Kinematics of Mechanisms and Machines
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Lecture – 30
Robot Velocity Analysis – III

We have been discussing about the Velocity Analysis of Robot manipulators. We have already discussed open chain robot manipulators, we have looked at the Jacobian; it's a interpretation and singularity of the Jacobian. We have looked at the consequences of singularity of the Jacobian.

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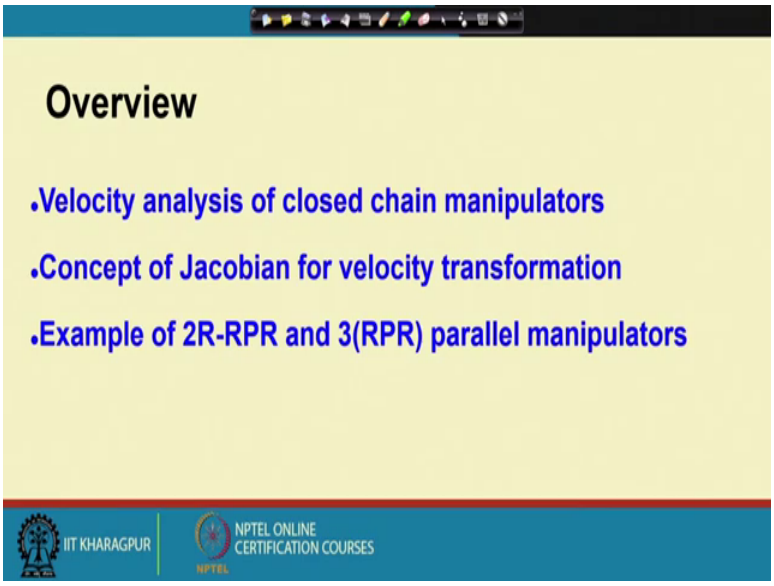

In this lecture we are going to embark upon discussions on closed chain manipulators. So, we are going to look at velocity analysis of closed chain manipulators and I will take 2 examples the 2 R-RPR and 3 RPR parallel manipulators; these are planar manipulators. We have already looked at the displacement analysis of both these parallel manipulator chains.

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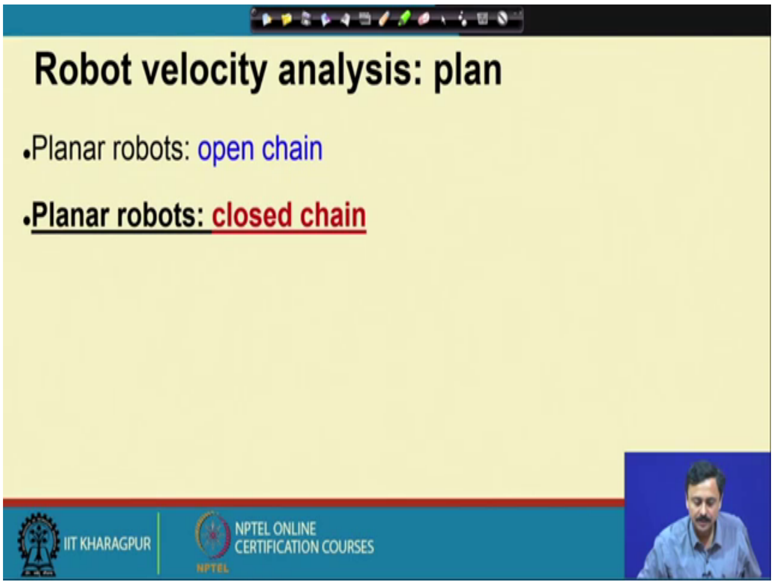

So, in this lecture we are going to look at closed chain planar manipulators.

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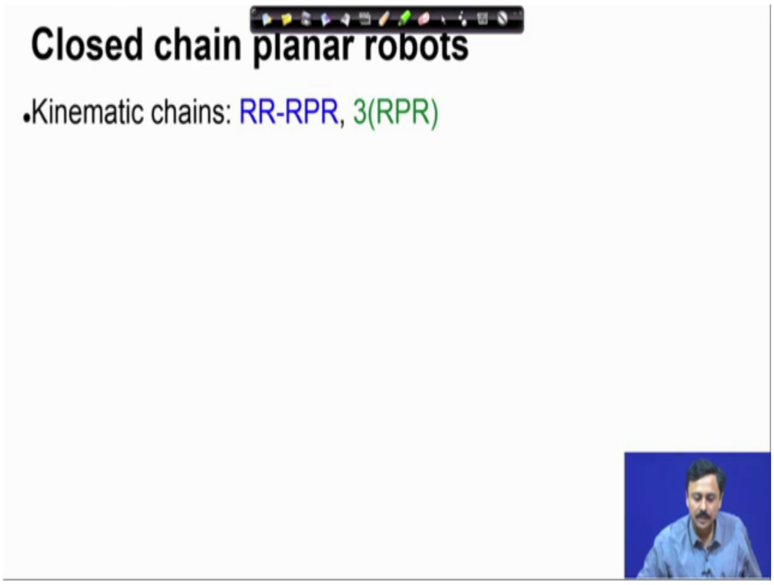

Going to discuss this RR-RPR and 3 RPR chain.

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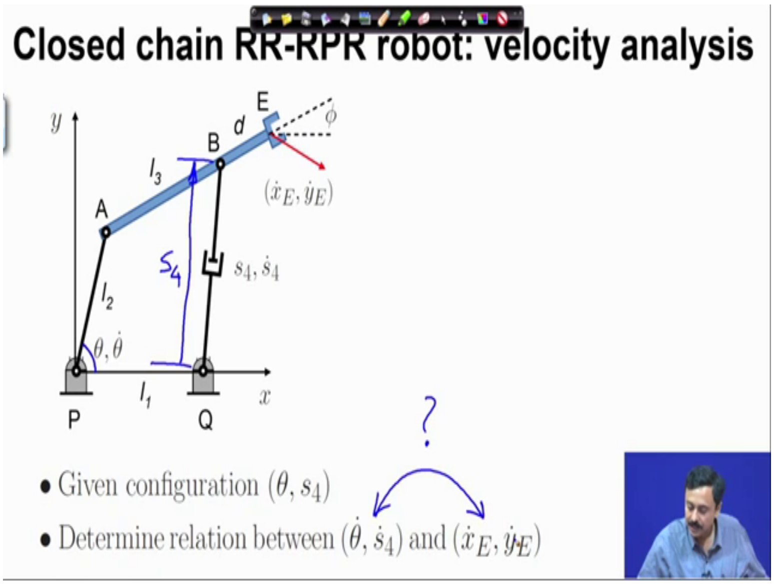

So, let us begin with the RR RPR planar closed chain robot. So, in this manipulator the configuration is specified by this angle theta and the extension of this prismatic pair which I have called s 4. So, theta and s 4 they specify the configuration of the manipulator; now the problem is to determine the relation between their velocities. So, we have theta dot and s 4 dot as the joint velocities or the actuator velocities and x E dot and y E dot as the end effector velocities; the velocity vector, so we would like to find out a relation between them.

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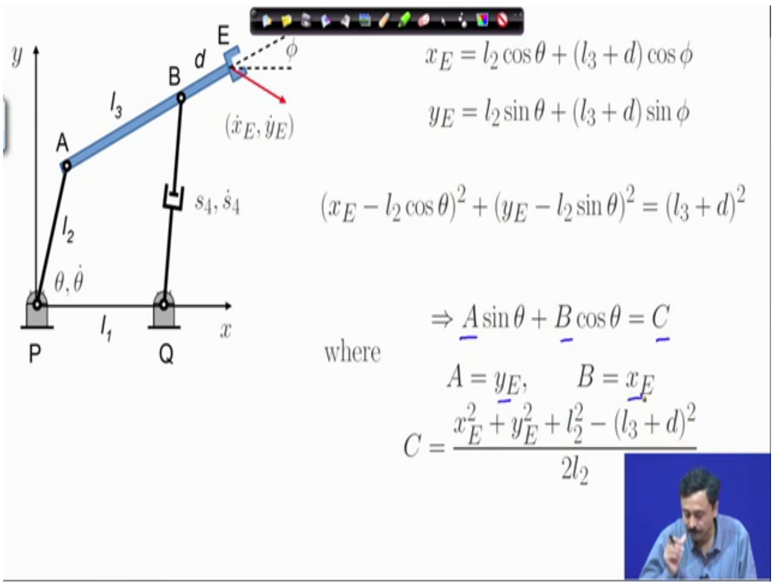

Here I have written out the displacement relations which we have already discussed in a previous lecture. The angle phi; the angle phi is this configuration, this orientation of the end effector; we have also seen in the displacement analysis that given theta and s 4 we can find out x E y E and phi.

So, we can find out everything; now if you eliminate phi between these 2 relations, we ultimately arrive at relation involving x E y E and theta. Here this A B and C they are functions of x E and y E; you can interpret this as given x E and y E; its a relation for theta its an equation from where we can solve for theta.

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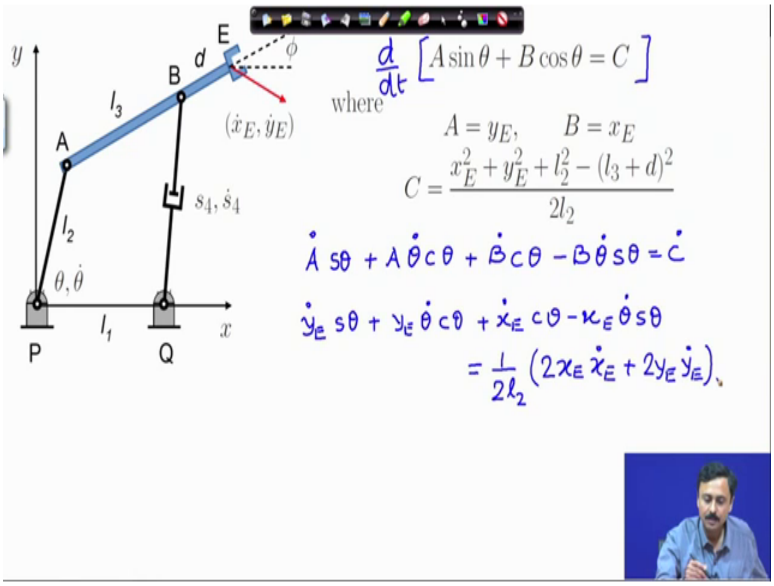

So, given x E and y E we can solve for theta. Now what we are going to do with this relation is that we are going to differentiate this with respect to time. So, this relation we are going to differentiate with respect to time. Then what we have is A dot sine theta S theta represents sine theta plus A theta dot cosine theta plus B dot cosine theta minus B theta dot sine theta is equal to C dot.

Now, if you use these expressions of A B and C; then you have these relation this is equal to, now I can collect terms of x E dot y E dot and theta dot. From here I can collect terms of x E dot y E dot and theta dot and write in a simplified form which I will show you directly.

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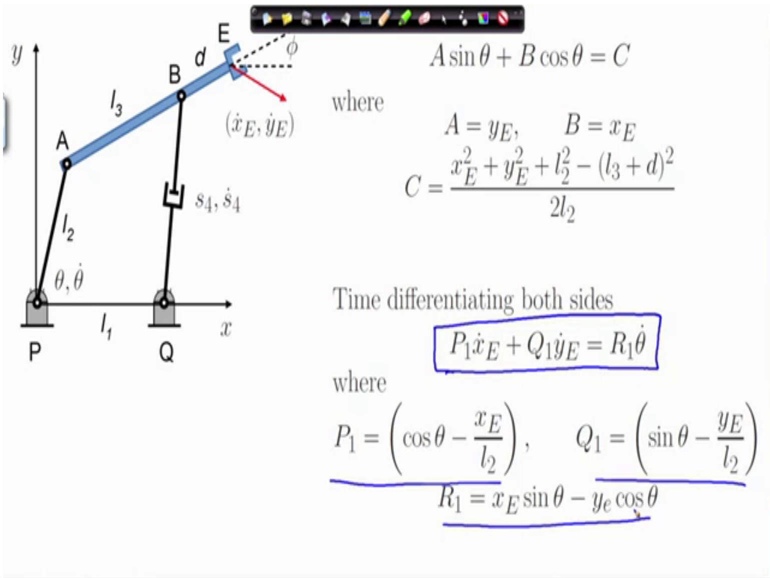

So, I obtain a relation like this by collecting terms of x E dot y E dot and theta dot. So, here P 1 is this expression Q 1 and R 1 which you can very easily derive by simplifying the steps that I have shown you. So, this equation relates x E dot y E dot and theta dot, but this is not enough I need one more relation, I must also relate s 4 dot to x E dot and y E dot. So, let us move on. So, here I have one relation next I must bring in s 4 dot.

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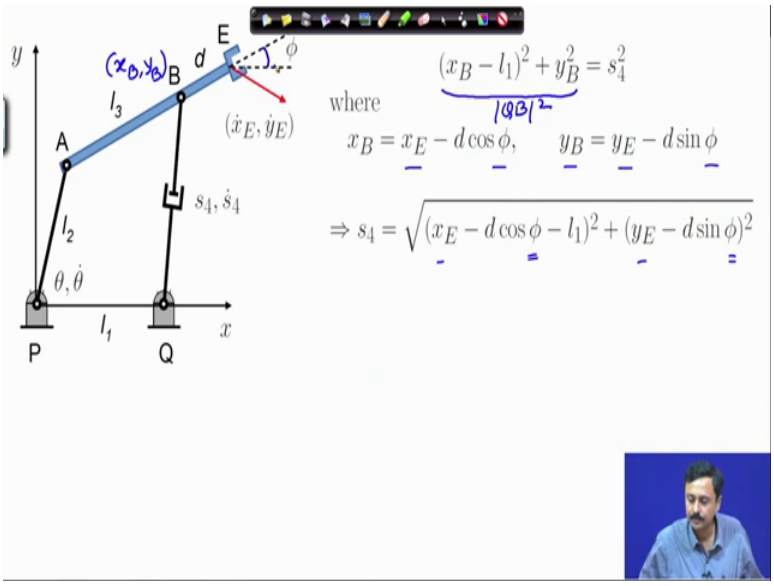

So, that is what we are going to look at. Now from the previous expressions; so, if you look at the coordinates of point B. So, coordinates of point B; x B and y B and the distance between B and Q is nothing, but s 4. So, therefore, the distance square this is the distance square. So, this is QB square is equal to s 4 square and x B can be related to x E and phi; similarly y B can be related to y E and phi. So, if you use these expressions s 4 can be related to x E y E, but you also have phi. Now, phi remember is the orientation angle of the end effector.

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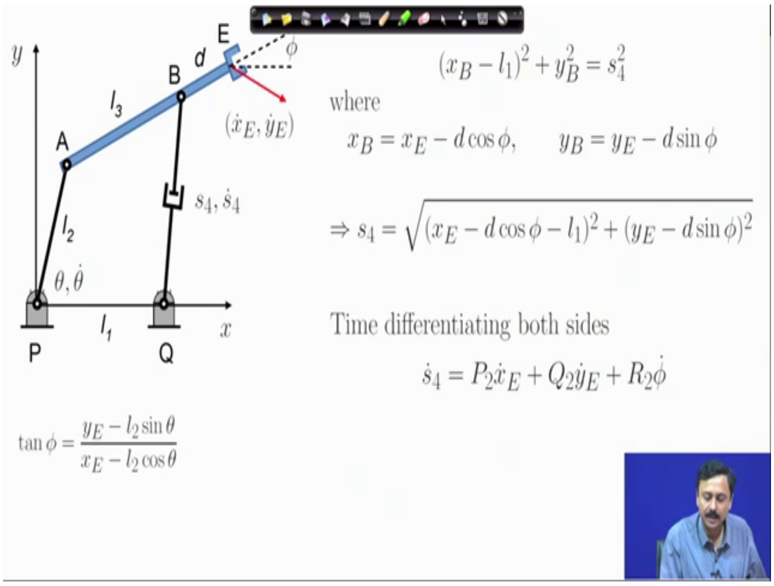

Now, if you differentiate with respect to time both sides of this equation, then you can imagine that you will have derivatives of x E, you will have derivatives of phi and you will have derivatives of y E. And the coefficients corresponding coefficients are P 2 Q 2 and R 2 which can be very easily found by differentiating this.

And this is going to be little complicated, but it is otherwise straightforward; I mean there will be a little bit of algebra involved. So, the steps are straightforward if we just differentiate the expression of s 4 with respect to time; now we have involved phi. So, let us see how we can find out phi and we have also involved remember this cosine phi and sine phi; we also have this cosine phi and sine phi as well as phi dot.

So, this P 2 Q 2 R 2 these will be functions of x E y E and phi which are the configuration which specify the configuration of the manipulator ok. Now this tan phi you can very easily write tan phi in terms of theta. So, tangent of this angle comes from the displacement relations that we have looked at before.

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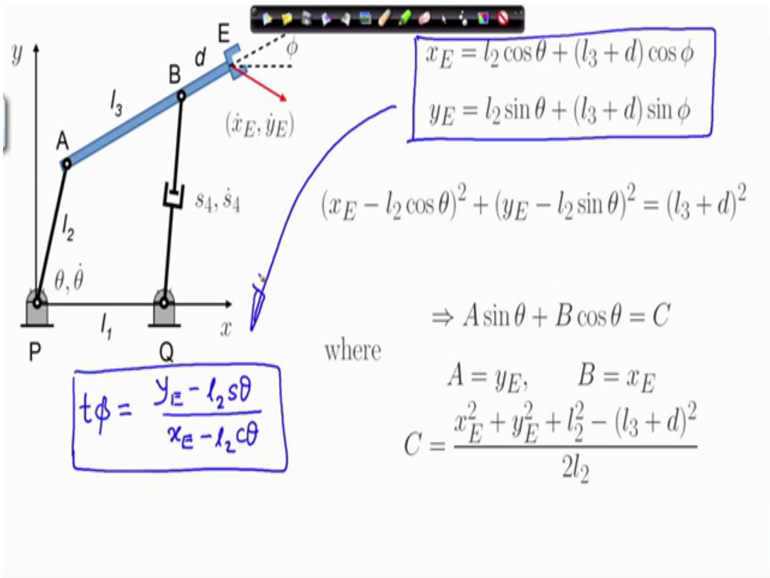

So, these are the relations; you can find out tangent phi. So, tan phi is as equal to y E minus l 2 sine theta by x E minus l 2 cosine theta right; from here we can obtain tangent phi and this is what we are going to use.

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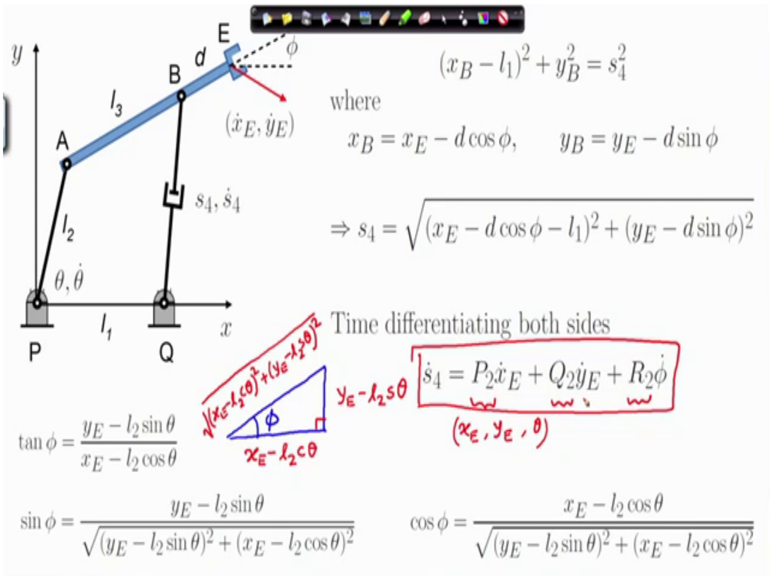

Here, so this is the expression of tan phi. Now once I have tan phi I can also find out cosine phi and sine phi. So, sine phi is this expression, so, how do I find this? Suppose this angle is phi then tan phi is this ratio.

So, if I say that this is y E minus l 2 sine theta and this base of the triangle is x E minus l 2 cosine theta; then you can very easily see that tangent phi is this ratio. So, this hypotenuse of this right angle triangle is nothing, but square root of the base square and the height square. Therefore, you can relate sine phi and cosine phi which I have written out. Now in these relations what I have done therefore, is I have replaced sine phi and cosine phi in terms of x E y E and theta.

So, once I have done that these terms P 2, Q 2 and R 2 they are only functions of x E y E and theta. P 2 Q 2 and R 2 are functions of x E y E and theta and this relation is between the velocities s 4. So, s 4 dot x E dot y E dot and phi dot; I have eliminated phi. The next step is to eliminate phi dot, then I would have a relation between x E dot y E dot and s 4 dot; now how to determine phi dot.

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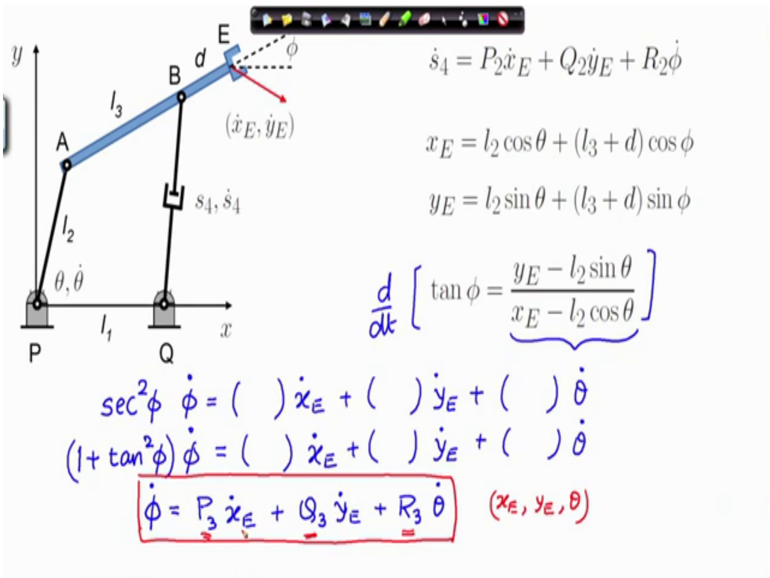

So, here I have re written that expression and I have also rewritten the displacement relation and this expression of tan phi. Now if I differentiate with respect to time I have second square phi into phi dot is equal to the time derivative of the right hand side of this ratio.

Now, if you think a little bit; it will be absolutely clear that the time derivative of this ratio will have a form or can be expressed in the form something times x E dot plus something times y E dot plus something times theta dot. Now this time derivative is straightforward, but there will be a little bit of algebra in this. Now second square phi can be again written in terms of tangent phi. So, I have this relation; now tangent phi is already known in terms of x E y E and theta.

So, therefore, finally, I must have let me call it some P 3 x E dot plus Q 3 y E dot R 3 theta dot; where remember that P 3 Q 3 and R 3 are solely functions of x E y E theta. So, these are solely functions of x E y E and theta. So, I have found phi dot in terms of x E dot y E dot theta dot x E y E theta; where this P 3 Q 3 R 3 are complicated expressions which you can easily find out by a little bit of algebra.

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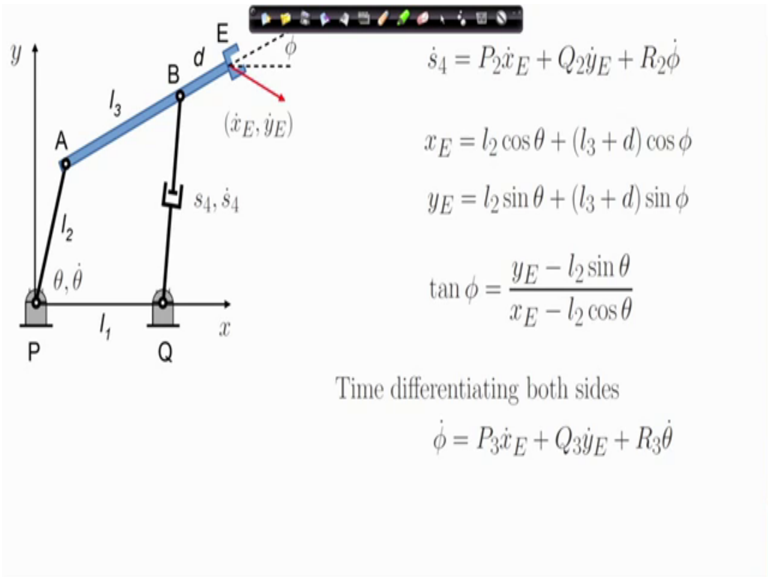

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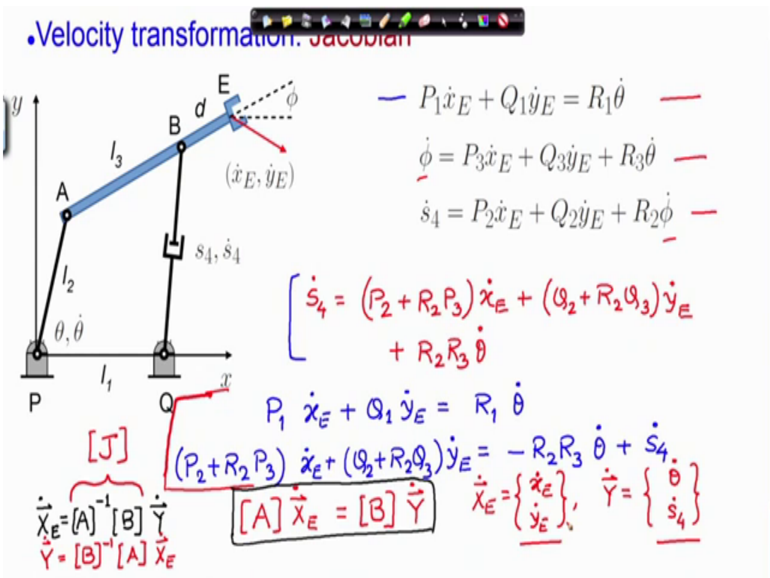

So, here I have collated all the expressions that we had derived; we have derived this first and then these two expressions were derived. Now here I have phi dot which appears in the expression of s 4 dot. So, therefore, I can eliminate phi dot in the expression of s 4 dot and write this as; so, when I substitute phi dot here I will get this times x E dot plus this term times y E dot plus R 2 R; R 3 times theta dot.

Now I can rewrite this as; so let me write out the first equation here. So, P 1 x E dot plus Q 1, y E dot is R 1 theta dot. And now I will write this equation after a little bit of manipulation as that is equal to minus of R 2 R 3 theta dot plus s 4 dot. Now this I can combine and write in a compact form as let us say some matrix A times X E dot is equal to matrix B times the vector Y dot, where X E dot of course, this vector X E dot is nothing, but x E dot y E dot the vector Y dot is theta dot s 4 dot.

So, the joint velocity vector and the matrices A and B; you can very easily read out from these two equations; so, this is our velocity relation. Now it looks a little different than what we had derived earlier, but you can always invert A and rewrite this as; such that this is our Jacobian for the manipulator. So, A inverse B is the Jacobian for this manipulator; if you invert B then you have the inverse Jacobian. So, if you write Y dot as B inverse times A X E dot, then you have the inverse Jacobian.

So, we have finally, determined the velocity relations. So, our joint velocity vector is theta dot s 4 dot. So, from here I have related I found a relation to x E dot and y E dot which is the end effector velocity.

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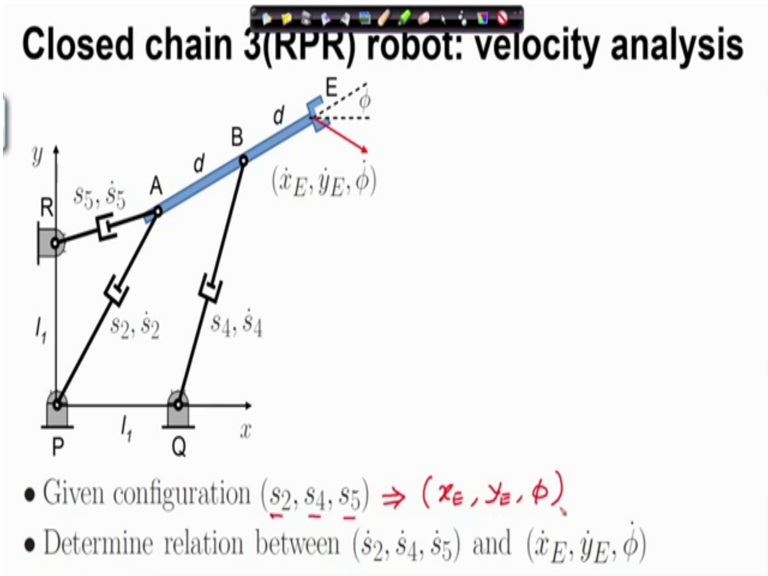

So, let us move forward and discuss the 3 RPR planar closed chain robot manipulator. Here we need to recall that we had already discussed the displacement analysis the configurations s 2 s 4 s 5. So, given s 2 s 4 s 5; the configuration of the manipulator is specified and we have we had found x E, y E and phi.

So, given s 2 s 4 and s 5 you can find out x E y E and phi; so, these are 3 degree of freedom manipulator. So, you have 3 inputs s 2 s 4 and s 5 and we can find out x E y E and phi. Now the velocity analysis requires the specification of the relation between s 2 dot s 4 dot s 5 dot and x E dot y E dot and phi dot. So, that is the velocity analysis problem.

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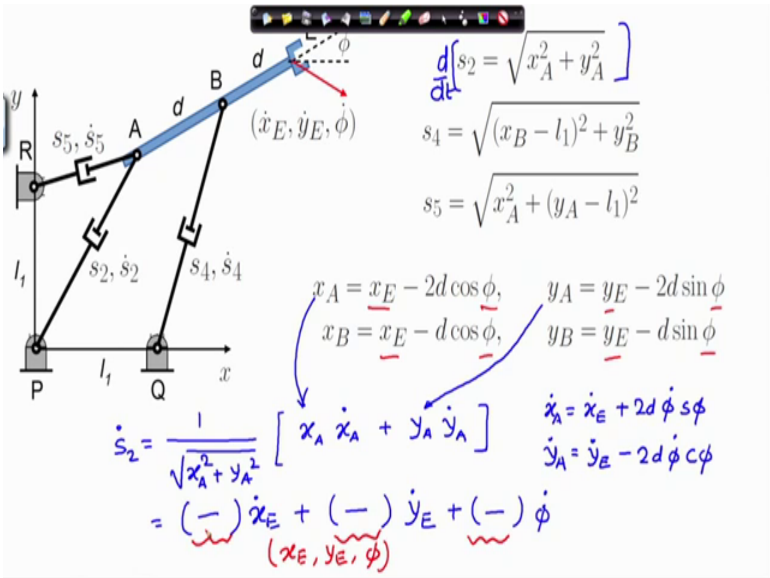

Now, we have derived these relations; these are very easy to see s 2 is the throw of this actuator. Similarly s 4 and s 5 and they are related in terms of the coordinates of point A and B and the coordinates of P Q and R.

So, these relations we have already looked at. So, here you can relate x A y A; so coordinates of point A and similarly coordinates of point B. In terms of the coordinates of the end effector and the orientation of the end effector phi; so therefore, you can just substitute them here and start differentiating. So, for example, if you differentiate the first equation, which I have written out after some simplification; so this is what you have.

And now you can replace x A y A in terms of x E y E and phi and you can find out x A dot as x E dot plus 2 d phi dot sine phi. Similarly y A dot is y E dot minus 2 d phi dot cosine phi; so these also you can replace. So, finally, S 2 dot will be something times x E dot plus something times y E dot plus something times phi dot. Now these bracketed quantities can be easily determined after some algebra; you substitute these expressions of x A y A x A dot and y A dot and you can find out this relation.

Now this bracketed quantities are functions of only x E y E and phi. So, this bracketed quantities are functions of x E y E and phi. So, if you know the configuration of the manipulator; these things are already known to you. So, which means these bracketed quantities are known to you; in that case I have found a relation between the expansion rate of s 2 which is s 2 dot and the velocity of the end effector which is x E dot y E dot and phi dot.

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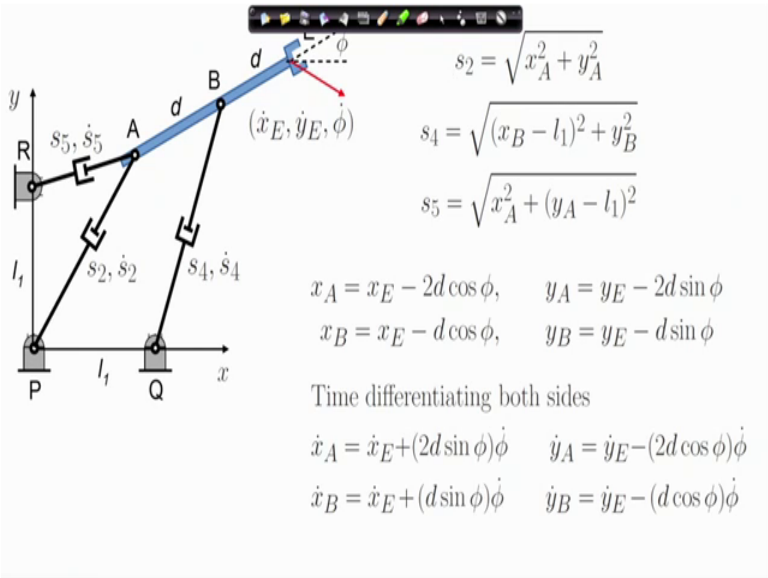

Now, if you continue. So, this is the time derivative.

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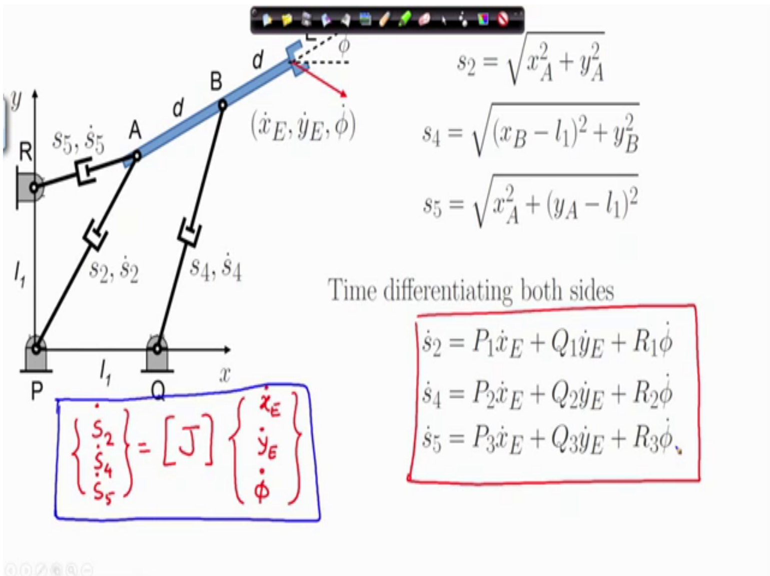

So, if you substitute all these expressions; then finally, you obtain these relations between s 2 dot s 4 dot s 5 dot and x E dot y E dot and phi dot. So, which means I have s 2 dot s 4 dot S 5 dot is equal to a Jacobian matrix times x E dot y E dot and phi dot and that is the velocity relation that we are seeking; the Jacobian can be easily read out from these detailed relations.

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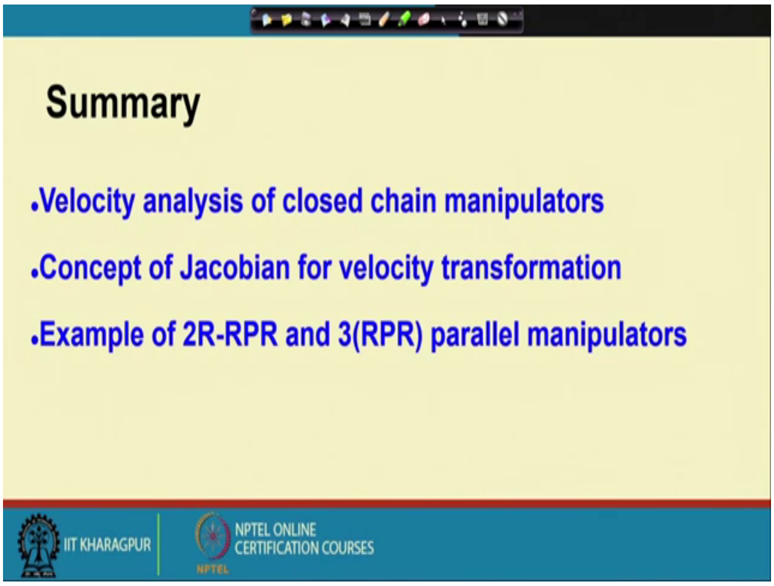

So, to summarize what we have discussed in this lecture; we have looked at the closed chain parallel planar manipulators. We looked at the velocity analysis problem, we have found the Jacobian of the Jacobian matrix for these manipulators. I have outlined methods for determining this Jacobian. We have looked at these 2 examples of 2 R RPR and 3 RPR closed chain planar a parallel manipulators.

So, with this we have now discussed the velocity analysis of both open chain and closed chain manipulators. Now this paves way for discussions on motion planning or path planning; path generation problem for these manipulators. So, this lecture I will close here.