Kinematics of Mechanisms and Machines
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Lecture - 28
Robot Velocity Analysis – I

In this lecture I am going to discuss the Velocity Analysis of planar robot manipulators open chain.

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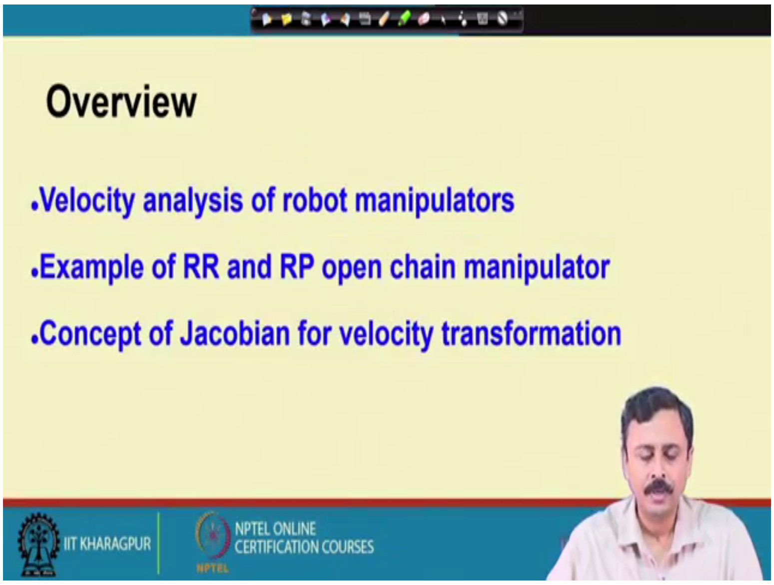

So, we are going to look at the velocity analysis problem for robotic manipulators. I am going to look at two examples RR and RP open chain manipulators and I will also introduce the concept of the Jacobian for velocity transformation.

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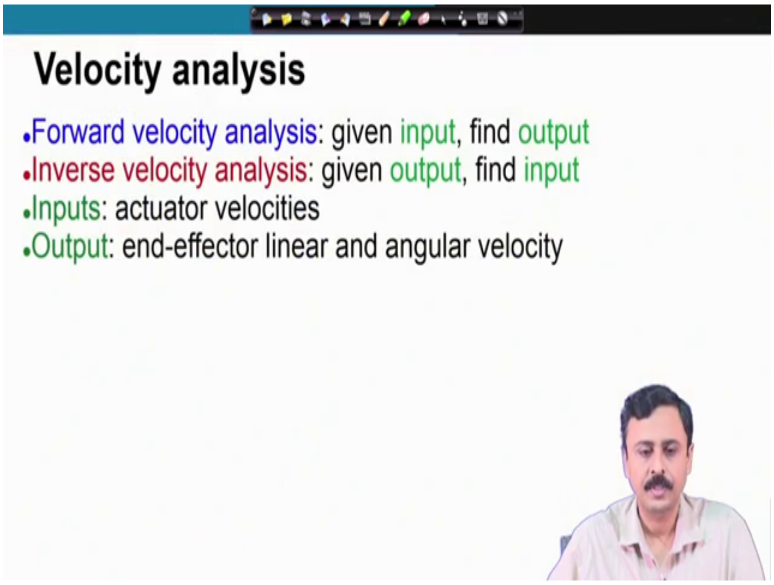

Now, as we have discussed before the forward velocity analysis problem is about finding out the velocity of the output link in the case of robotic manipulators this is the end effector. So, the velocity vector at the end effector the orientation rate and of the end effector. In terms of the input velocity which are the actuator rates. In the case of rotary motor it will be the angular velocity, in the case of linear motors it is going to the expansion rate. The inverse velocity problem is about finding out the expansion rates or the actuator angular velocity rates. So, given for a given output velocity at the end effector.

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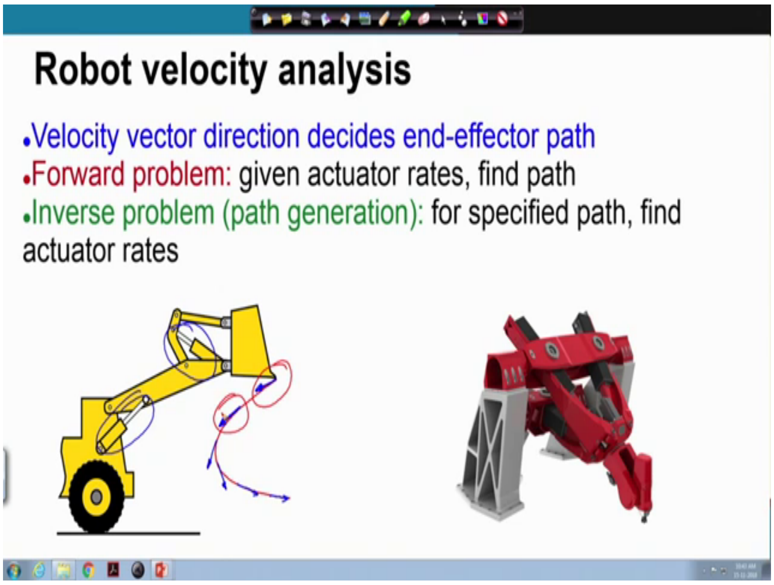

Now, this velocity problem for robots is intimately connected to the motion planning or path planning of the robot here let us say this excavator it has to follow this path. So, at each point of the path the velocity vector is going to be tangent to the path. Now as the bin moves on this path its velocity is decided by this tangent vector, now given this position of the bin and given this tangent vector what should be the actuator expansion rates. So, that this velocity is achieved at this point this velocity is achieved at this point. So, as the bin moves in successive positions how should you change the actuator expansion rates.

So, this comes from the inverse velocity analysis problem. So, this is the path generation problem the forward problem of course, is easier in which we are given the actuator expansion rate and we have to find out the path. So, essentially at every point we know the velocity vector given the actuator rates we know the velocity vector, if I know the velocity vector at a particular point we know how the bin or the tip is going to move and as I integrate that we are going to get the path of motion of that tip. So, this velocity analysis problem for robots is important from motion planning or path planning point of view as well.

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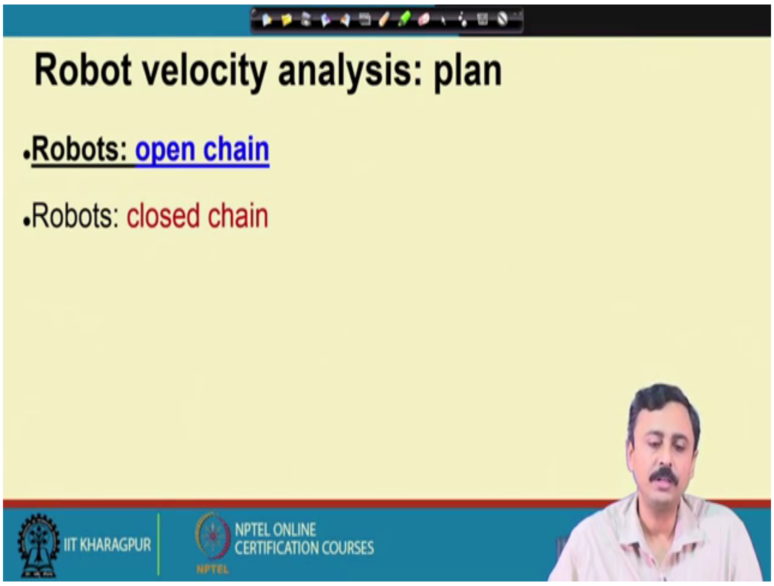

So, our plan for velocity analysis is as follows you are going to first discuss open chain robots subsequently we are going to discuss closed chain robots robotic manipulators.

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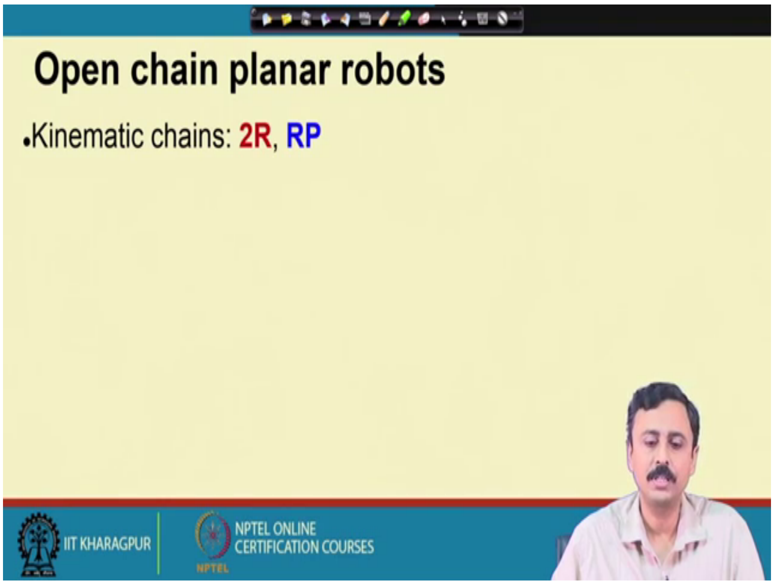

In this lecture I am going to discuss two kinds of chains; one is 2 R the other is RP.

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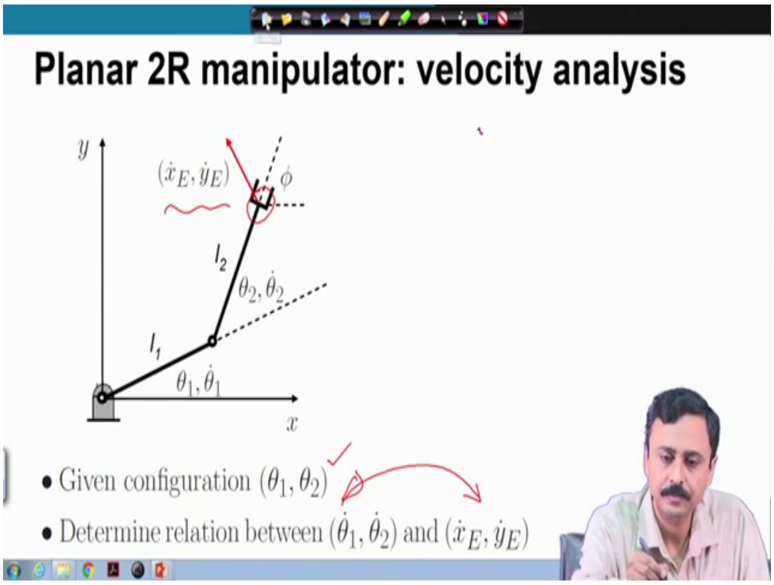

So, this is a 2 R manipulator we are given the configuration. So, configuration needs to be given this comes from the displacement analysis we have to find out a relation between the joint rates the joint angular rates and the end effector velocity vector. So, the velocity of the point, this point of the end effector so, which is specified by this vector x E dot y E dot.

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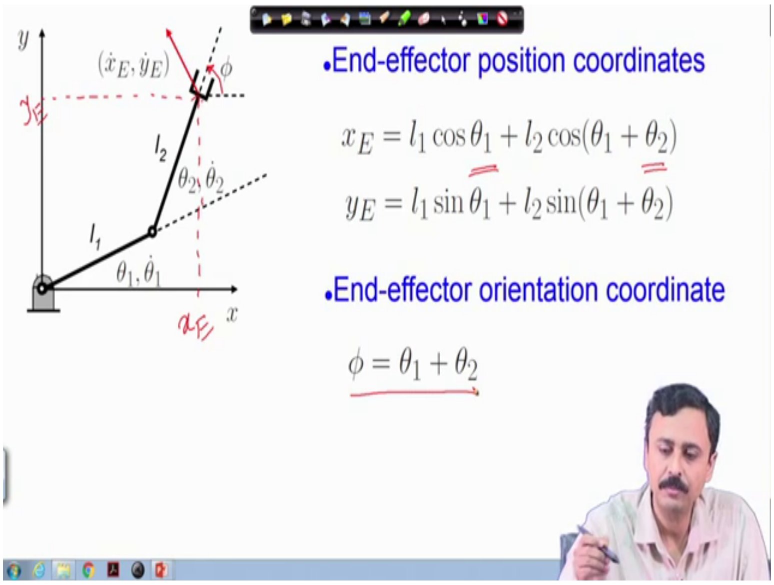

We start with the forward kinematic relations which we have discussed before. So, the end effector positional coordinates which are given in the coordinates shown as xE and yE and we had related this end effector coordinate in terms of the angles theta 1 and theta 2. Now, if you look at this angle phi which is the orientation of the end effector and that can be written as theta 1 plus theta 2.

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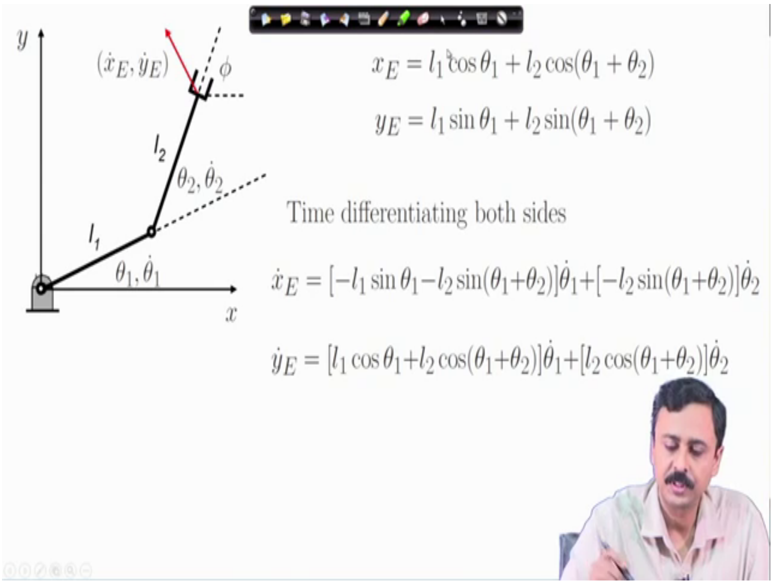

Now, starting with these forward kinematic relations if you differentiate with respect to time then you relate the rates at which xE is changing with the rates at which theta 1 and theta 2 will change similarly for yE.

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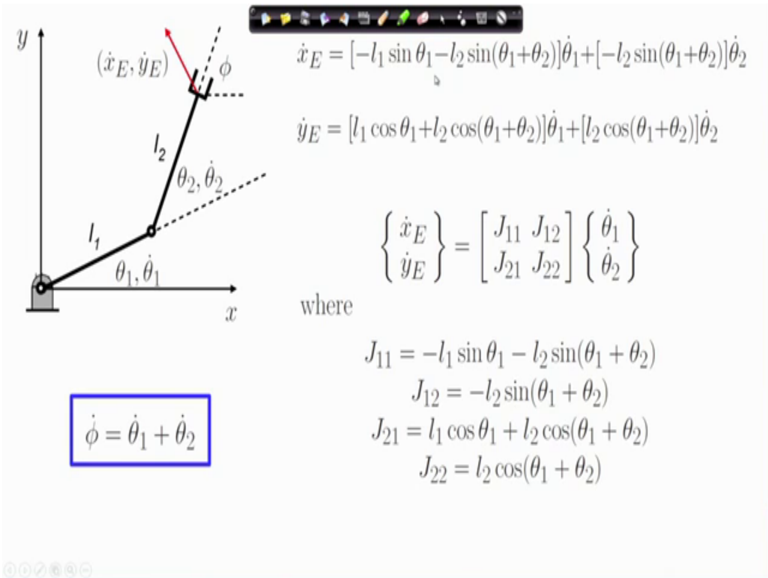

Therefore, we can assemble this and write it as a product of matrices. So, this is the end effector velocity vector and we have this matrix 2 cross 2 matrix whose elements can be read out from these relations and that gets multiplied by the joint rate vector. The orientation rate of the end effector is therefore, related by in terms of theta 1 dot and theta 2 dot that comes by directly differentiating the orientation angle with respect to time.

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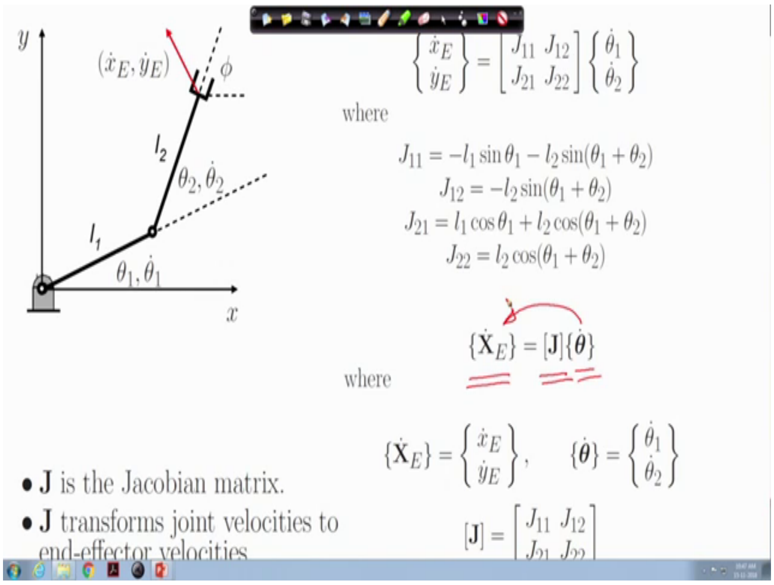

Now, this matrix J that relates the end effector velocity vector with the vector of the joint velocities is known as the Jacobian; the Jacobian matrix. Now Jacobian matrix therefore, transforms the joint velocities to the end effector velocity.

So, there is a relation this provides a relation between the joint velocities and the end effector velocities. Therefore, given the joint velocities at a particular configuration of the manipulator I can find out the end effector velocity vector and if I moves move these joints. So, subsequent velocity vectors and the joint angles will be known to me. So, joint as I move given the joint angular rates I can integrate very easily to find out the joint angles as it evolves with time and given the end effector velocity vector I can integrate to find out the displacement of the end effector and that is how we are going to determine the path.

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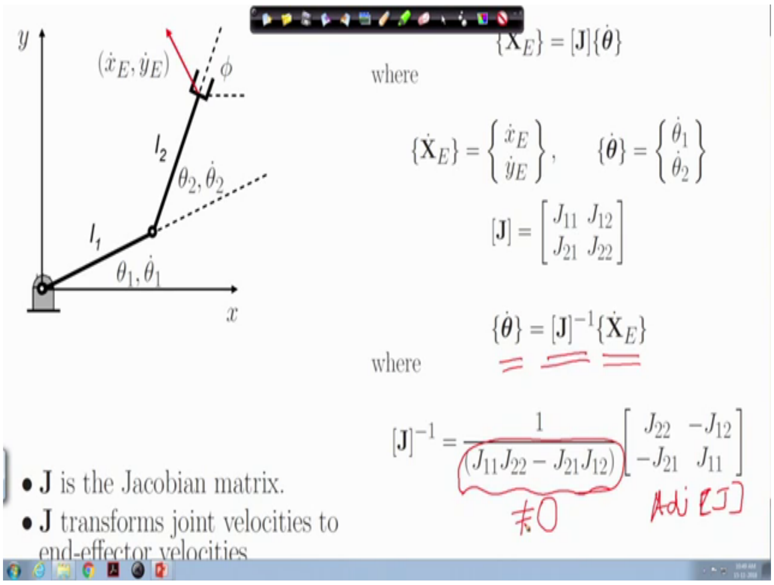

Now, given so, so that is the forward problem now given the end effector velocities how to find out the joint velocities? This is just by inverting the Jacobian matrix, but something has to be borne in mind in order to invert the Jacobian matrix which relates.

So, the inverse of the Jacobian matrix relates the end effector velocity vector with the joint velocity vector, but in order to invert that the matrix must be invertible which means the determinant of this matrix which sits in the denominator in this expression must be non zero only then we can invert this is the adjoint of the Jacobian matrix.

So, the determinant of the Jacobian matrix must be non zero in order for us to be able to invert it and relate the end effector velocities with the joint velocities. Now these non singularity of the Jacobian matrix or singularity of the Jacobian matrix has certain implications which we are going to discuss in subsequent lectures.

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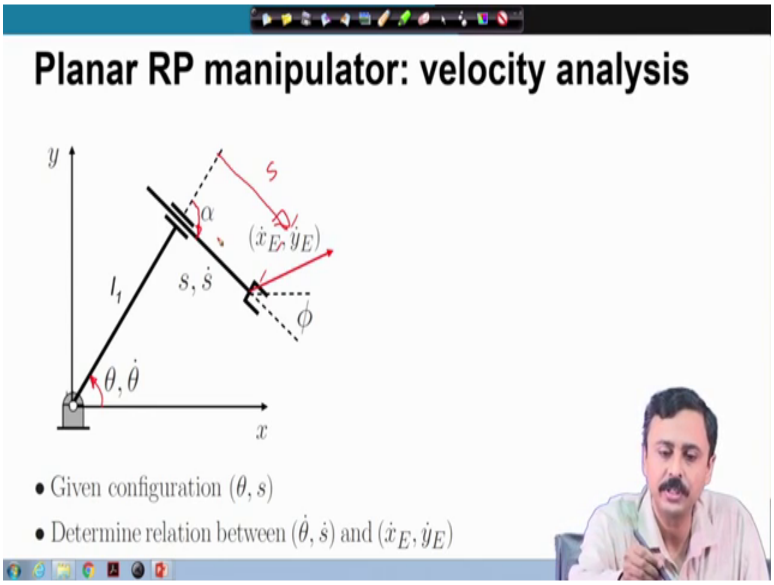

Next we are going to look at the RP manipulator in this manipulator we have this angle as our variable and the through of this prismatic actuator as the other variable. The angle alpha is fixed note that angle alpha is being measured in the clockwise sense therefore it is to be treated as a negative angle.

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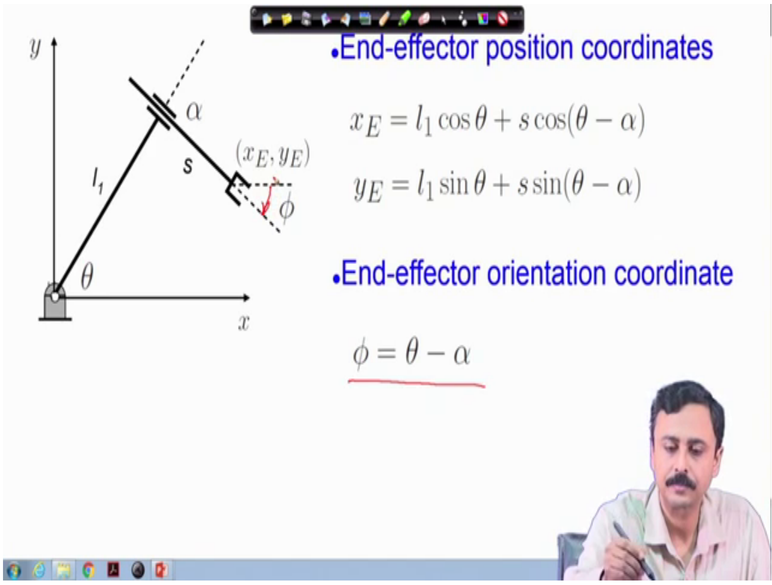

So, the end effector position coordinates xE and yE are related by these expressions which we have already discussed here as I mentioned this angle we in order to find out xE and yE. So, let us say the expression of xE it is l 1 cosine theta plus s which is this length cosine of theta minus alpha, this being the clockwise sense the alpha so, therefore, its theta minus alpha. Now if you differentiate this with respect to time this is the end effector orientational coordinate. So, here also you find that phi is theta minus alpha so, that is the orientational coordinate.

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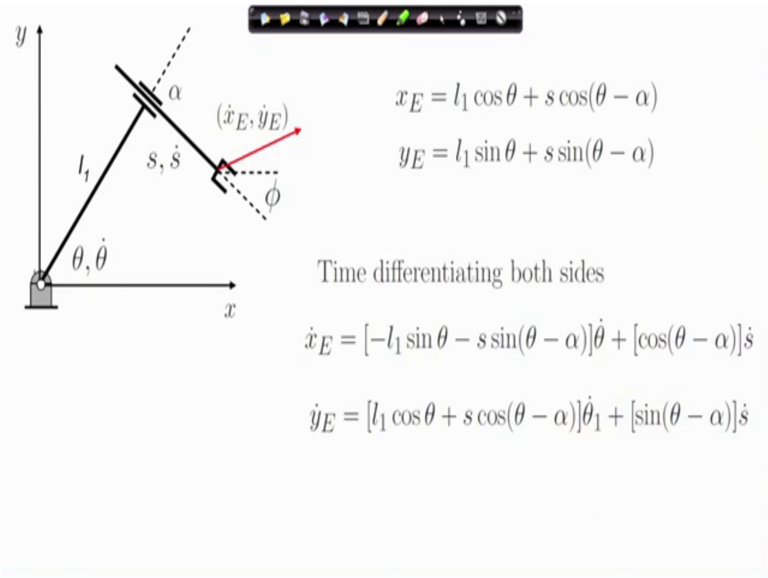

If you differentiate these expressions with respect to time we are going to get the velocity relations with the joint rates.

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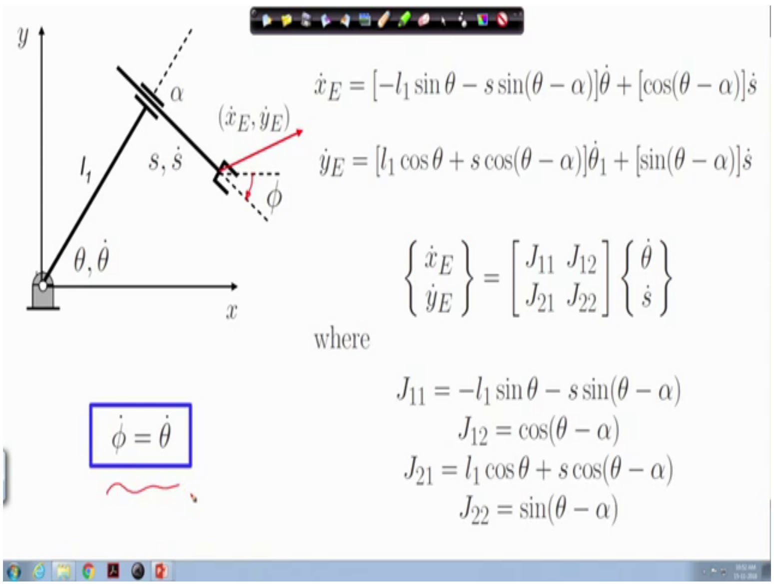

Here I have differentiated these expressions with this involves the joint angle rate and the expansion rate of the prismatic actuator and this can again be assembled in terms of this Jacobian matrix whose elements can be read out from the above expressions. Here the orientation rate of the end effector is just the joint rate of the manipulator this is because the angle alpha is a constant.

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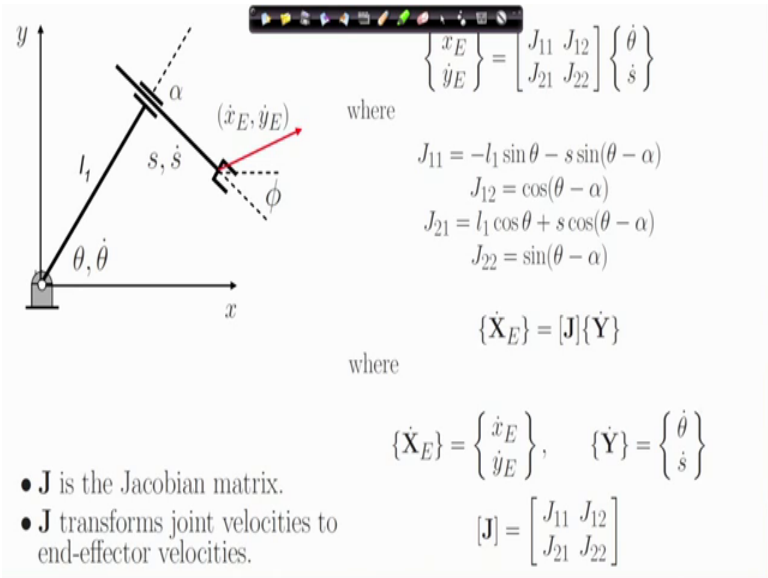

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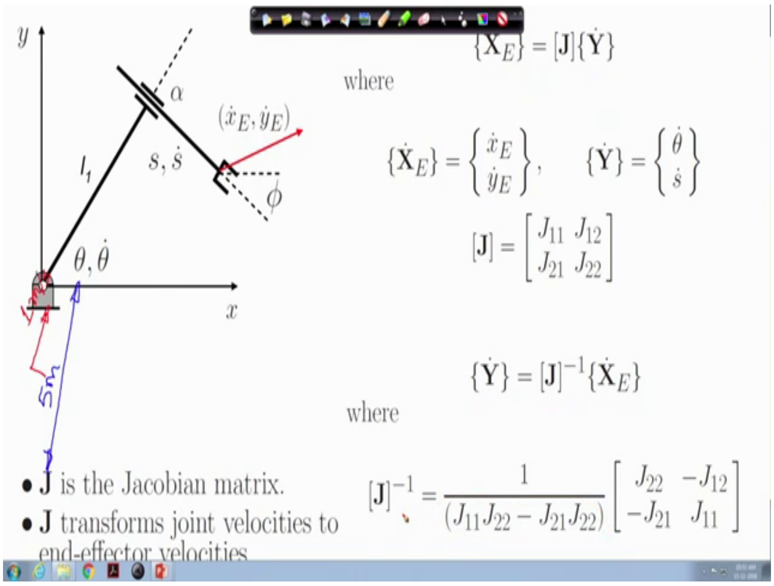

Therefore we can write out the Jacobian matrix. And if you want to find out if you want to relate the end effector velocity with the joint rates you require the Jacobian and if you want to relate the joint rates in terms of the end effector velocities then you need to invert this Jacobian. And once again the same consideration comes up the Jacobian must be invertible or in other words is determined must be nonzero. So, if the determinant is 0, then we cannot invert which means we are losing some degrees of freedom and this is what we are going to discuss in a subsequent lecture.

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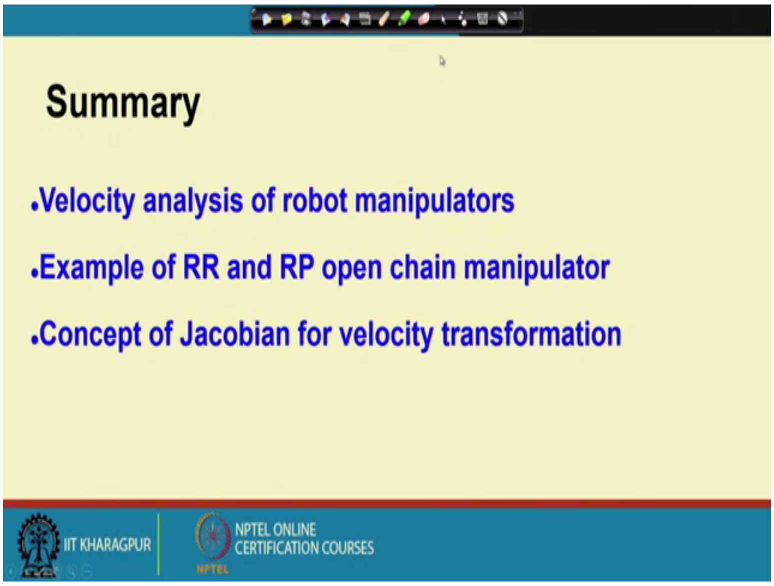

So, let me summarize you have discussed the velocity analysis of robot manipulators I have discussed about how the velocity analysis problem of robotic manipulators is intimately connected to the motion planning or the path planning of the manipulator which we are going to again look at separately in a future lecture. I have looked at the examples of RR and RP manipulators open chain planar manipulators and discuss the concept of Jacobian and how it relates the joint velocities with the end effector velocities.

And the inverse of the Jacobian which relates the end effector velocities and the joint velocities and the inversion of the Jacobian is key to determining the joint rates in terms of the end effector rates the consequences of an non invertible Jacobian or where the or under what conditions the Jacobian becomes non invertible this we are going to discuss in a future lecture. So, with that I will close this lecture.