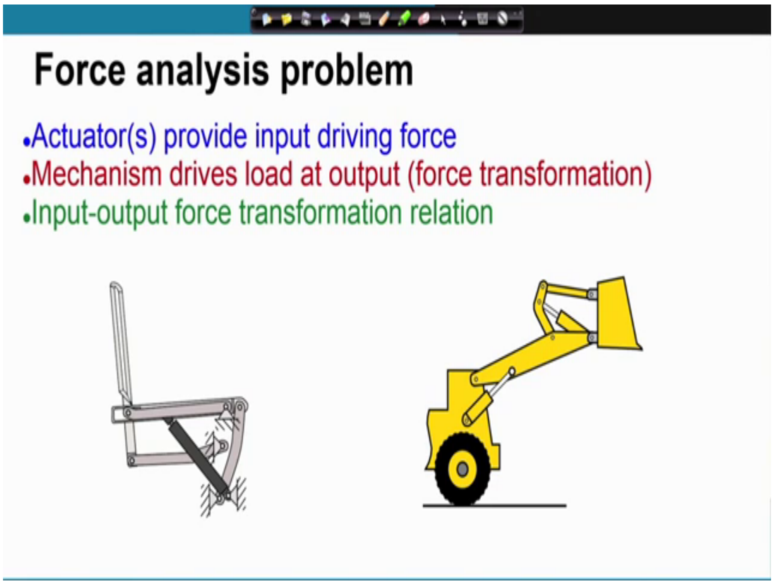

So, let us recapitulate what we have discussed, the force analysis problems, the problem is about finding the input output force relation for a mechanism. In a mechanism or in a robot the actuators provide the input driving force and the mechanism drives certain load at the output. So, the mechanism is responsible for the force transmission and transformation. So, the objective of force analysis is to find this input output force relation.

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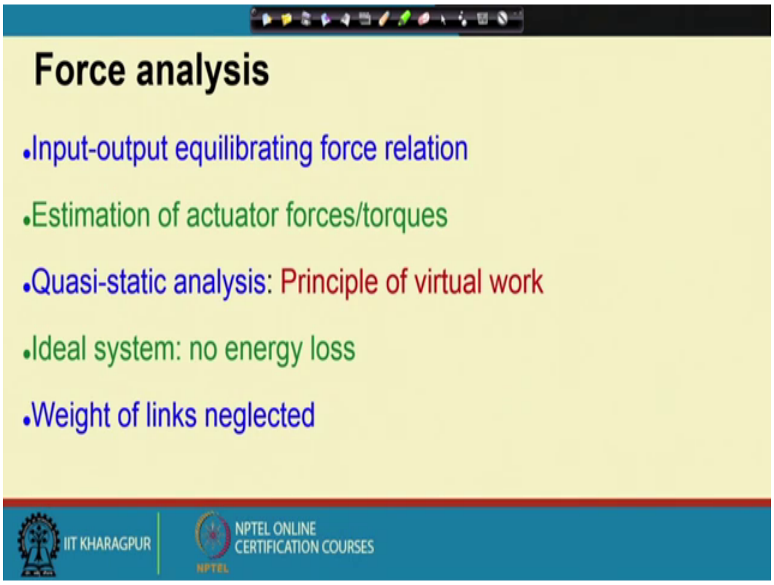

We have discuss that, the input output equilibrating forces is what we are interested in this helps in us in estimating the actuator forces or torques required. We will use the quasi static analysis using the principle of virtual work; that means, there is no dynamic forces in our analysis. We will consider ideal systems, which means that there is no energy loss, there is no dissipation. The weight of the links are neglected in this analysis.

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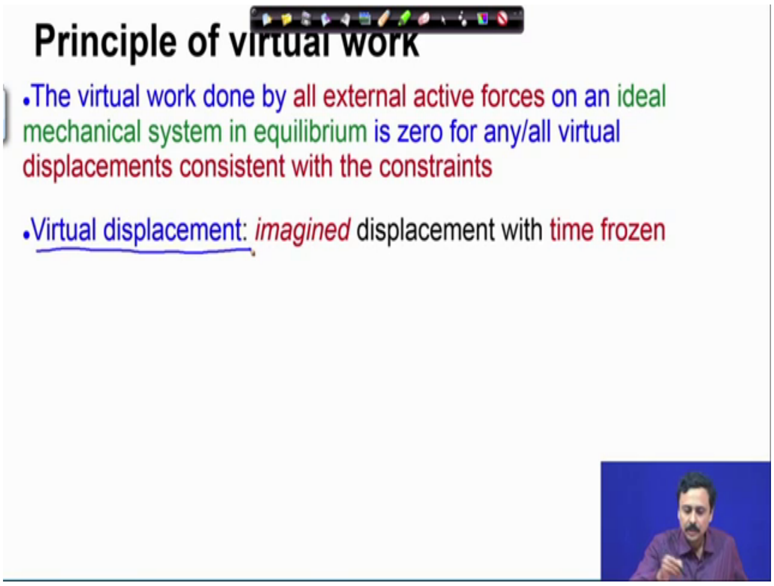

Now, let us review the principle of virtual work, the statement of the principle of virtual work states, that the virtual work done by all external active forces on an ideal mechanical system in equilibrium is 0, for any or all virtual displacements consistent with the constraints. So, here a very important term is this virtual displacement.

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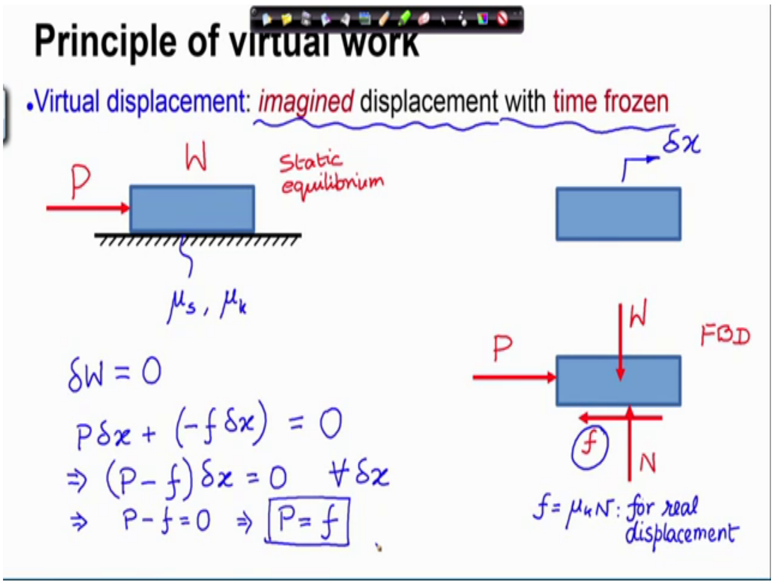

So, we will look into this notion of virtual displacement through an example, we will review this consider a block of weight W on a rough surface being pushed by a force P. But, we will assume that this system is in equilibrium, it is in static equilibrium. So, this block remains in static equilibrium. Here, I have the free body diagram of the block, we have the weight W the applied force P, the reaction force normal reaction force on the ground N and the friction force f. Through this example I would demonstrate what is meant by virtual displacement.

Since, this surface is rough we will assume the static friction coefficient and the kinetic friction coefficient mu s and mu k. Now, virtual displacement is an imagined displacement with time frozen. Suppose this block is given a virtual displacement delta x. Now, what is this virtual displacement, what is the difference between virtual displacement, and real displacement? In a real displacement if I give a real displacement to this block this f the friction force will immediately go to the value mu k times N.

So, this is under real displacement. So, for real displacement; so, if I give a real displacement to the block, then the because of sliding the force of friction will immediately become mu k times N as we know. However, I am interested in finding out f under static equilibrium and that is definitely not mu k times N. So, under virtual displacement f will still remain f as unknown. So, under virtual displacement the force of friction will remain f, unknown force f. So, that is the difference between real displacement and virtual displacement. So, this is an imagined displacement with time frozen. So, if there are time varying forces, then the force values will be frozen at the instant of time we are trying to find out the equilibrating force relation. So, that is the significant of time being frozen. So, in time varying systems, in systems where the forces maybe functions of time.

We will freeze the time; that means, the value of the force the vectors the force vector will get frozen and then we will give an imagine displacement which is consistent with the constraints. For example, here the constraint is that the block will never leave the table ok. Then, the statement of the principle of virtual work states that the work done, the virtual work done must be 0. The net virtual work done by all active forces must be 0 under virtual displacements. Now, if I have a virtual displacement delta x in the direction I have shown, then the virtual work done by the force P by the applied force P is P times delta x.

The virtual work done by f is minus f times delta x. The reason being that f and delta x are in opposition delta x is to the right, but f x to the left. W and N which are the weights and the normal reaction from the ground, they do not do any work under the displacement delta x as you can see. So, therefore, there is no other active force which is doing virtual work. So, this must be 0. So, from here it follows that P minus f delta x must be equal to 0. Now, this should hold for all arbitrary virtual displacements delta x, this statement must hold for arbitrary displacements delta x then it immediately implies that P minus f must be equal to 0, which means that P must be equal to f.

This is a statement which you have also easily derived by taking writing out the equilibrium equation in the horizontal direction. Then, why do we use virtual work? The reason is that we need to only consider the input and output process, we do not need to consider the internal forces, and this advantage will get cleared as we look in to apply this principle to mechanisms.

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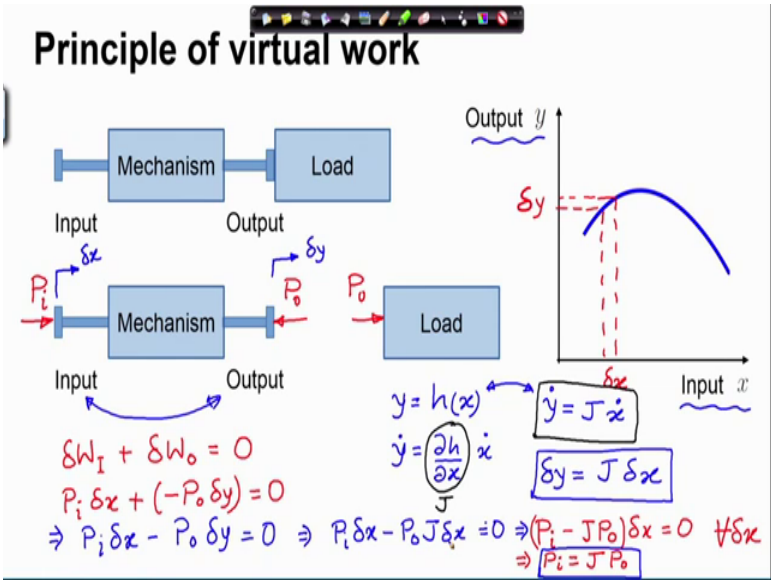

So, let us look at this once again through a schematic figure as I have shown here. Here, I have a mechanism and imagine mechanism shown as a block which has an input and a output at the input let us say we have applied a force P i, which is being transformed and transmitted to the load as P o, which is doing certain work on the load. Therefore, the mechanism feels a negative force P o so, P o in the opposite direction. So, that is the reaction force on the mechanism.

So, we have P i at the input and P o in this direction on the output. Now, if I have to apply the principle of virtual work then I must know this input output relation, the input output relation suppose, the input output relation is known in this form.

So, that at the particular configuration x, if I give a virtual displacement input displacement delta x. If, I obtain a virtual output displacement delta y and if I am able to find the relation between delta x and delta y, then I can proceed to apply the principle of virtual work. So, the principle of virtual work says that, the work done at the, virtual work done at the input plus the virtual work done at the output must vanish for this mechanism. So, this is the net work net virtual work done; work done at the input and some work being done at the output. Now, work done at the input you can easily see, if I give a displacement delta x and if I have virtual displacement delta y at the output.

So, these are corresponding to one another through the input output relation. In that case the virtual work done at the input is P i times delta x, you can see that P and delta x are in the same direction plus. Now P o and delta y are in opposition. So, therefore, and this must vanish. So, that implies P i delta x minus P o delta y must be equal to 0. Now, let us look at the input output relation, we have studied this input output relation under displacement analysis. So, we can have y as a function of x. We have also studied input output velocity relations y dot it is some Jacobian times x dot. Now, you can very easily relate this input output relations by differentiating this displacement relation with respect to time and that is what we have done.

And therefore, this term is nothing, but the Jacobian. So, we have the velocity relation, input output velocity relation for the mechanism and from there I can directly find out the virtual input and virtual output relation. So, if I consider the time is being frozen. So, then I can write delta y as J times delta x. So, I am imagine the time is being frozen. So, that any input displacement delta x is related to the output displacement delta y through the Jacobian.

So, therefore, using this I can rewrite this as P i delta x minus P o and J times delta x and that and that is equal to 0, which implies P i minus J times P o delta x must be equal to 0. So, therefore, and this should be satisfied for all virtual displacements at the input delta x. This immediately implies that P i is equal to J times P o and this is our force relation, this is our force relation. So, thus we have related the input output forces for the mechanism.

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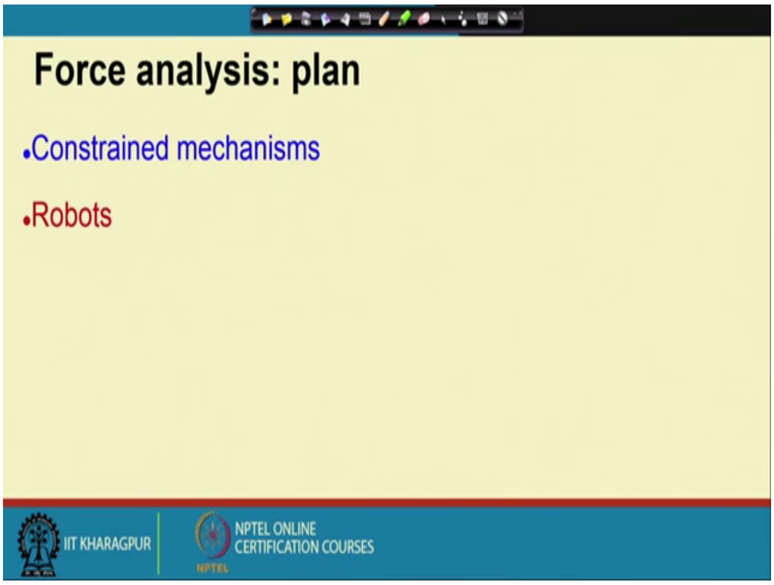

So, our plan is to study constrained mechanism and robots.

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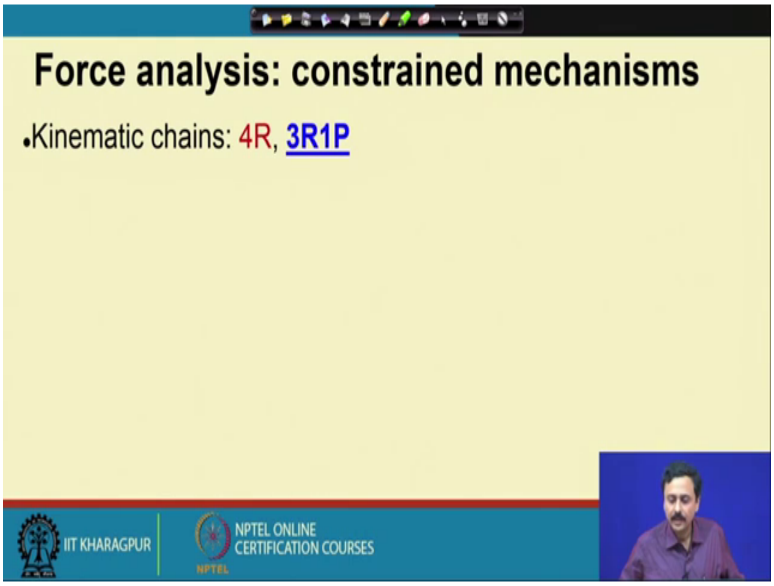

In the constrained mechanism we are going to look at 3 R 1 P chains.

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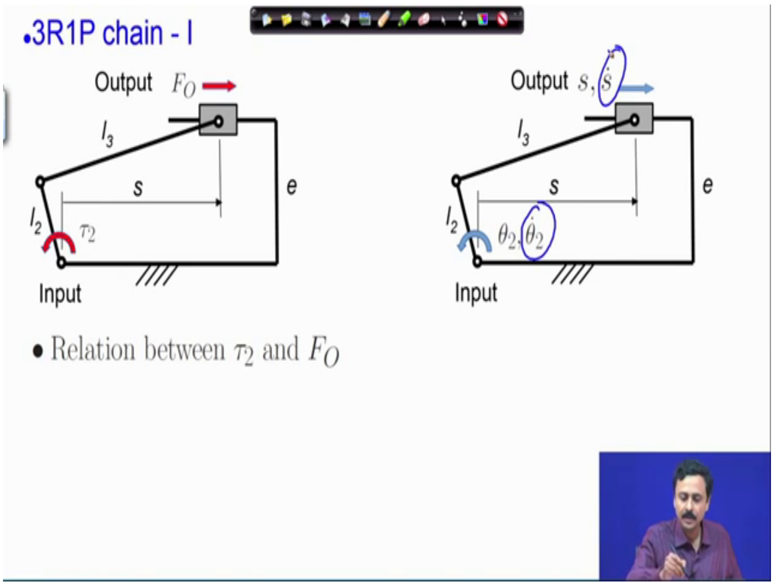

So, here I have shown you 3 R 1 P chain of type 1 and as you can see I have marked the input and the output. So, at the input I have a torque tau 2 and at the output I have a force F O. The force and the output and we have to find the relation between tau 2 and F O. Now, from the kinematics from the velocity relations we had already related theta 2 dot and S dot. So, these 2 have been related through velocity analysis which we have discussed previously at a specific configuration.

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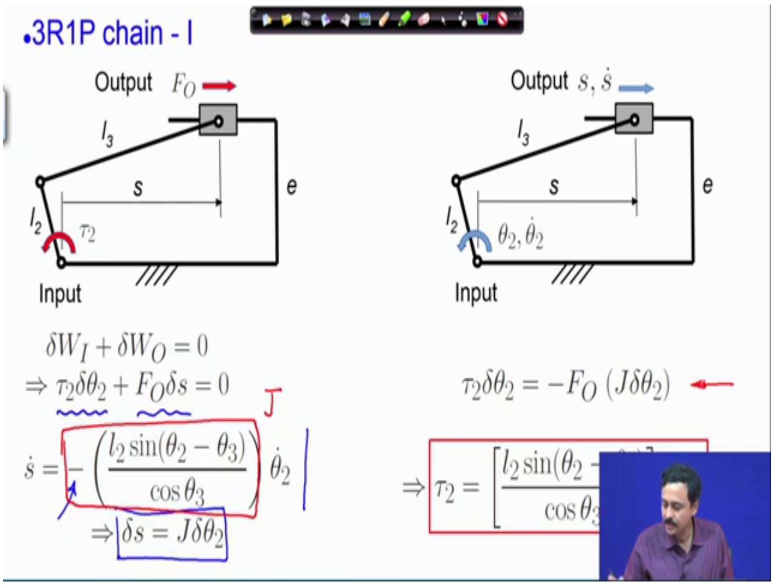

So, then if we proceed further the statement of principle of virtual work says that delta W I plus delta W I equal to 0 where delta W I is the virtual work at the input a virtual work at the input, you can say is tau 2 times delta theta 2. And, virtual work at the output is F O times delta S, you can very easily come to this statement by noticing that tau 2 and theta 2 dot, they are the same direction and F o and S dot have been considered in a same direction.

Now, from the previously discussed velocity relation we have already obtained this velocity relation, where this negative sign takes care of the velocity direction at the output, because of a counterclockwise theta 2 dot. So, therefore, the relation between the virtual displacements at the input and output are obtained as delta s is equal to the Jacobian. So, here the Jacobian so, this is the Jacobian. So, again form the principle of virtual work by replacing delta S using the virtual displacement relation, we have obtained this. And therefore, because delta theta 2 must be arbitrary therefore, I obtain finally, this relation between the input torque tau 2 and output force F O.

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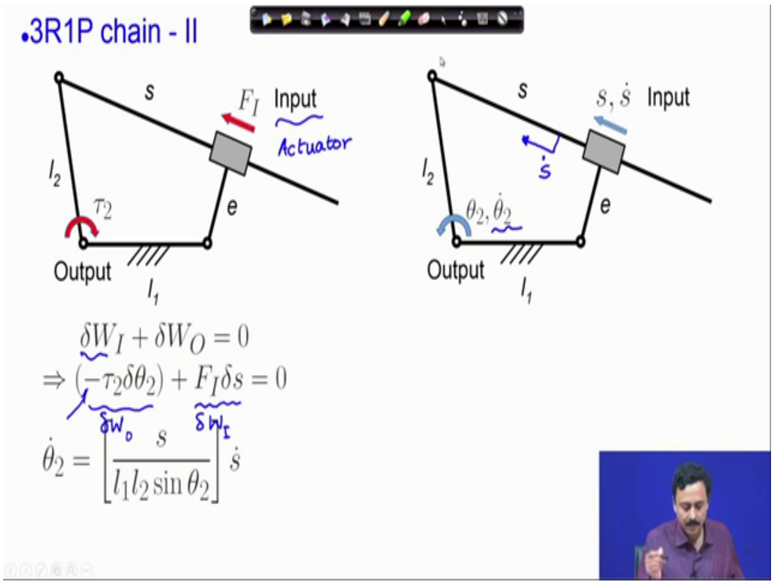

Now, if you look at the 3 R 1 P chain of type 2, we have the input here. So, we considered maybe a hydraulic or a pneumatic actuator and the output is the torque tau 2. So, we have to find out the relation between the input force at the actuator and the output torque tau 2. Now, the motion relation so, at the input because of the actuator force we considered that this link moves out at a rate s dot. So, that is the expansion rate of the actuator and theta 2 dot which is the rotation of this link is counterclockwise.

So, the principle of virtual work states that the network must vanish now at the input. At the input we have a F I as the force and delta S as the displacement virtual displacement and at the output. So, this is delta W I at the output we have tau 2 as the output torque and delta theta 2 as the virtual displacement at the output, now they are in opposition as you can see. So, that is why you have this negative sign sitting here, the work done being negative. Now, we have the velocity relations with which we have derived earlier.

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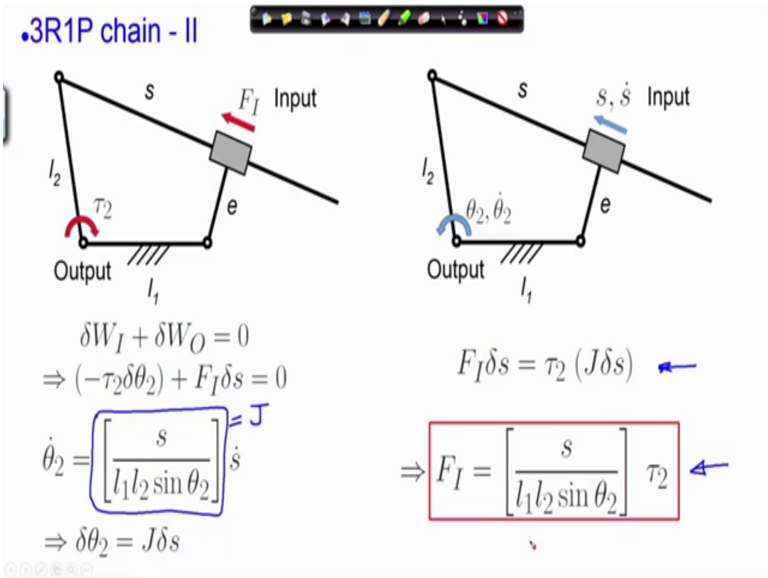

And from here follows the virtual displacement relations delta theta 2 is Jacobian times delta S, now here we have the Jacobian. So, therefore, from the statement of virtual work by replacing this delta theta 2 I have obtained this relation.

And finally, since this relation must hold for arbitrary delta s we must have a F I related to the torque through this relation. Now, you can see that the Jacobian has come into the relation.

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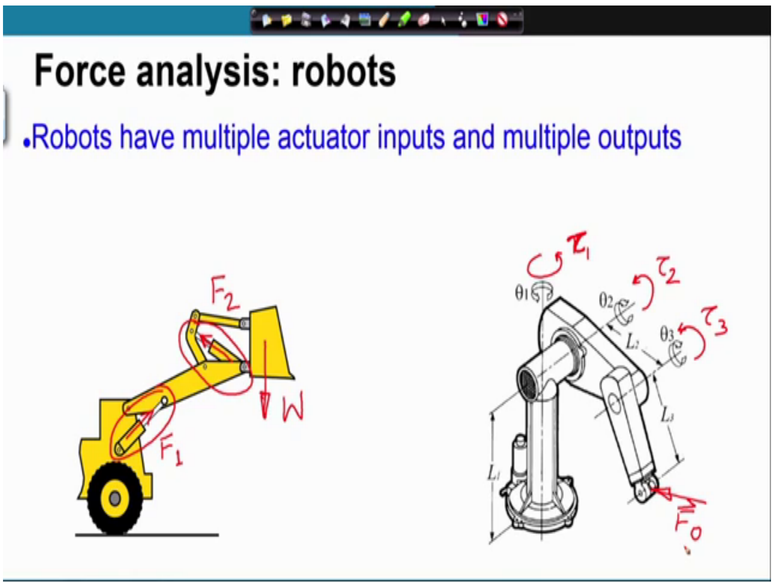

Now, we will look at the force analysis for robots, as you know robots have multiple actuators as inputs and there can be multiple outputs. For example, here we have this excavator we have a load maybe W and we have 2 actuators, which are applying forces F 1 and F 2 and that is responsible for equilibrating the load W.

In this example of the puma robot; suppose there is some force at the end effecter that must be equilibrated by input torques tau 1, tau 2, and tau 3. Now, this force is a vector here. So, therefore, I have multiple input torques and force as a vector. So, there are 3 components. So, there are multiple outputs.

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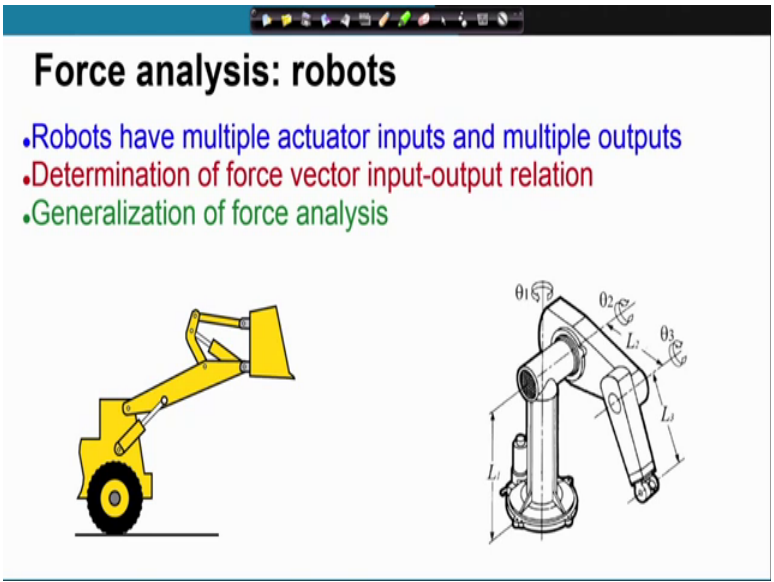

So, we have to find out this force vector input output relation. So, the joint forces and the output forces these are both are vectors. So, when you do generalize our force analysis that we have been looking at. So, till now we had considered only scalar forces, in 1 dimension analysis, but now when we have multi input multi output system, then the forces the joint forces and the output forces both are vectors. So, we need force relations between vectors. So, we need to generalize our force analysis.

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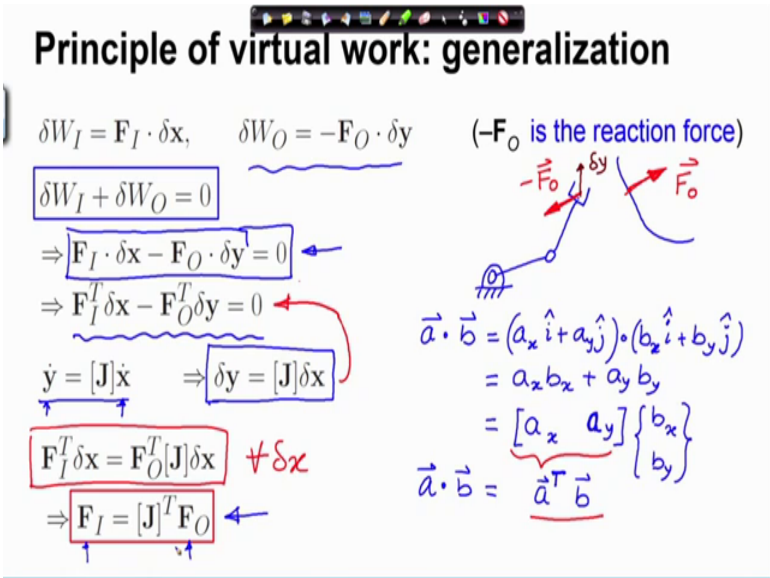

So, in order to generalize our force analysis I have written out the virtual work done at the input delta W I is F I dot delta x. So, delta x is the virtual displacement at the inputs and F I is the force vector at the input. At the output we have F naught as the force being applied. So, if I consider for example, a manipulator the 2 are manipulator let us say, which we are going to look at very soon.

Suppose it is a applying a certain force F O. So, on the manipulator there must be a reaction force, now since this is a vector I must write it as minus F O. So, F O is the force being applied by the manufacture on the external world through the end effecter and minus F O is the reaction force that the manipulator feels.

So, therefore, if delta y is the virtual displacement at the output, if delta y is a virtual displacement at the output therefore, delta W O which is the work done virtual work done at the output is given by minus F O dot delta y. So, minus F O being the force experienced by the robot manipulator at the output so, F O is the force being given to the external world by the robot. So, minus F O is the reaction force.

Now, the principle of virtual works states that delta W I plus delta W O must vanish. So, if I substitute these expressions, I obtain this relation. Now, I need the input output relations for the manipulator. Here, I have written out this statement, which is a dot product, which uses dot product of vectors as matrix product for our further simplification.

So, to understand this let us consider 2 vectors in dot product so, a dot b. So, is nothing, but a x i cap plus a y j cap dot b x i cap plus b y j cap, and as you know that this is a x b x plus a y b y. Now, I can write this as a matrix product like this, which is a vector transpose into b vector. So, the transposition converts a column vector to a row vector as I have written here.

So, the position of a vector which is considered to be a column vector. So, the transposition makes it a row vector. So, as a matrix product this becomes a transpose b. So, a dot b can be written as a transpose b and that is what I have used here now let us look at the input output relations. Suppose the output velocity is related to the input velocity or the joint velocities through the Jacobian like this. Therefore, when we considered freezing time and giving virtual displacements at the input and looking at the virtual displacement at the output they are related to the Jacobian as delta y is J times delta x, where J is the Jacobian matrix.

So, using this relation in here rather in here so, using the Jacobian relation in here I obtain this relation or this should be true for all virtual input displacements delta x. So, therefore, I must have this as a relation between the input and output forces. So, you should note that the input output forces are related through the Jacobian transpose. So, on the left we have the input force, on the right we have the output force, while the velocity in the velocity relation it is the output velocity at the left and input or the joint velocities and on the right. Here we have input forces on the left and the output forces on the right. So, this now generalizes our force analysis. So, let us proceed further.

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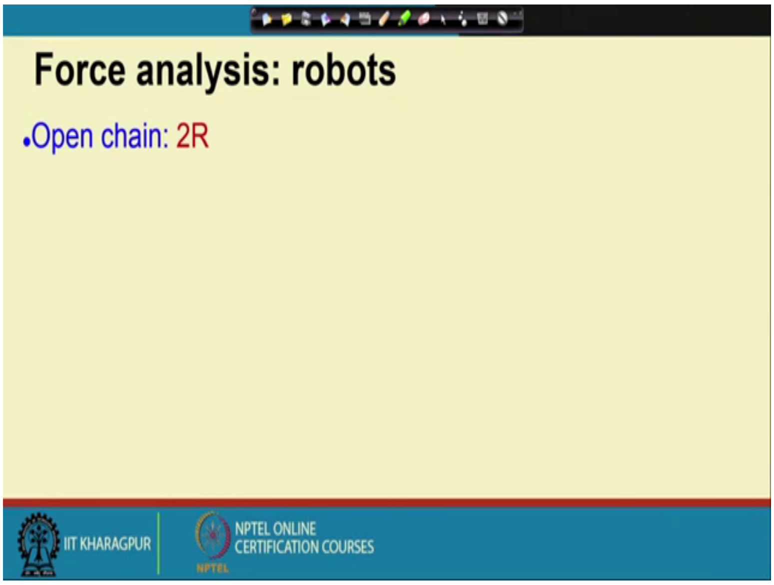

We will look at open chain 2 R manipulators.

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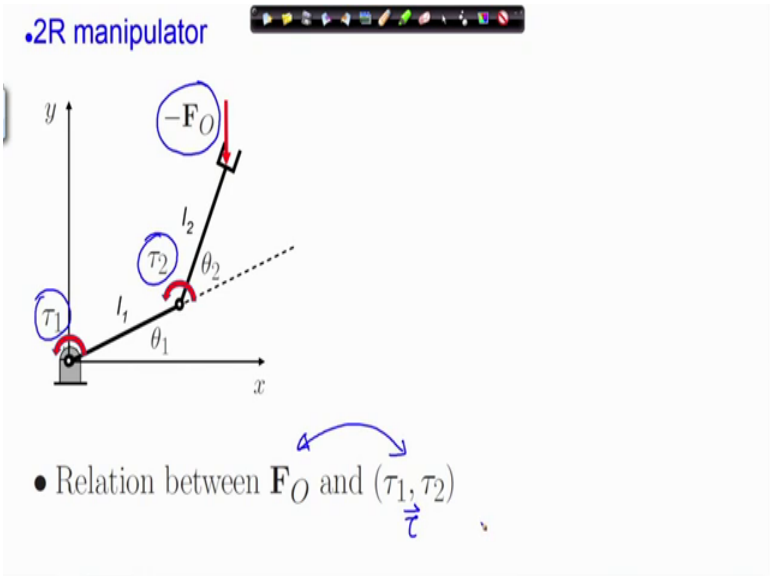

So, here I have a schematic for the 2 R manipulator we have these input forces which are the joint torques tau 1 and tau 2. And, the reaction force that is experienced by the manipulate the at the end effecter which is minus F O, I have to find the relation between this F O and therefore, vector and the joint torque vector tau. Now, we go through the velocity analysis that we have already discussed.

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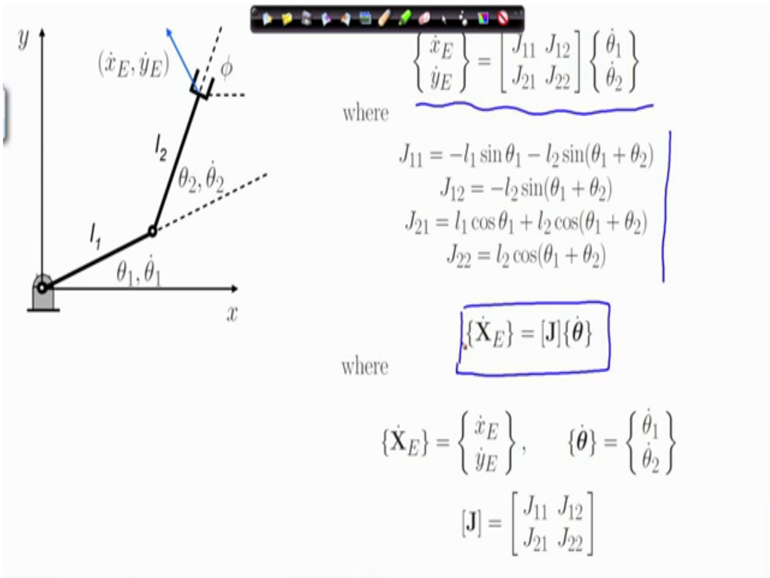

So, we know that the velocity vector at the output is related to the joint velocities at the input through this Jacobian relation, where the Jacobian elements are given here. So, this we wrote in a compact form as X E vector dot is J times theta dot.

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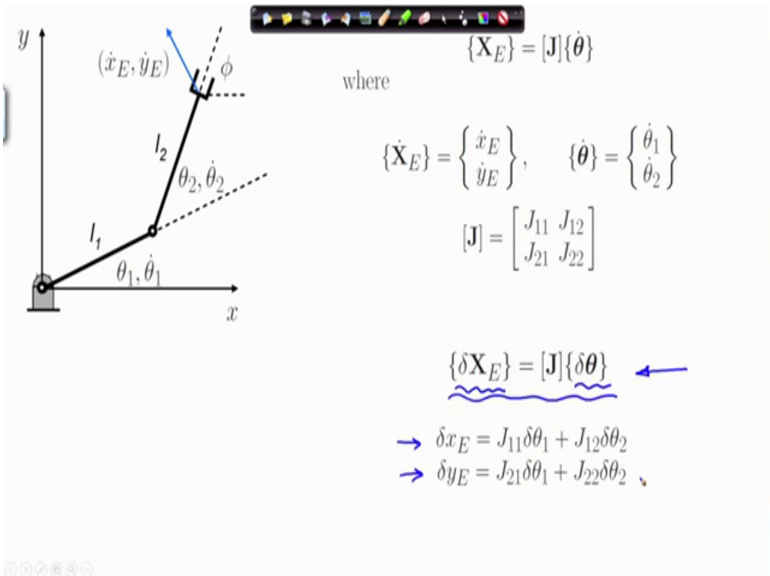

Now, starting from there we can now write the virtual displacement relations when we freeze time and provide an arbitrary input virtual displacement delta theta. And, obtain the corresponding output virtual displacement delta X E. And, they are related through the Jacobian as we observed here. So, the implication is like this that delta x E is J 1 1 times delta theta 1 plus J 1 2 times delta theta 2 and delta y E is J 2 1 times delta theta 1 plus J 2 times delta theta 2.

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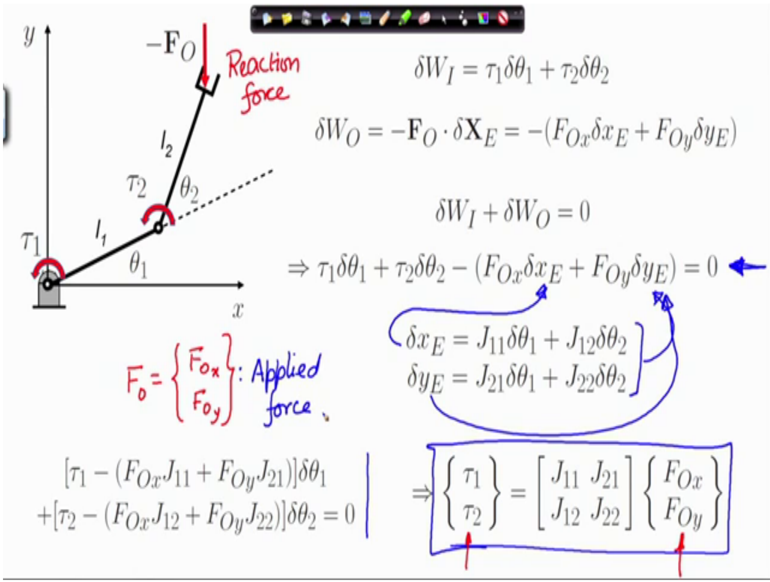

Now, we compute the virtual work done at the input which is tau 1 times delta theta 1 plus tau 2 times delta theta 2. The virtual work done at the output because of this reaction force minus F O is minus F O dot delta X E so, which is obtained here. Now, the statement of principle virtual work states that the net virtual work must vanish. So, therefore, I have this relation following from the statement of principle of virtual work. Now, since delta x E and delta y E we have already found the expressions we will just plug in these expressions in here and collect terms of delta theta 1 and delta theta 2.

So, when you plug in delta x E here and delta y E here, we have the principle virtual work in terms of only delta theta 1 and delta theta 2. Then, we collect terms of delta theta 1 and delta theta 2 to obtain this relation, which can be easily done. And, finally, when we assemble we obtain this relation between the joint torques on one hand and the output forces on the other. So, these are the joint torques and these are the forces at the end effecter. So, remember that minus f o is the reaction force. So, therefore, this F O is the applied force. So, this is the applied force on the external world by the manipulator.

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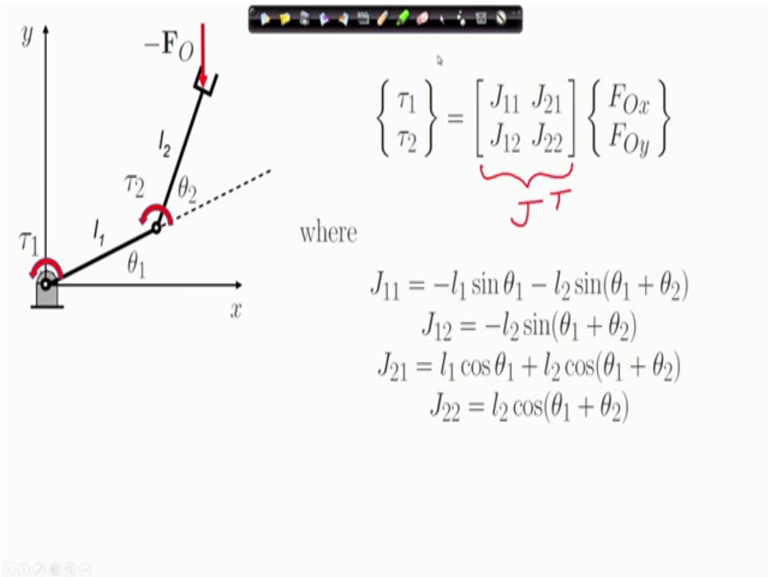

So, when you write out the elements of the Jacobian J 1 1 J 2 1 J 1 2 J 2 2 here you note one thing that this is nothing, but the Jacobian transpose.

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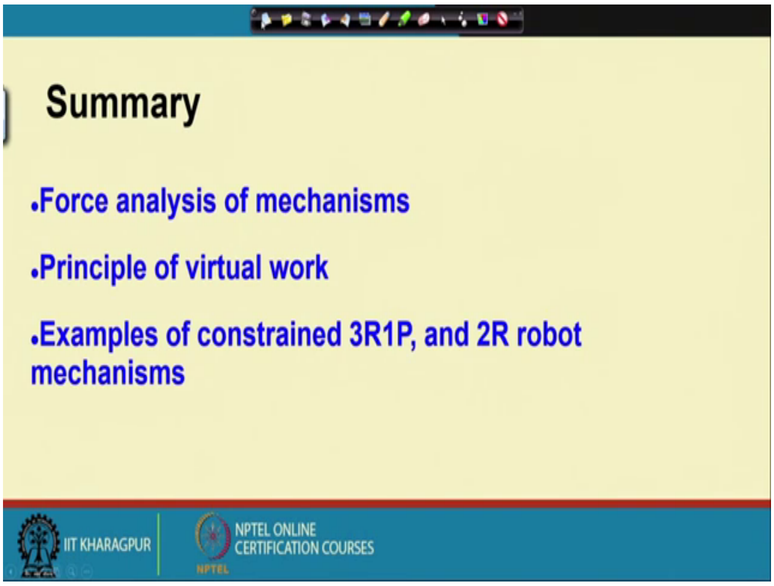

So, let me summarize what we have studied in this lecture, we have looked at the force analysis problem for mechanisms and robots using the principle of virtual work. We have considered the example of 3 R 1 P constrained chains of 2 types, type 1 and type 2. And, we have also considered the planar open chain manipulator 2 R manipulator and discuss the force analysis for this manipulator. So, with that I close this lecture.