Kinematics of Mechanisms and Machines
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Lecture – 33
Acceleration Analysis - II

In this lecture we are going to continue our discussion on Acceleration Analysis. So, in the last lecture we had looked at 2 examples of constrained mechanisms, we are going to discuss on this further and take examples of constrained mechanism as well as robots.

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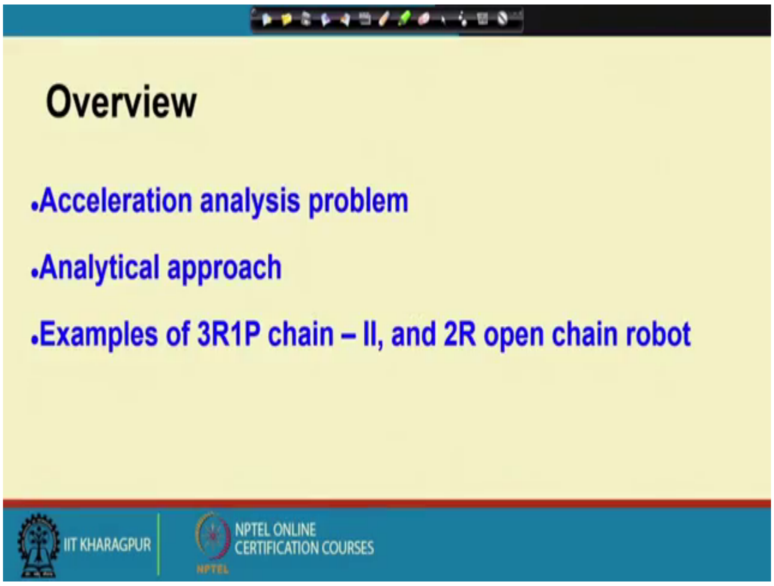

To give you an overview of today's lecture we are going to discuss the analytical acceleration analysis problem with examples of 3R1P chain of type II and 2 R open chain planar manipulator.

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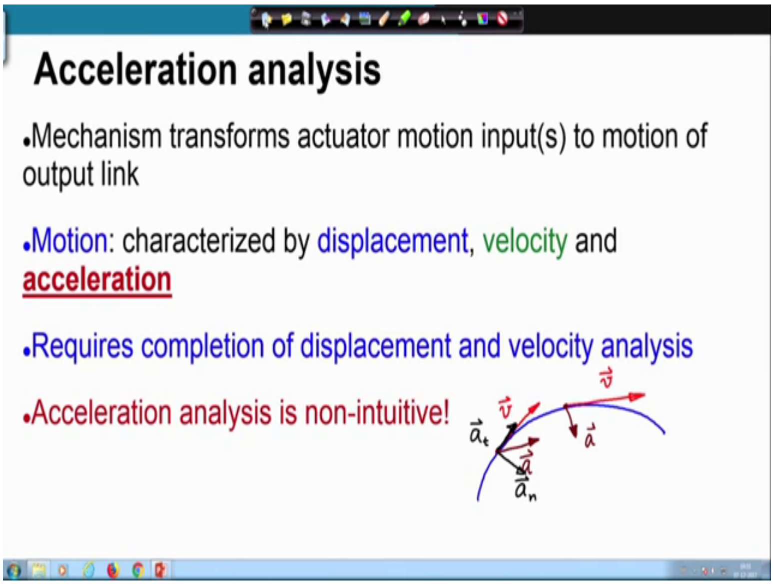

Let us review the things we had discussed before, we know that a mechanism transforms the actuator input motion to the motion of the output link. Now, motion as we know is characterized by displacement velocity and acceleration and in these lectures we are now discussing the acceleration part. We have already discussed the displacement and velocity analysis of mechanisms and robots, this acceleration analysis will now require the displacement and velocity analysis as a starting point. Now, acceleration analysis is non intuitive, so we have discussed this point let me reiterate once more. You know that when a particle is moving on a path the velocity of the particle at any point on the path is tangential to the path, the velocity vector is always tangential to the path.

However, the acceleration vector does not have any such restriction. So, I am showing the acceleration at 2 points at 2 instants of a particle moving on a curved path, as you know that this has 2 components; the acceleration has 2 components: one is tangent to the path, the other is normal to the path. The tangent acceleration if you call it a t the tangential component of acceleration goes on to change the magnitude of the velocity which means it changes the speed and the normal component is because of the path curvature. So, the normal component of acceleration in this case is because of the path curvature. So, acceleration can have an arbitrary direction for a particle moving on a curved path there was another example that I discussed.

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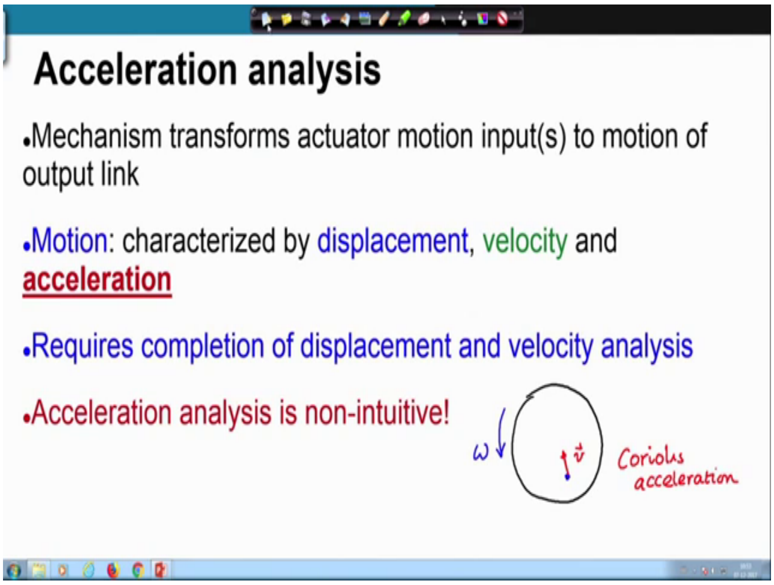

And, that is of a disc that is rotating at a constant speed and a particle that is moving with respect to the disc also at a constant speed. Let us say so even though speeds here are all constant, the angular speed is constant, the linear speed of the particle with respect to the disc is constant, the particle still suffers acceleration and this is the coriolis acceleration as you know. So, the particles suffer coriolis acceleration. So therefore, the acceleration analysis is non-intuitive.

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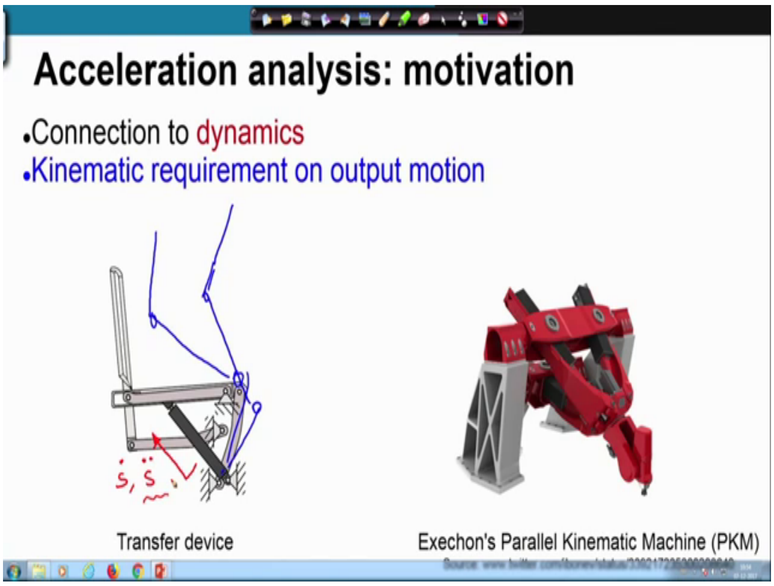

Now, the motivation for I mean doing acceleration analysis is that it is connected to dynamics. For example, Newton's second law relates the acceleration of a particle or a rigid body centre of mass of a rigid body to the forces acting on the particle or the body, there is another requirement that we have and that is the kinematic requirement on the input output motion.

So, here we have this transfer device which I might require to have constant speed while it goes from the sitting to the standing position, in that case not only the velocity of expansion of the actuator is important but also the acceleration of expansion is also important. So, if I want to produce certain acceleration or possibly 0 acceleration at the output I may require a non-zero acceleration at the input, so this is what we need to find out. So, we want to find out or relate the output acceleration with the input acceleration.

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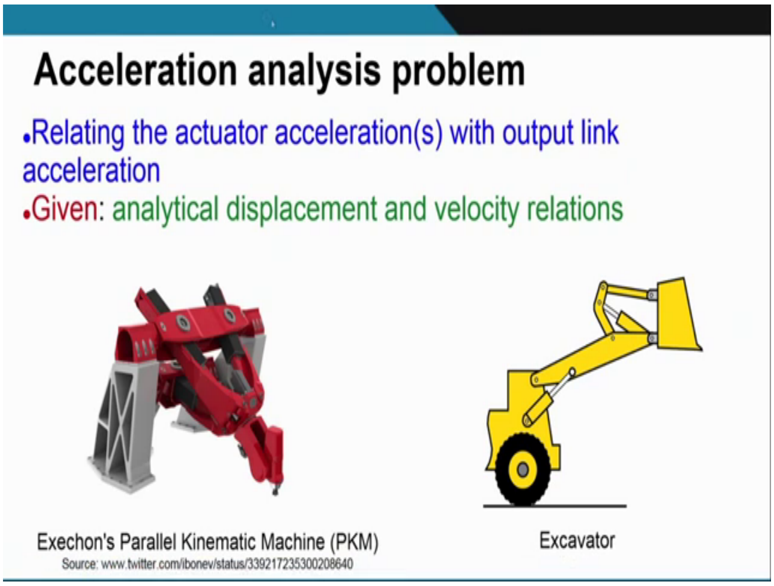

So, the acceleration analysis problem is about relating the actuator acceleration in input with the acceleration of the output link and we are given the displacement and velocity relation. So, we will start with these as inputs.

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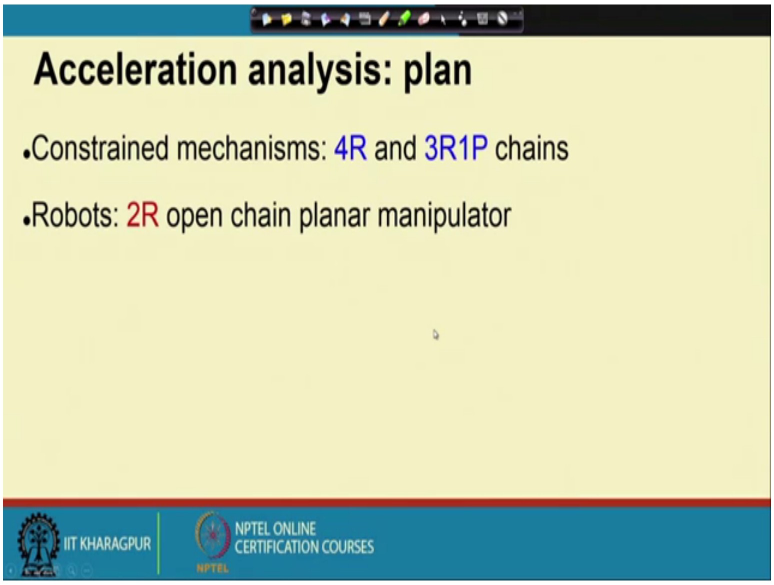

So, as we have discussed before our plan is to first discuss the constrained mechanism and subsequently go over to robotic manipulator. So, we will discuss the example of a 2 R open chain planar manipulator.

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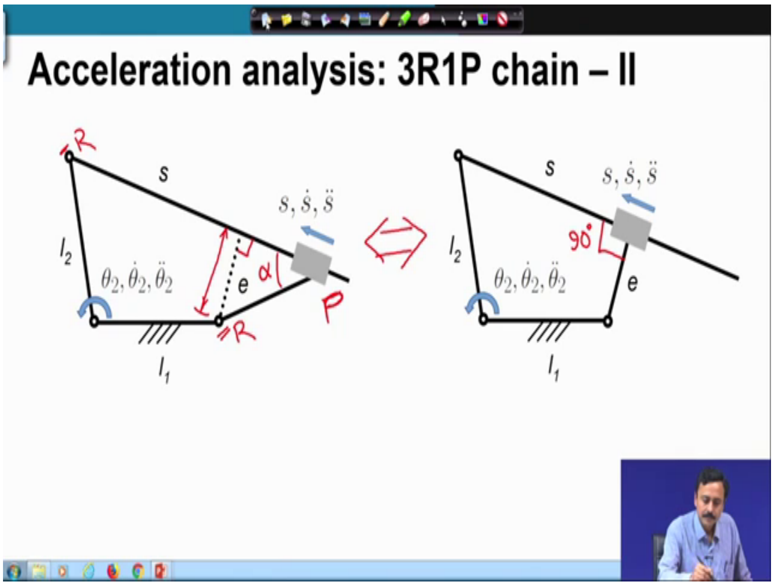

We will start with the acceleration analysis of a 3R1P chain of type 2; we have discussed this point before that for a 3R1P chain of type 2 here we have the offset. So, this is the distance between the 2 revolute pairs connected to the ends of the links with the prismatic pair. So, here we have the prismatic pair and these are the 2 revolute pairs connected to the links which are connected through the prismatic pair. So, the distance between these 2 revolute pairs measured in a direction perpendicular to the direction of the P pair is the offset, now this angle let us say alpha is a constant.

So therefore, I can consider an equivalent 3 R1P chain with this angle being 90 degree. The reason we are doing this is that it simplifies certain expressions and we can always go from 1 chain to the other. So, if I now the velocity acceleration analysis of the chain on the right I can always recomputed the velocity and acceleration of the chain on the left.

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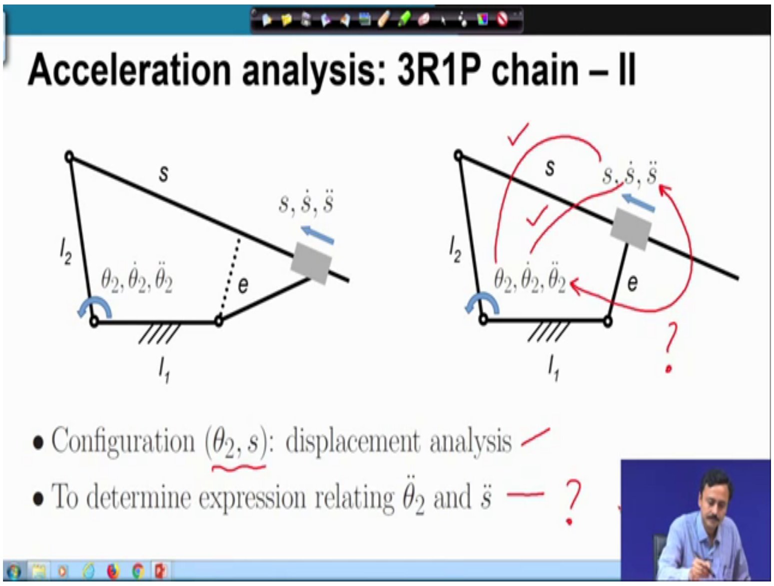

So therefore, we are going to analyze the chain shown on the right, we are given the displacement relations. So, we already know the displacement relations, we know the angle theta 2 and s. So, if we are given theta 2 we can find out s and vice versa. We also know the velocity relations. So essentially we know this relation we know the velocity relation what we have to find out is the acceleration relation, so this is what is our objective in this lecture: to determine the relation between theta 2 double dot and s double dot.

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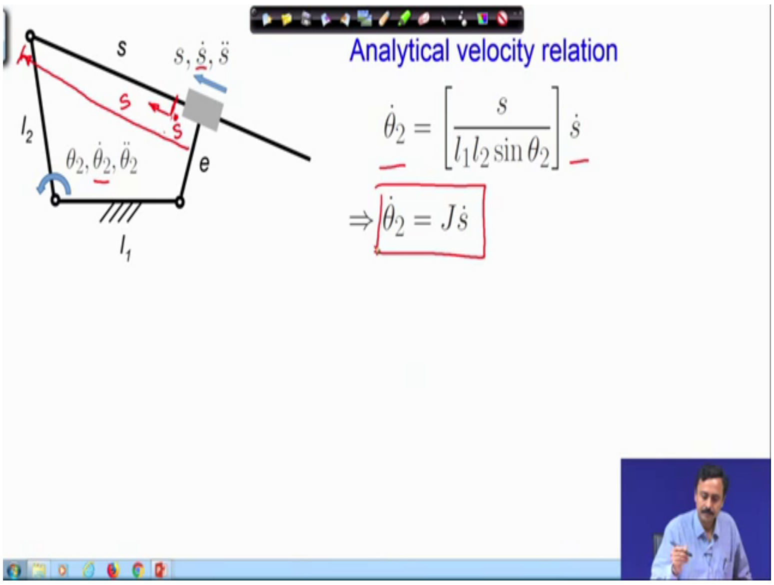

So, we will start off with the analytical velocity relation which we have derived earlier for this chain, I have mentioned that this is the angular speed at the output link and the linear speed at the input of the prismatic pair. So, essentially this distance which is s, we know the rate at which this link is sliding out of the prismatic pair so this is s dot. So, I know the rate at which this link is sliding out of the P pair and that relation is given here, this relation we have derived earlier when we discussed the velocity analysis.

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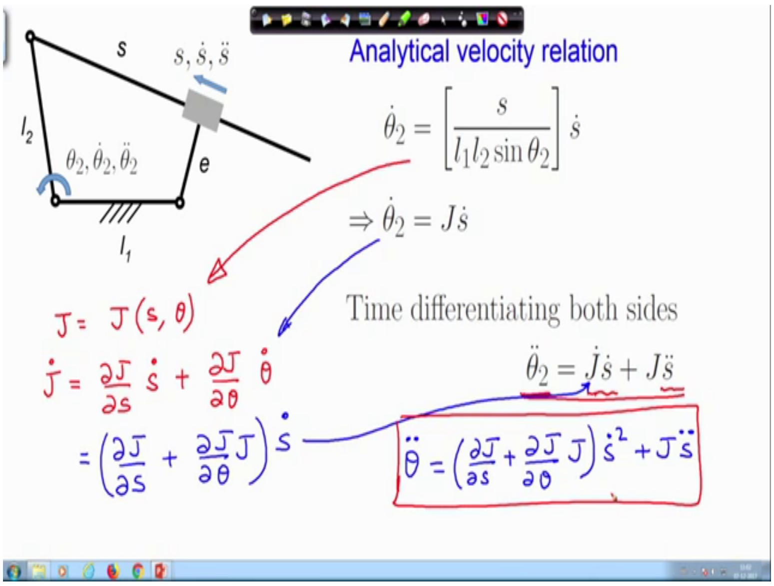

So, if you differentiate with respect to time both sides of this velocity relation you will obtain this acceleration relation. So, on the left I have the acceleration at the output and on the right I have a term where I have the time derivative for the Jacobian and the rate of expansion of the actuator and a term which involves the acceleration of expansion of the actuator.

Now, once again this Jacobian is a function of s and theta. Therefore, the time derivative of Jacobian can be obtained easily using chain rule. So, del J del s s dot plus del J del theta theta dot. Now you will observe that theta dot is nothing but J times s dot. So therefore, this expression can be rewritten by taking s dot out in this form. Finally, if you substitute this expression of J dot in here you will obtain the expression of theta double dot. So, this expression relates the angular acceleration of the output link with the input acceleration and velocity of the prismatic pair expansion.

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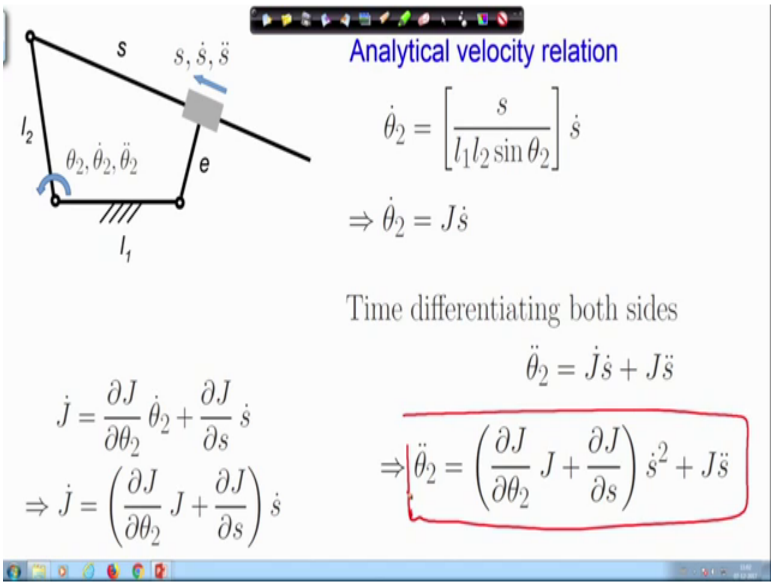

So, let me show this formally, so this is our final expression that we have.

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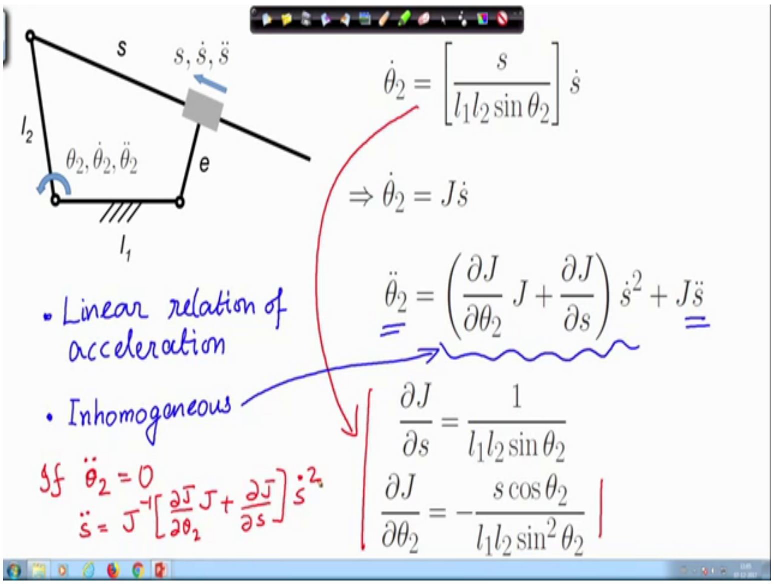

So, let me rewrite these expressions and determine what is del J del theta 2 and del J del S. So, these are the relations which can be easily obtained from here by differentiating with respect to s I have del J del s and with respect to theta 2 I have this expression of del J del theta 2. Now what we observe here is that the acceleration relations are linear. So, between theta 2 double dot and s double dot I have a linear relation however this relation is inhomogeneous because, of the presence of this term which is configuration and velocity dependent.

So, this is the inhomogeneous term which depends on the configuration and the rate of expansion of the actuator the velocity of expansion of the prismatic actuator. Further more if I want to move the output at a constant speed in other words if I want to have theta 2 double dot as 0 which means the output moves at a constant speed I must have acceleration at the input and that is obtained in this form. So, this is the acceleration that I must have at the input in order to produce a constant output acceleration.

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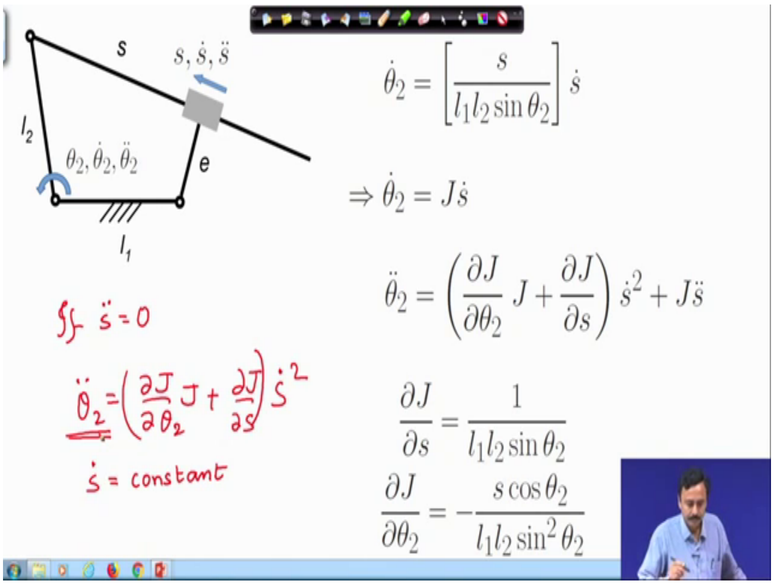

Similarly, if I have the input as a constant which means if s double dot is 0, that means the prismatic actuator is expanding at a constant rate in that case I have acceleration at the output. If I am moving at a constant if I have the prismatic actuator expand at a constant rate which means s dot is constant then the output has an angular acceleration.

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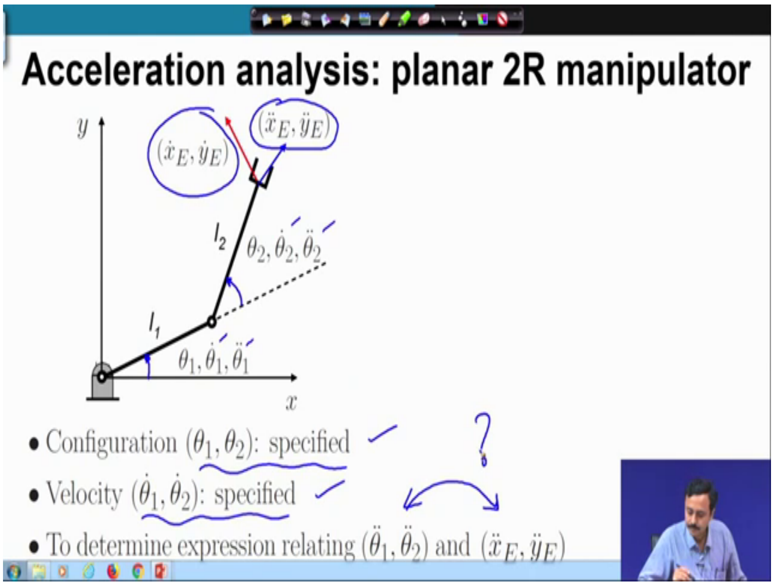

Next let us look at the acceleration analysis of a planar 2 R manipulator. So, here I have shown a 2 R manipulator which is moving at a certain configuration it is moving with velocity theta 1 dot theta 2 dot and acceleration theta 1 double dot and theta 2 double dot.

So, this has a velocity at of the endeffector and x E dot y E dot and an acceleration x E double dot and y E double dot. Now the configuration of the manipulator is specified the velocity is specified we need to relate the actuator accelerations the joint accelerations and the endeffector accelerations.

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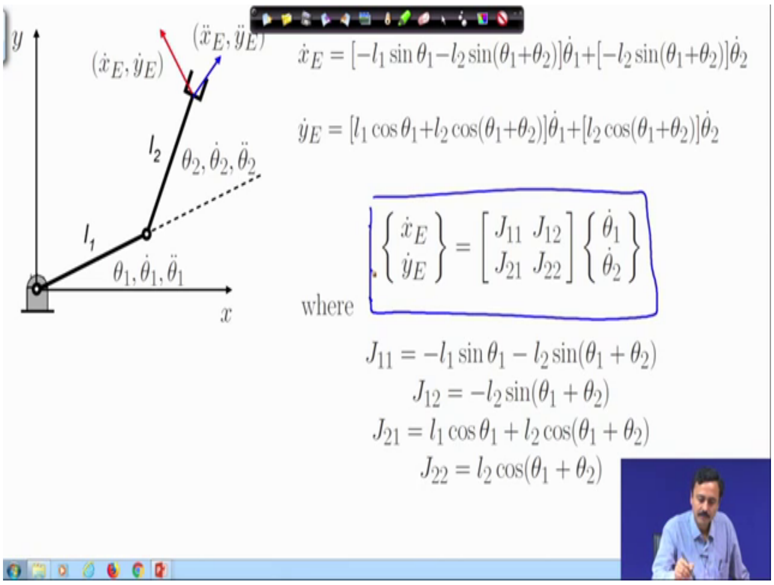

So, here I have written out the velocity relations which we have already derived earlier. Now if you assemble them in this form we have the relation of the endeffector velocity and the joint velocities through the Jacobian which we have discussed.

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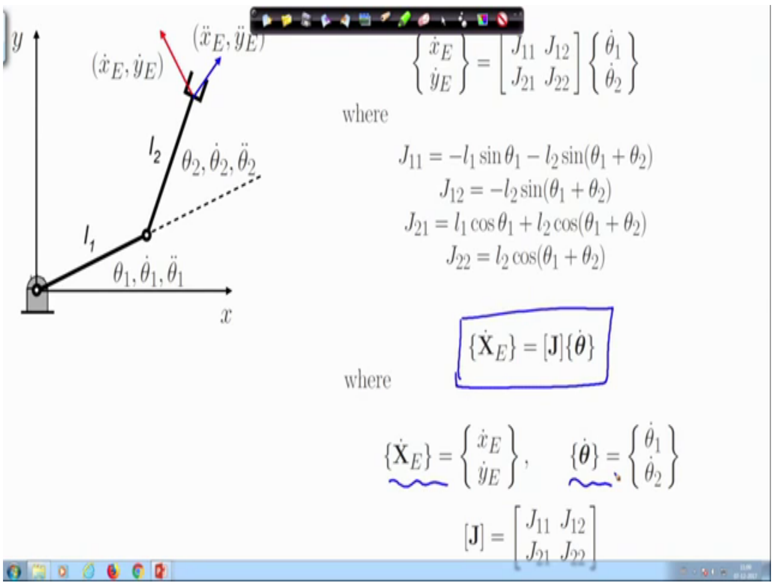

This can be written in a compact form as shown here where x E dot this is the endeffector velocity vector and this theta dot vector is the joint velocity vector.

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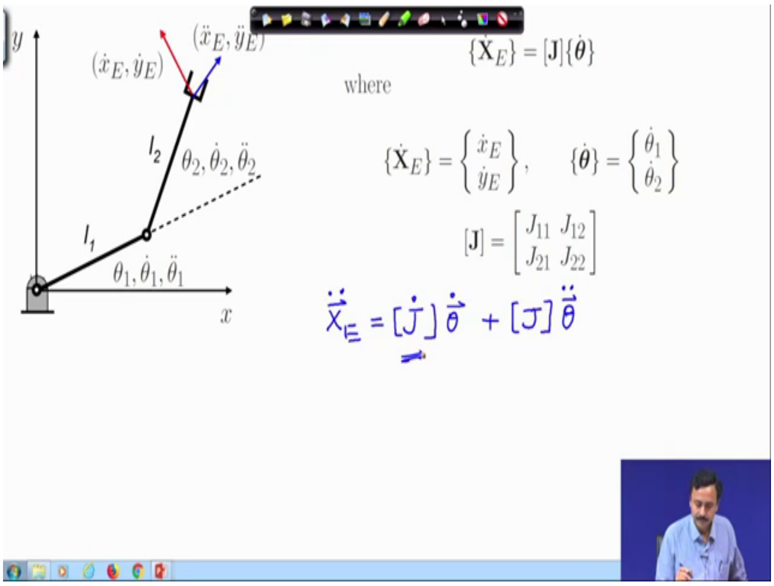

Now, if you differentiate this velocity relation with respect to time as we have been doing. So, we will obtain x E double dot is equal to the time derivative of the Jacobian remember that the Jacobian is the function of the configuration. So, time derivative the Jacobian times the joint velocity vector plus Jacobian matrix times the joint acceleration vector. Now, this Jacobian derivative is now little more complicated.

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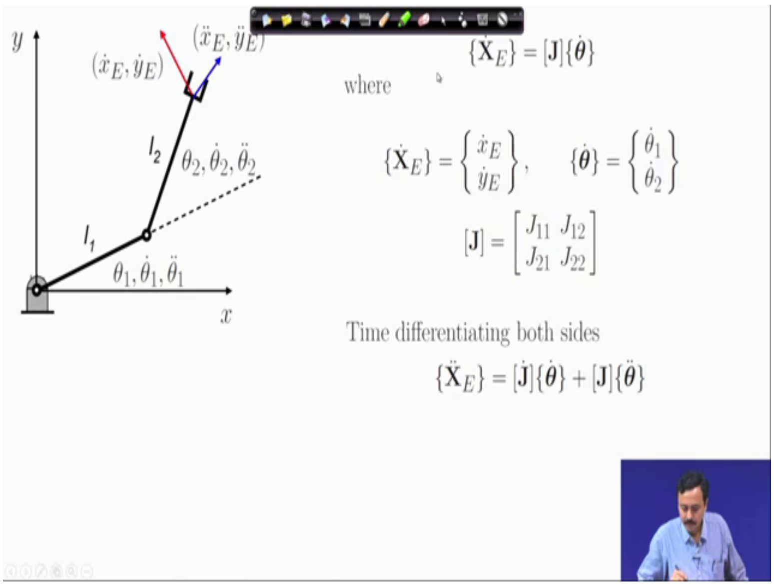

So, we are going to look at that so here I have written out this derivative of the endeffector velocity which gives us the endeffector acceleration.

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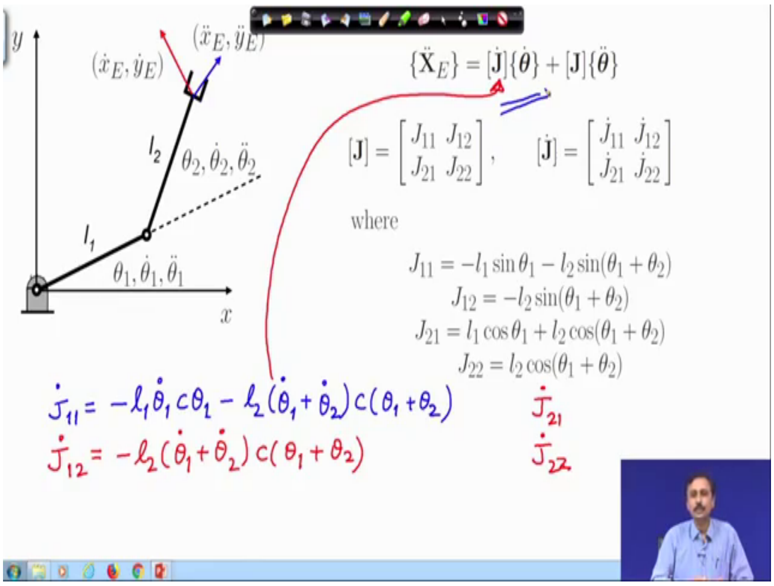

And the derivative of the Jacobian is the derivative time derivative of individual entries of the Jacobian remembers that the Jacobian has these entries. So, if you calculate for example, J 11 dot then you have minus l 1 theta 1 dot cosine theta 1 I am writing c theta 1 in place of cosine theta 1 and minus l 2 theta 1 dot plus theta 2 dot times cosine theta 1 plus theta 2 so that is J 11dot.

Similarly, you can write J 12 dot which is nothing but minus of l 2 theta 1 dot plus theta 2 dot times cosine of theta 1 plus theta 2, in this manner you can find out all the time derivatives you can find out J 21 J 2 2 dot J 21 dot and J 2 2 dot etcetera. So, these are straight forward but now the expressions are little complicated they are little lengthy. So, these expressions go into this J dot and then we have the relation between the input or the joint accelerations and endeffector acceleration.

So, here again we find that in terms of acceleration these relations are linear, but there is this term J dot theta dot which makes this relation inhomogeneous. Now in this case of robot manipulators the acceleration analysis has one more significance, we have discussed this path generation problem and we have found that in the path generation problem when the manipulator moves close to the singular singularity or the singular configuration then the input velocities can be have very high values. The reason being the inverse of the Jacobian becomes goes close to singularity which means the entries of the inverse of the Jacobian become very large the entries become very large.

There in the inverse expression in the expression of the inverse of the Jacobian we have the determinant of the Jacobian sitting in the denominator. So, when the denominator goes to 0 that is when the Jacobian becomes singular the terms in the inverse the Jacobian they become very large, because the denominator is going to 0. And, hence the required input velocity to generate the finite output velocity becomes very large the required input velocities or the joint velocities become very large.

Now, if the joint velocities become very large, that means this will have very high accelerations. Now, very high joint accelerations would require very high torques on the part of the motor this needs to be restricted. So, any motor will have a finite torque producing capacity or torque specification it cannot produce torque more than a certain value further more. If you do not want to damage the motors you would like to have the accelerations restricted to certain values. So therefore, from the acceleration expressions we can now find out what will be the required joint acceleration near singularity and we can (Refer Time: 26:38) the acceleration.

So, in order to understand how we should restrict accelerations and what effect it is going to produce on the our output motion we need this acceleration expression which we are derived here. So, if you want to restrict the joint accelerations to within certain range, then you can correspondingly calculate what will be the acceleration on the endeffector. So, this one accrue certain errors and you can estimate those errors.

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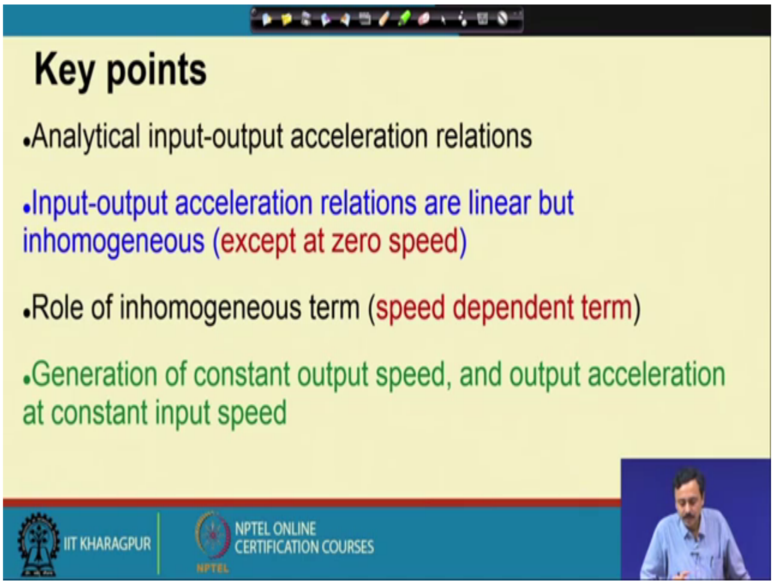

So, the key points of our discussion in this lecture we have derived the analytical input output acceleration relations, they are found to be linear but inhomogeneous we have looked at the role of the inhomogeneous term which is speed dependent. And, we have also discuss the generation of constant output speed constant and the corresponding acceleration at the input that is required to produce constant output speed. And, we have also seen that if I have a constant input speed then we have acceleration at the output and we can calculate all these things from our acceleration relations.

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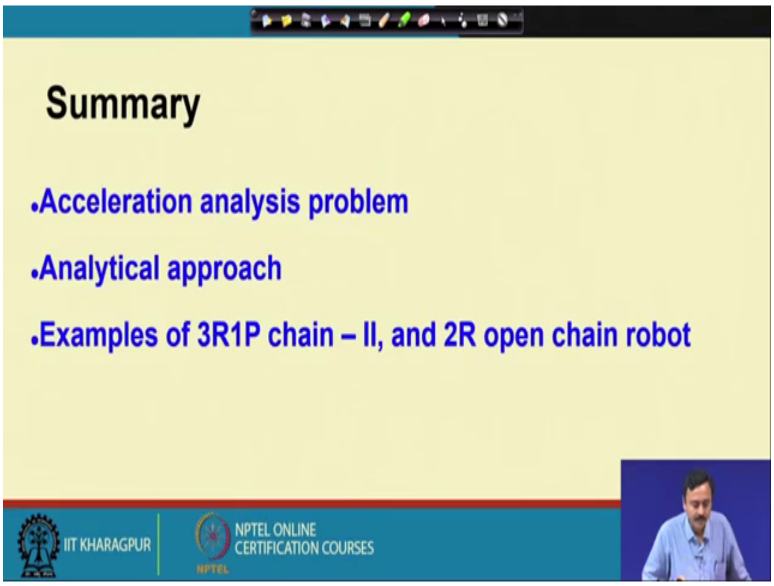

So, to summarize we have looked at the acceleration analysis problem using the analytical approach you have looked at this examples of 3R1P chain of type 2 and a 2 R open chain planar robot manipulator. So, with that I will conclude this lecture.