Kinematics of Mechanisms and Machines
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Lecture - 31
Robot Path Generation

We have been discussing about the velocity analysis of Robot manipulators. We are looked at open chain and closed chain robot manipulators. So, the problem of velocity analysis is to relate the actuator velocities to the end effector velocities, today we are going to look into the problem of Path Generation.

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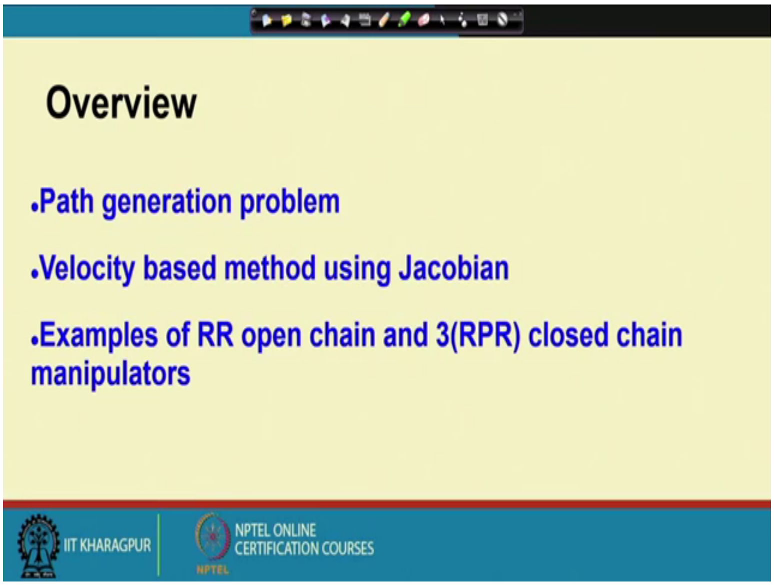

So, to give you an overview of what we are going to discuss in this lecture, we are going to look at the path generation problem using the Velocity based method. We look at examples of the RR open chain planar manipulator and 3 RPR closed chain planar manipulator.

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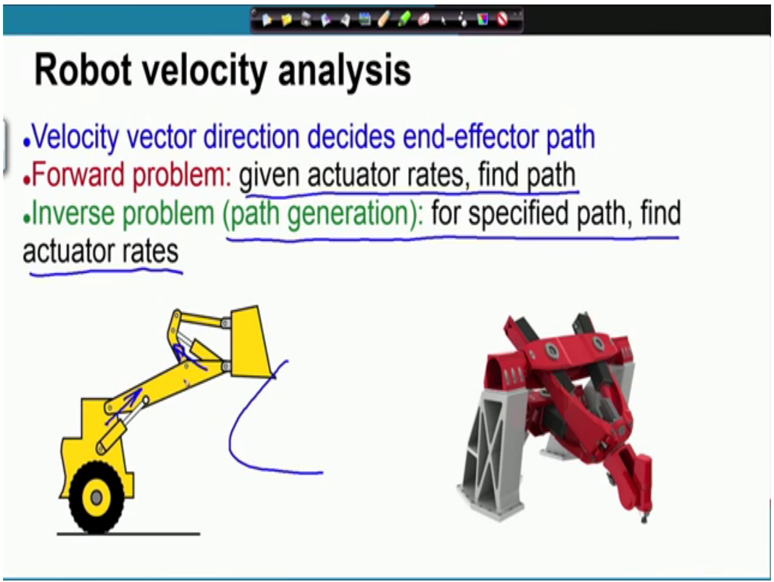

So, as I have mentioned that the velocity analysis problem in the case of robots is also closely related to the path generation problem, so given the actuator rates or the rates at which the actuators are expanding or rotating, we can find the path that the end-effector takes.

So, given the actuator rates we can find out the path of the end-effector, so that is a forward problem. The inverse problem which is very relevant for various applications is the path generation problem, in which for a given path of the end-effector we have to find out the rate of expansion of these actuators. So, that is the inverse problem which is the path generation problem.

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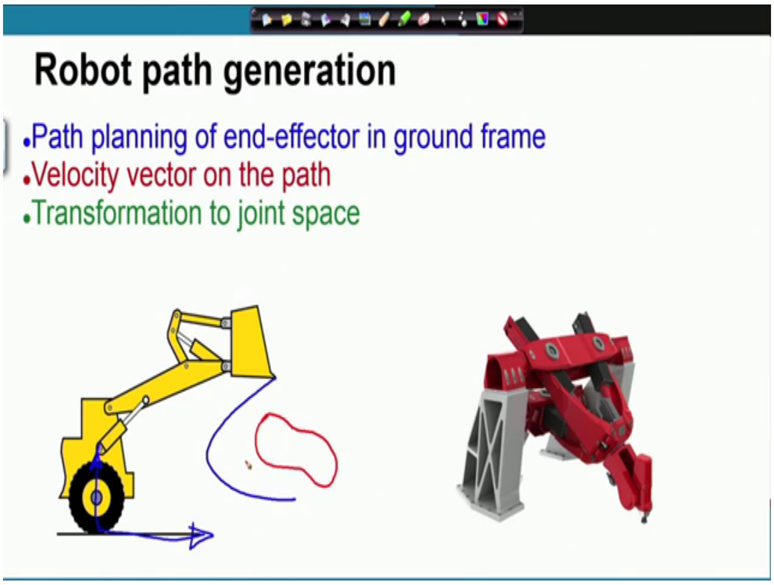

We have discussed briefly what is the path generation problem. Now the path generation problem has 3 components in it, let us go through them 1 by 1, first is the path planning of the end-effector in the ground frame which means that given this configuration for example this excavator, I would like to know in a certain ground reference frame what is the path that I desire. So, essentially is the representation of the path, now this representation of the path will take into account for example, obstacles which might represent or it might be that there is a bin on which this must be dropped.

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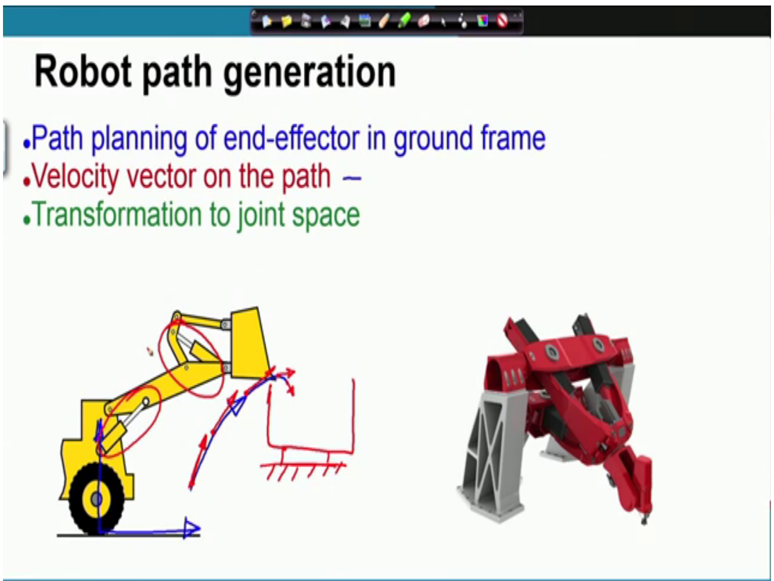

In that case it might go something like this, so based on our requirement we have to first plan the path in the ground reference frame. The second point is the specification of velocity on the path. So, if I have to generate this path how do I generate? I specify the velocities that I desire for the end-effector on this path, something like this. Once I have the end-effector velocity at every point on the path I transform that velocity back to the joint space. So, essentially I find out the actuator rates to produce the corresponding endeffector velocity. So, that is the plan of this path generation problem, so that is how this will proceed.

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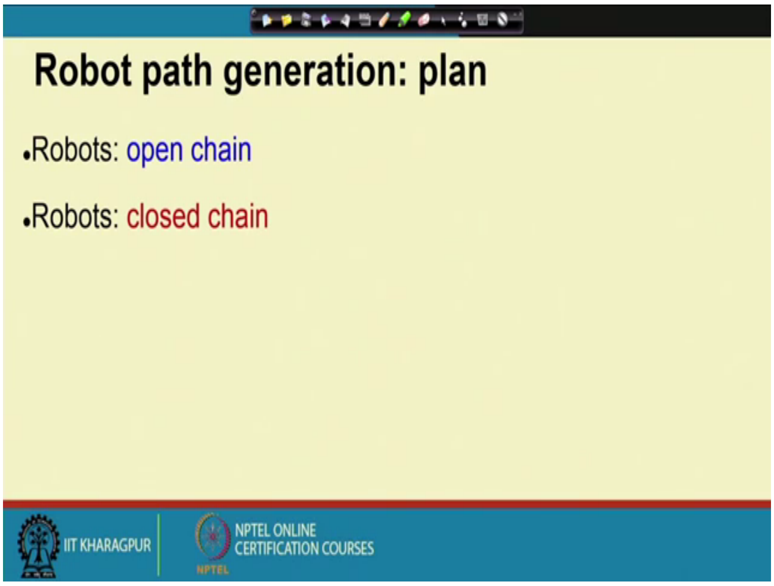

So, we will look at open chain and closed chain manipulators.

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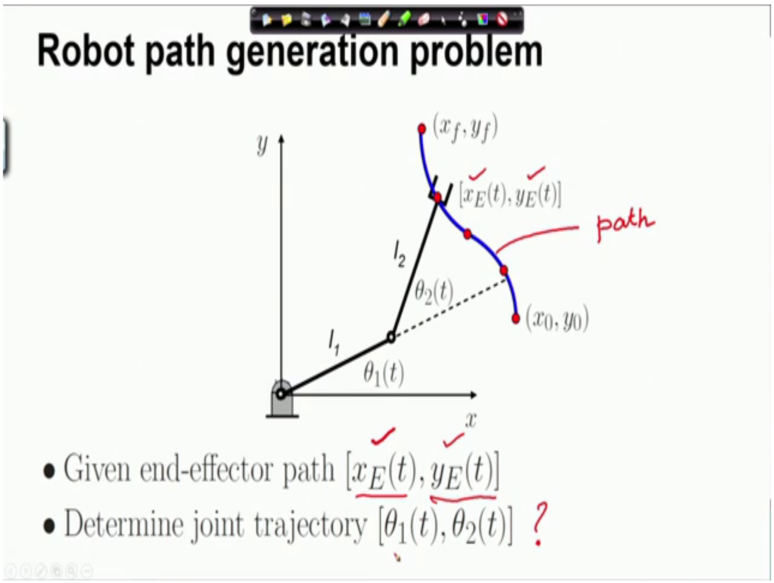

So, this is the problem of path generation, so essentially this is the second and third path. So, what we have is we have the path already specified or determined. Now we have to specify the velocities at each point on this path. So, we are given the end-effector path in terms of x E and y E as a function of time so these are specified, so as the function of time I know how x E and y E weld. We have to determine the corresponding joint trajectory or the joint velocity.

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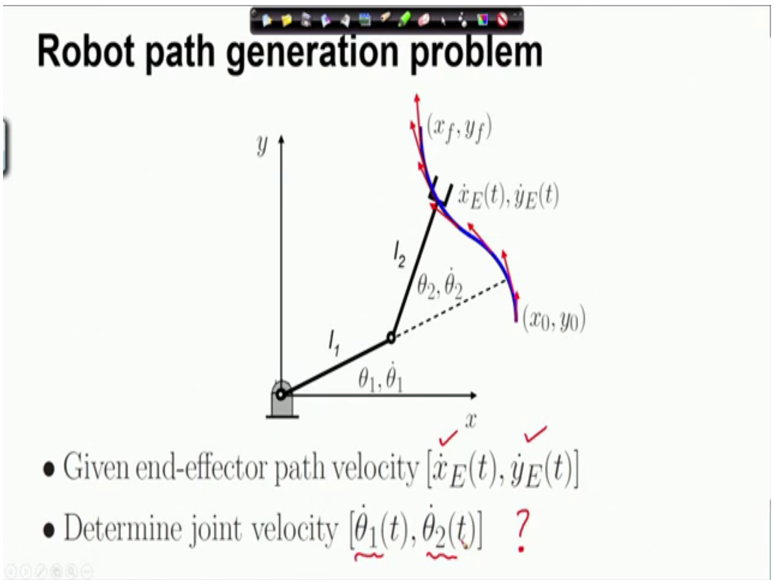

So, the way we will proceed is that given the path we can differentiate at every point and determine the velocity of the end-effector at a specific point on the path. So, here we can differentiate and find out the velocity as a function of time and using the velocity analysis we are going to determine the joint velocity which comprises in this case is theta 1 dot and theta 2 dot.

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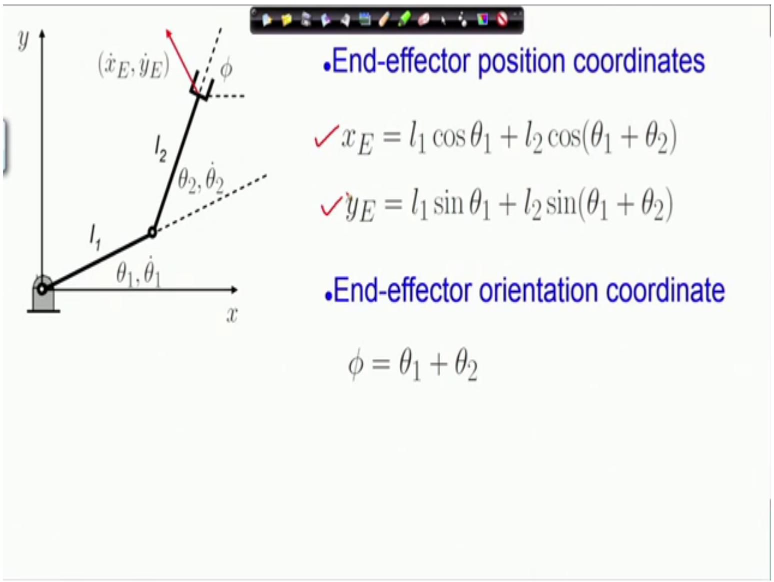

So, let us review what we had discussed in the velocity analysis problem, so we were given the displacement relations.

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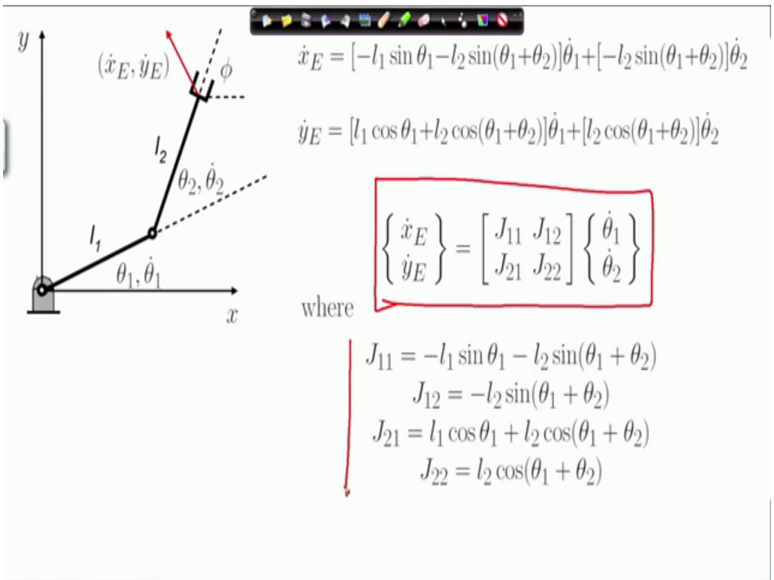

Then differentiating the displacement relations we arrived at the velocity relation, input output velocity relation. So, on the left I have the end-effector velocity vector and on the right I have relation through the Jacobian to the joint velocity vector. Whether an element of the Jacobian for the 2R manipulator has been written out, this we have discussed already.

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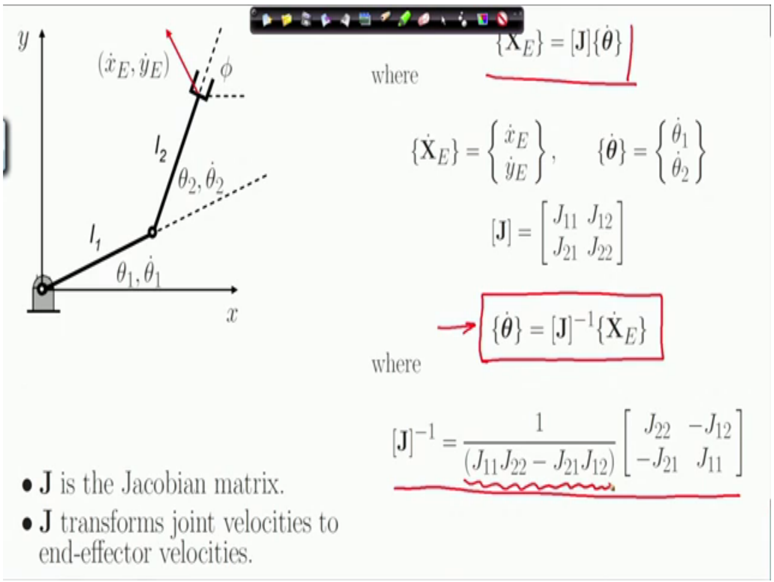

So, we had these relations and finally we obtain the velocity relations in the forward kinematic forward velocity analysis and this is the velocity relation in the inverse velocity relation. So, this is the inverse velocity relation and this is the forward velocity relation. So, we have noted that in the inverse velocity relation we required the inverse of the Jacobian and the inverse of the Jacobian has this denominator term, this is the determinant of the Jacobian. So, this determinant must be nonzero in order for the Jacobian to be invertible.

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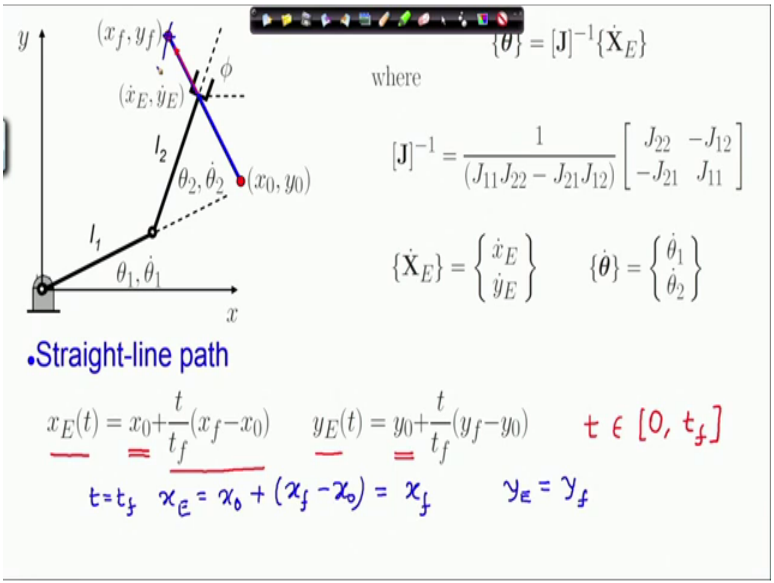

So, this is our inverse velocity relation, now let us look at the example of a straight line path generation. So, here I have 2 points x o y o and x f y f and I would like to connect them by a straight line. Now every path planning problem can be considered to be a specialization of this problem, because any path can be thought of as straight lines between infinitesimal points. So, if I can find this then I can generate any path.

So, let us look at this straight line path generation, first we are going to represent the path. So, the first problem will be to represent the path. You can very easily represent this X E and Y E at any point on the path as a function of time. So, here since we start with x naught so this the x coordinate of the end-effector. So, we start at x naught and we move linearly in time because of this additional term. So, we can see a time t equal to 0 I am at x naught and similarity at time t equal to 0 I am at y naught.

This path has to be traversed in time t f, so essentially this time goes from 0 to t f. So, at time t equal to 0 I am at x naught y naught a time t f when t is equal to t f then this factor becomes 1, so this is x naught plus x f minus x naught so that becomes x f. Similarly y E become y f so at time t equal to t f we are at the final point x f y f.

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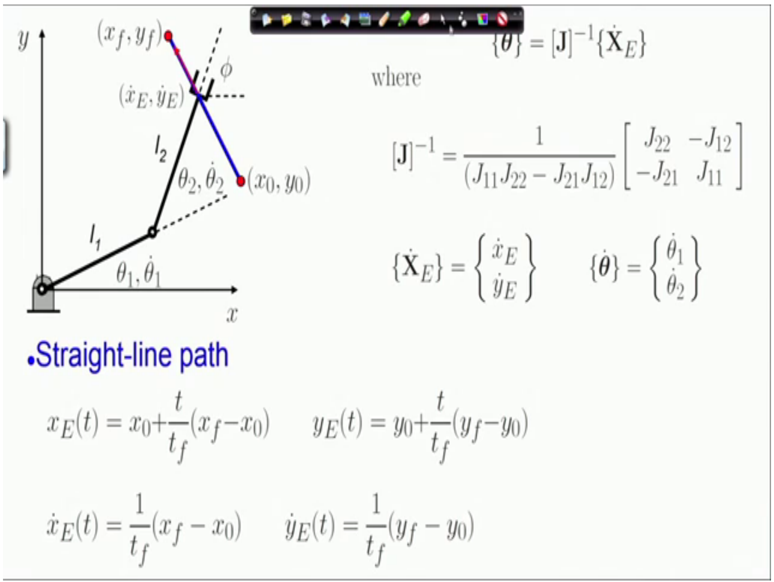

So to differentiate this function then you have the velocity expression. So, this is the velocity of the end-effector point in the Cartesian coordinate in the ground based Cartesian coordinate system. So, this is the velocity that we are going to use, so once I have the velocity now I can very easily, so I have the right hand side here this is the right hand side, the velocity of the end-effector point forms the right hand side of this inverse velocity relation and through the inverse Jacobian I obtain the joint velocity rates, so joint velocity vector.

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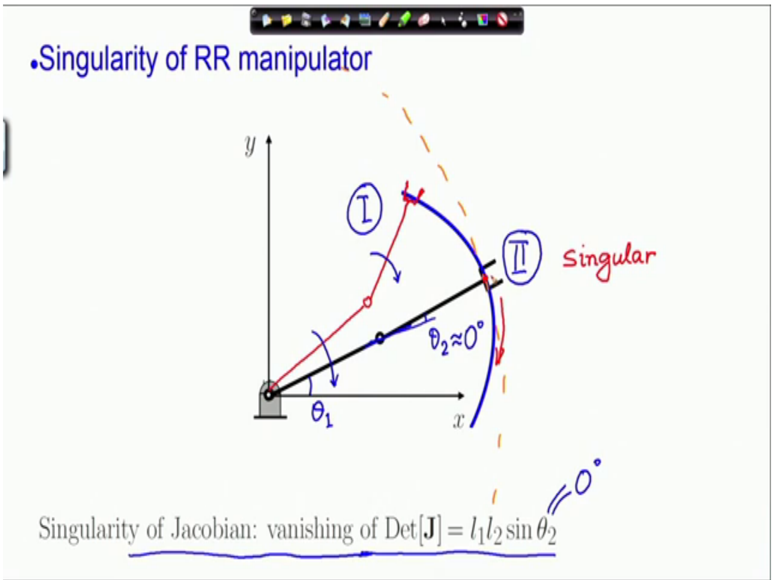

Now, there is this inversion problem of the Jacobian as we have seen here. So, the inverse of the Jacobian has this determinant of the Jacobian setting and the denominator. So, you must ensure that the determinant of the Jacobian is non zero, but there are instances where this might go to 0 or this might go very close to 0.

So, let us look at this situation what we have here let us say a manipulator which traverse this path up to this point, this manipulator moved from configuration 1 to this configuration 2 the black configuration. Now here as you can see that this angle has straightened out, so this angle theta 2 is roughly 0 degree. So, this is theta 1 measure from the x axis and theta 2 is the angle made by the second link with respect to the first link.

Now, here theta 2 is roughly equal to 0 and we have discussed that the Jacobian is singular when the determinant goes to 0, which in this case it is because theta 2 is almost equal to 0 or 0 at this configuration. So, in that case the determinant of the Jacobian vanishes and therefore the Jacobin is no longer invertible, so this is a singular configuration.

Now when the manipulator passes through a singular configuration there are various possibilities, in the sense that I can generate this path if this path is such that it passes through the; so this point let us say on the path, the manipulator passes through the singular configuration and the path then again comes inwards as you can see the manipulator is completely extended.

So, this point on the path is actually on the boundary of the workspace. So, this is roughly the boundary of the workspace which we have discussed before. So, this point of the path lies on the boundary of the workspace. So therefore, the manipulator goes to a singular configuration. At this configuration if you want to proceed further on the path which again actually comes into the workspace as you can see, it touches the workspace boundary and then again comes inward into the workspace, so therefore it can be continued.

So, at the workspace boundary either you can actuate theta 1 or you can just actuate theta 2, which means that one actuator can be held fixed the other actuator can be moved in order to produce a velocity which is tangent.

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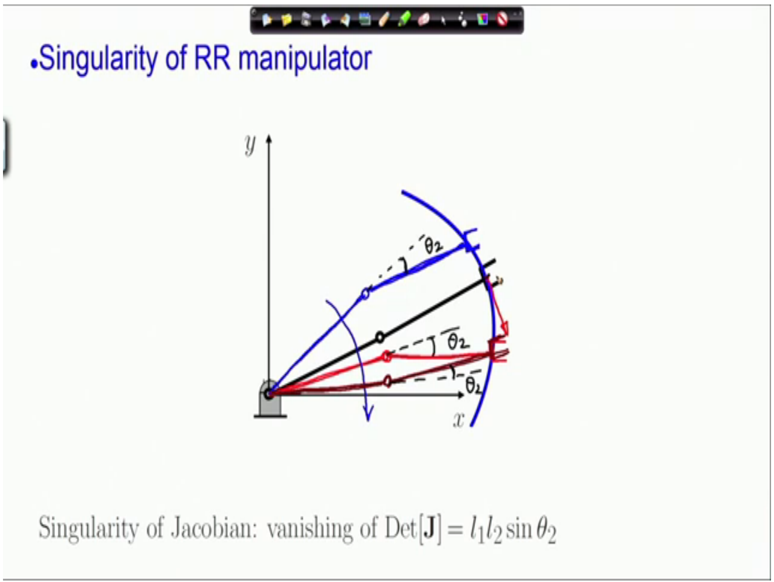

Because at this boundary point we need to generate a velocity vector of the end-effector which is tangent to the path and since this is also the boundary, so it will be tangent to the boundary. Now once you cross then you can come again, the configuration like this. So, you can move from a configuration which I have shown in blue. So, you can move from the blue configuration through the black configuration to the red configuration, there is another possibility that you can move when you go past the singular point you can move to a configuration like this.

So, this is also a possibility that the angle theta 2 here it was negative goes to 0 and then again goes to negative or theta 2 was negative goes to 0 and flips to the positive side in the ground configuration, as you can see here. So, there is a flipping of configuration through the when the manipulator passes through the singular configuration. Now usually near the singular configuration because the Jacobian is so in conditioned.

So, we have this numerical problem; we have numerical problems because the inverse of Jacobian will involve very large quantities and therefore once you have very large quantities sitting in the inverse of the Jacobian then the joint velocities become very high, this cannot may not be supported by your actuators, the actuators may not be able to produce such high joint velocities.

So, you always face a problem, so the path has to be planned initially such that it does not go very close to a singular configuration of the manipulator. If at all it is required that the path has to go very close to the singular configuration, then the inverse problem has to be solved carefully. You have to put additional constraints on the velocity of the actuators or the actuator rates, so that your motors or actuators do not get saturated. In that case you loose on the path or you can loose on the velocity that you desired on the path. So, there is a trade off, if the path is too close to a singular configuration then you will lose accuracy on that path.

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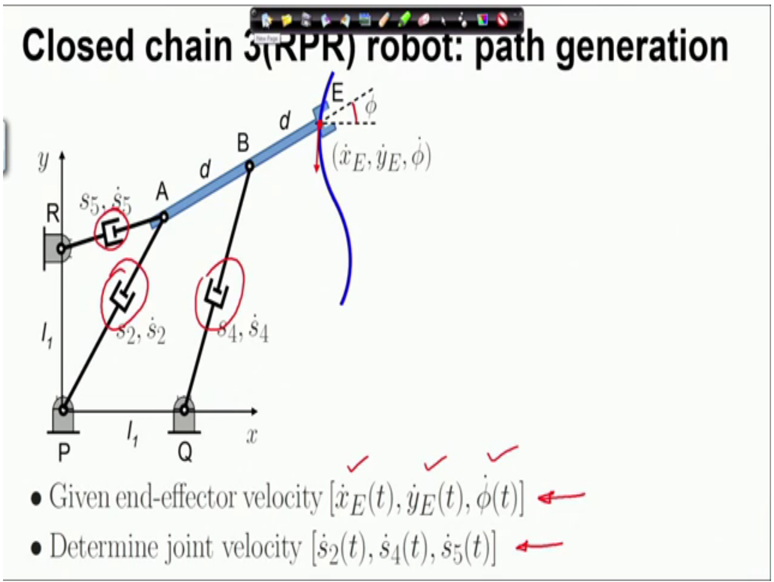

Now, let us look at the close chain 3 RPR manipulator, here this manipulator has 3 degrees of freedom, we are specified the velocity of the end-effector point, the velocity trajectory of the end-effector point though x E dot y E dot and phi dot.

So, phi dot so phi is the orientation angle and x E and y E are the positional coordinates in the ground based Cartesian coordinate system. What we need to find out is the actuator expansion rates. So, this vector S 2 dot S 4 dot S 5 dot so here at the actuators you need to find out their expansion rates for a specified end-effector trajectory or velocity. So, ones given the endeffector trajectory we can always determine or calculate the end-effector velocity.

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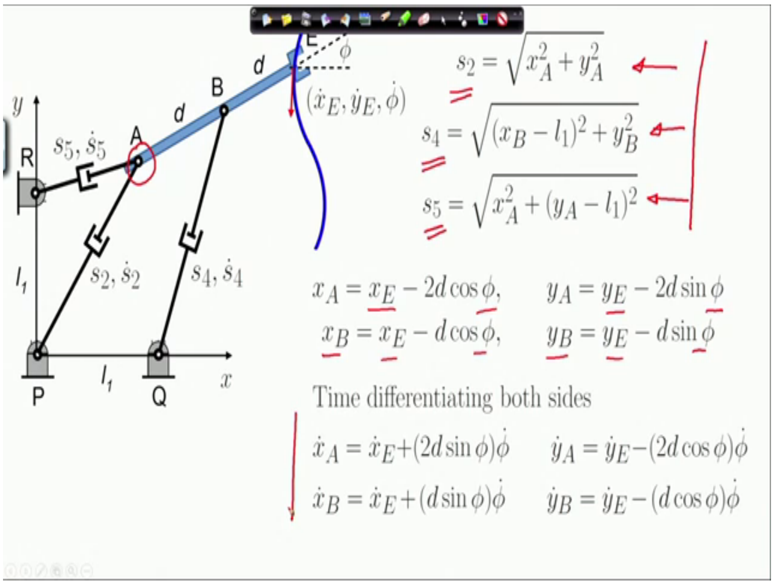

So, let us recapitulate what we are discussed about the velocity analysis problem for this manipulator, we have these displacement relations in terms of the coordinates of point A. So, here we have point A, so we have related the coordinates of point a in terms of the end-effector coordinate and the orientation angle phi. Similarly the coordinate x B y B they are related again to the end-effector coordinates and phi.

Now if you time differentiate these relations you will get the velocity relations which you use in this set of 3 equations which relate to the actuator expansion. So, once I differentiate these relations with respect to time and use the velocity relation that I have here.

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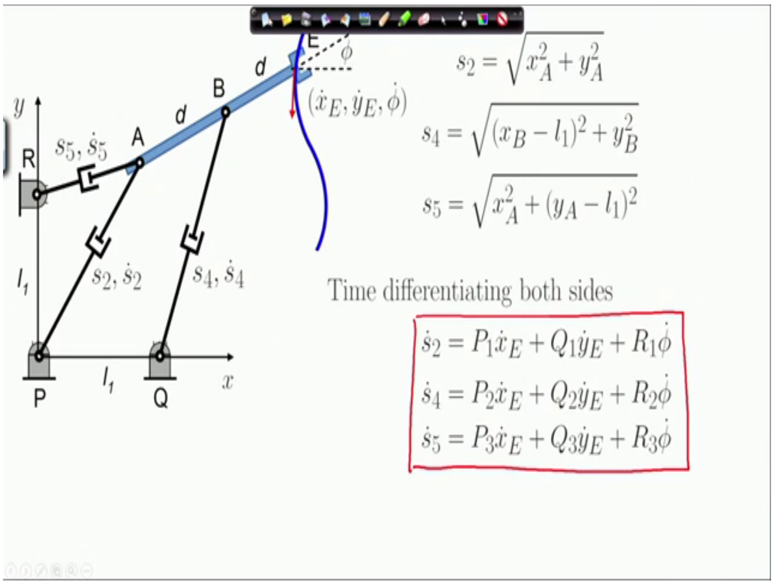

Then I obtain the velocity relations between the end-effector velocity vector and the actuator expansion rates.

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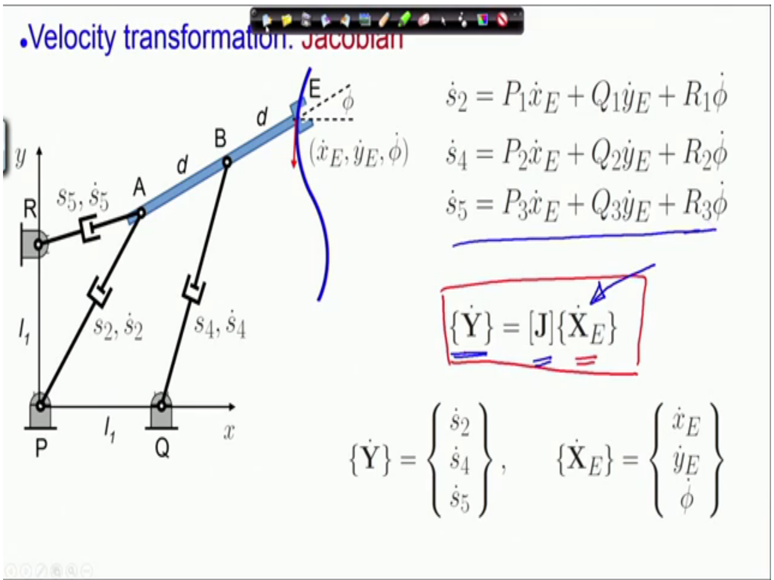

So, here we are defined the Jacobian which relates the actuator expansion rate and the end-effector velocity vector. Here this is quite straight forward because we will have the end-effector velocity vector already specified through the path. So, this vector on the right X E dot will be specified this is known to us, given the path we can find this vector once we plan the time of motion. Therefore, we can directly find out the expansion rates of the actuators. Now here again the Jacobian which might look very simple, but these P1 P 2 P 3 Q 1 Q 2 Q 3 and R1 R2 R3 they have certain denominator terms which might go to 0 and make the Jacobian singular.

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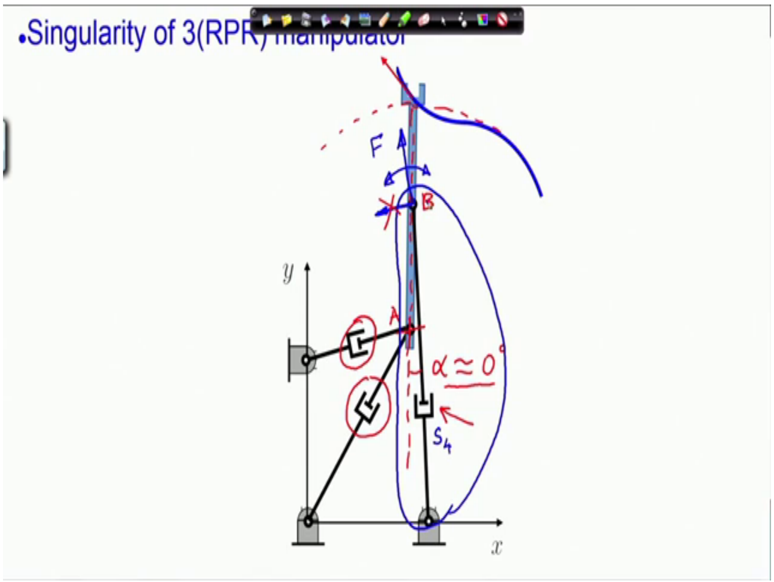

So, let us look at one of these singular configurations, here I have drawn a singular configuration or a configuration very close to singularity, this manipulator is very close to singularity because, the output link and this actuator angle, this angle let me call this alpha is very close to 0. Now why this is singular? Because when alpha is exactly 0 it is a singular configuration and this configuration that I have drawn is very close to singularity.

Now why this configuration is closed to singularity or when alpha become 0 why is this singular? The reason is imagine that I have fixed these 2 actuators. So, this point gets fixed this point gets fixed, now this output link can only rotate about point A point A is fixed. Now this end-effector link can only rotate about point A; now who prevents the rotation? This actuator this set of links, this should be able to constrain the output link so that it cannot rotate.

However, when alpha is 0, when alpha becomes 0 this part this leg of the manipulator remember it can only apply force along the actuator, this leg of the manipulator can apply force only along the actuator, it cannot apply any force perpendicular to the actuator. In other words this actuator cannot prevent any rotation of, it cannot prevent any rotation of the output link about point A.

So therefore, when alpha is equal to 0 the rotation of the output link cannot be prevented by this actuator S 4. So therefore, there will be some rotation so the manipulator gains a degree of freedom, why because it can rotate though is very small amounts there is a possibility of very small rotations, you cannot constrain the output link using S 4 when alpha is equal to 0.

So therefore, this is the typical gaining of degree of freedom in mechanisms at singular configuration which we have seen, even for constraint mechanism it gains degrees of degree of freedom at the dead centre or singular configuration. So, this is precisely the situation here, so we have a singular configuration of a parallel manipulator at the dead centre or singular configuration of the manipulator.

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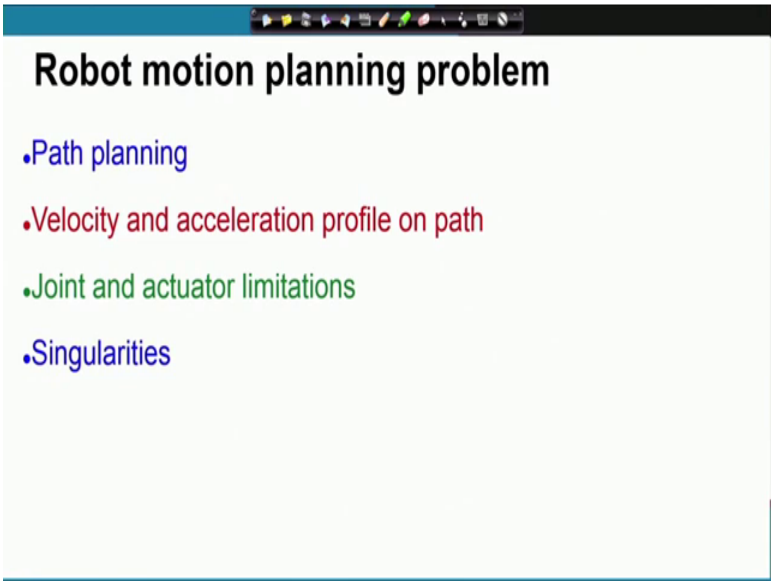

So, we have looked at the path planning problem. So, once you have the path which is planned based on obstacles that might be there you plan a path of the endeffector, you determine the velocity and acceleration profile on the path, you may have joint and actuator limitations. So, joint limitation because of physical construction actuator limitations because, it may not the actuator may not be able to produce very high velocity or very high acceleration.

Remember that since we are determining the joint velocity based on the end-effector velocity. So, the joint velocity can change very quickly very close to the singular configurations as we have seen. So, we can have very high joint velocities and therefore we will require very high acceleration, if you require very high acceleration then production of very high acceleration depends on the torque restrictions of the torque or force restrictions of the actuators.

So, near the singularities the motion planning is tricky. So, we will have very high actuator velocities and accelerations. So, you need more considerations or restrictions on the joint motion near singularities. Now once you have restrictions you will have errors on the path, so initial path planning has to be done carefully.

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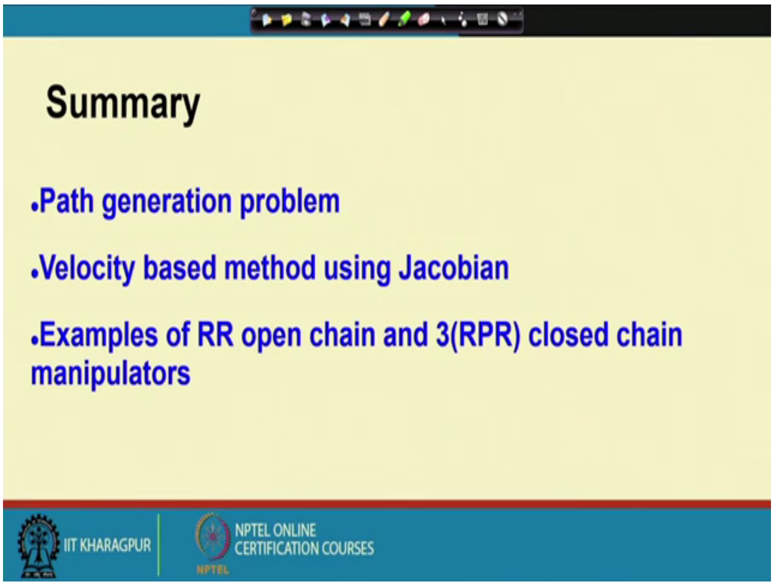

So, to summarize we have looked at the path generation problem using the velocity analysis problem, we have looked at 2 examples 1 is of the open chain 2RR manipulator and the other is the closed chain RPR manipulator. So, with that I will conclude the lecture.