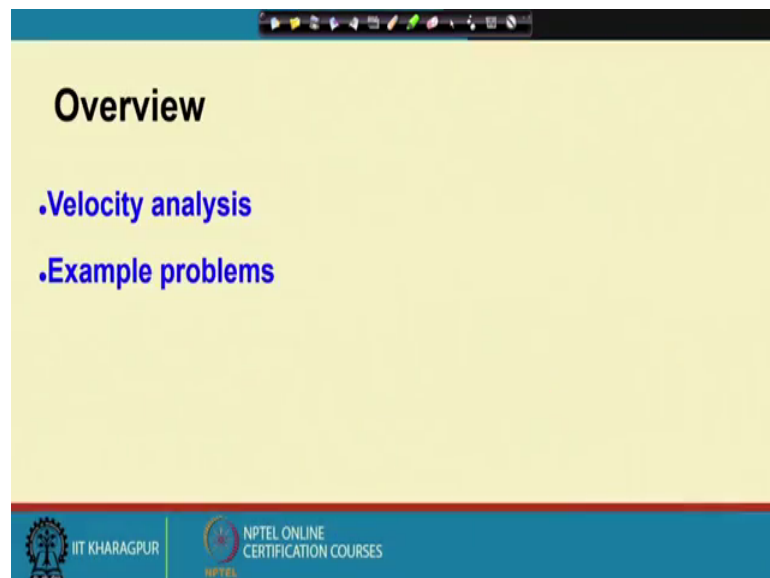


Kinematics of Mechanisms and Machines
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Lecture – 27
Velocity Analysis Examples

In this lecture, I am going to look at some examples related to Velocity Analysis.

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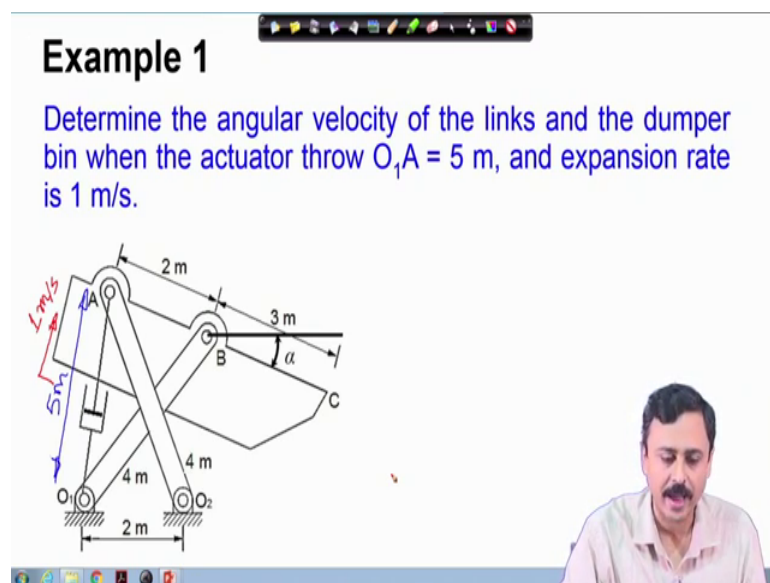


Overview

- Velocity analysis
- Example problems

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Example 1

Determine the angular velocity of the links and the dumper bin when the actuator throw $O_1A = 5\text{ m}$, and expansion rate is 1 m/s .

The diagram shows a mechanism with joints O_1 , A , B , O_2 , and C . Dimensions include 2 m , 3 m , 4 m , and 5 m . A velocity of 1 m/s is indicated at joint A . The angle α is shown at joint B .

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The first example is of the bin which we have lived before, we have discussed the displacement analysis of this bin. Here we have to determine the angular velocity of the links and the dumper bin when the actuator through O_1A , this is O_1A ; this is given as 5 meter and expansion rate is 1 meter per second. So, this actuator is expanding at 1 meter per second. So, throw is 5 meter and it is expanding at 1 meter per second you have to find out the angular velocity of the links and the dumper bin.

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Example 1

Define the vectors

$$\vec{O_1O_2} = l_1, \vec{O_1B} = l_2, \vec{BA} = l_3,$$

$$\vec{O_2A} = l_4 \text{ and } \vec{O_1A} = l$$

From displacement analysis

$O_1A = 5 \text{ m}$

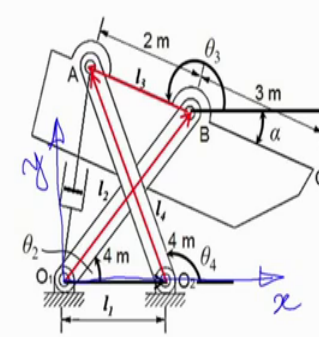
$\theta = 49.458^\circ, \theta_2 = 21.168^\circ, \theta_3 = 90.1^\circ,$
 $\theta_4 = 71.79^\circ.$

We have defined these vectors before during displacement analysis, let us just go through them again l_1 is O_1O_2 . So, this is the l_1 vector l_2 vector is O_1B l_3 vector is BA which is the coupler which is actually the bin O_2A is l_4 and O_1A is l O_1A is this vector.

So, this vector is the vector l ; from our previous discussions on displacement analysis for this given actuator throw we had found all these angles θ , the angle θ is this angle that angle is θ , that was 49.45 degree. And θ_2 , θ_3 , θ_4 we are all determined, θ_3 is the bin angle, but these were determined from the displacement analysis.

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Example 1



From O_1O_2A chain we had

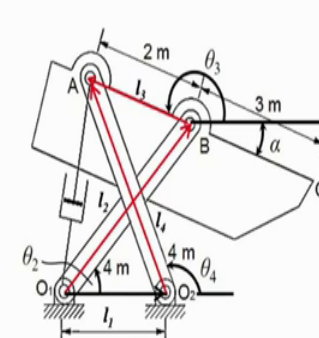
$$l \cos \theta = l_1 + l_4 \cos \theta_4$$

$$l \sin \theta = l_4 \sin \theta_4$$

Now, from the chain O_1O_2A which is this chain, we can write using the kinematics 1 cosine theta plus l 1 is equal to l 1 plus l 4 cosine theta 4, we are using the coordinate system x y the same that was used for displacement analysis. Now, if you differentiate this equation with respect to time here you note that l and theta and theta 4 these are functions of time.

(Refer Slide Time: 04:21)

Example 1



From O_1O_2A chain we had

$$l \cos \theta = l_1 + l_4 \cos \theta_4$$

$$l \sin \theta = l_4 \sin \theta_4$$

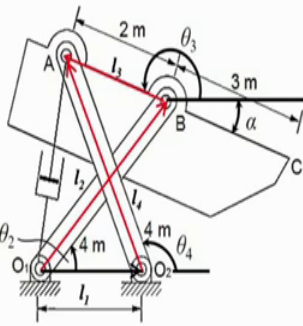
Differentiating w.r.t. time

$$\dot{l} \sin \theta + l \dot{\theta} \cos \theta = l_4 \dot{\theta}_4 \cos \theta_4$$

So, if you differentiate this with respect to time you obtain this relation, which relates which involves l dot theta dot and theta 4 dot, of this we only know l dot.

(Refer Slide Time: 04:42)

Example 1



$$\begin{aligned}
 l \cos \theta &= l_1 + l_4 \cos \theta_4 \\
 l \sin \theta &= l_4 \sin \theta_4 \\
 \Rightarrow \tan \theta &= \frac{l_4 \sin \theta_4}{l_1 + l_4 \cos \theta_4}
 \end{aligned}$$

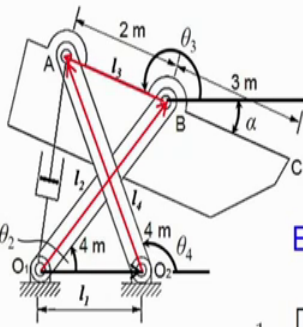
Differentiating w.r.t. time

$$\dot{\theta} = \frac{l_4 \cos^2 \theta (l_4 + l_1 \cos \theta)}{(l_1 + l_4 \cos \theta)^2} \dot{\theta}_4$$

Next, if you look at again these two expressions, these two equations and divide 1 by the other you can find out tangent of the angle theta which turns out to be this ratio. And, if you differentiate this with respect to time you have this expression relating now theta dot and theta 4 dot. In terms of the angles theta and theta 4, since theta, theta 4 are known to us therefore, we can find out we have this relation between theta dot and theta 4 dot.

(Refer Slide Time: 05:35)

Example 1



$$\begin{aligned}
 \dot{l} \sin \theta + l \dot{\theta} \cos \theta &= l_4 \dot{\theta}_4 \cos \theta_4 \\
 \dot{\theta} &= \frac{l_4 \cos^2 \theta (l_4 + l_1 \cos \theta)}{(l_1 + l_4 \cos \theta)^2} \dot{\theta}_4
 \end{aligned}$$

Eliminating dθ/dt and simplifying

$$\dot{l} = \frac{1}{\sin \theta} \left[l_4 \cos \theta_4 - \frac{l_4 l \cos^3 \theta (l_4 + l_1 \cos \theta)}{(l_1 + l_4 \cos \theta)^2} \right] \dot{\theta}_4$$

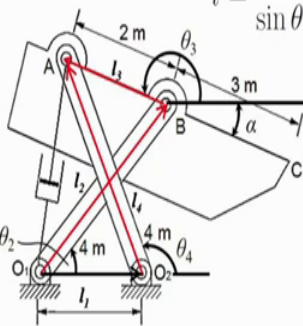
Now, let us look at the previous expression that we had determined. So, these are the two expressions, this involved theta 1 dot theta dot and theta 4 dot and here I have relation

between $\dot{\theta}_2$ and $\dot{\theta}_4$. Therefore, if I eliminate $\dot{\theta}_2$ then I have a relation between $\dot{\theta}_4$ and $\dot{l}$.

So, if you eliminate $\dot{\theta}_2$ and simplify then you have a relation between $\dot{l}$ and $\dot{\theta}_4$. Out of this $\dot{l}$ is given to us the expansion rate is given to us and from the displacement analysis we already have θ_4 and θ_2 .

(Refer Slide Time: 06:26)

Example 1



$$\dot{l} = \frac{1}{\sin \theta} \left[l_4 \cos \theta_4 - \frac{l_4 l \cos^3 \theta (l_4 + l_1 \cos \theta_4)}{(l_1 + l_4 \cos \theta_4)^2} \right] \dot{\theta}_4$$

From displacement analysis

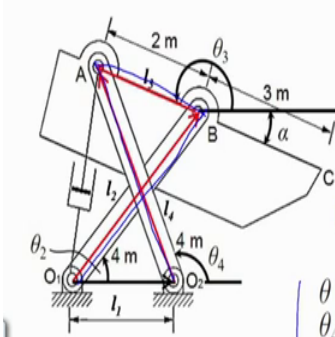
$\theta = 49.458^\circ, \theta_2 = 21.168^\circ, \theta_3 = 98.891^\circ, \theta_4 = 71.79^\circ.$

$\dot{l} = 1 \text{ m/s}, \Rightarrow \dot{\theta}_4 = -0.658 \text{ rad/s}$

Therefore we can determine $\dot{\theta}_4$, that turns out to be minus 0.654 radians per second for given expansion rate of the actuator as 1 meter per second. Now, once we have $\dot{\theta}_4$, now we can use the relation between $\dot{\theta}_4$ and $\dot{\theta}_2$ and find out $\dot{\theta}_2$.

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Example 1



From O_1BAO_2 -4R chain,

$$\dot{\theta}_4 = \frac{l_2}{l_4} \left[\frac{l_4 \sin(\theta_2 - \theta_4) + l_1 \sin \theta_2}{l_2 \sin(\theta_2 - \theta_4) + l_1 \sin \theta_4} \right] \dot{\theta}_2$$

From displacement analysis

$$\theta = 49.458^\circ, \theta_2 = 21.168^\circ, \theta_3 = 98.891^\circ, \theta_4 = 71.79^\circ.$$

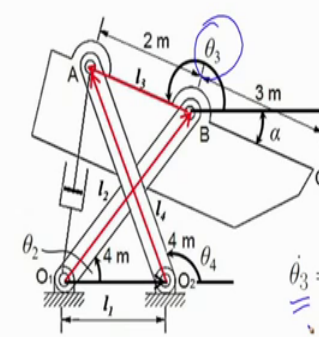
$$\dot{\theta}_4 = -0.658 \text{ rad/s}, \Rightarrow \dot{\theta}_2 = -0.316 \text{ rad/s}$$

This was our relation which we have found from the analysis of this 4R chain the 4R chain is O_1BAO_2 . This is the 4 bar chain whose analysis we have done previously and we have found this relation between $\dot{\theta}_2$ and $\dot{\theta}_4$. Now using that, since we know $\dot{\theta}_4$ now we can find out $\dot{\theta}_2$ which has been determined here.

So, from the displacement analysis we know these angles from the previous velocity analysis we know $\dot{\theta}_4$. So therefore, from the above expression we can now find out $\dot{\theta}_2$.

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Example 1



$$\tan \theta_3 = \frac{l_4 \sin \theta_4 - l_2 \sin \theta_2}{l_1 + l_4 \cos \theta_4 - l_2 \cos \theta_2}$$

Differentiating w.r.t. time

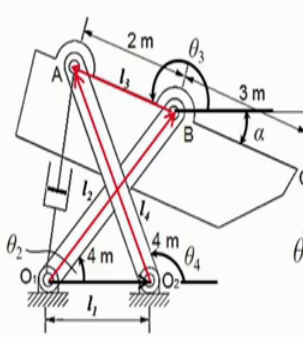
$$\dot{\theta}_3 = \cos^2 \theta_3 \left[\frac{l_2 \{l_2 - l_1 \cos \theta_2 - l_4 \cos(\theta_2 - \theta_4)\} \dot{\theta}_2}{(l_1 + l_4 \cos \theta_4 - l_2 \cos \theta_2)^2} + \frac{l_4 \{l_4 + l_1 \cos \theta_4 - l_2 \cos(\theta_2 - \theta_4)\} \dot{\theta}_4}{(l_1 + l_4 \cos \theta_4 - l_2 \cos \theta_2)^2} \right]$$

Finally, we need to find out the angular speed of the bin, the rotation of the bin. Now, this expression we had derived during displacement analysis of the 4R chain. So, tangent of this angle theta 3 is given by this ratio which is in terms of theta 2 and theta 4.

If you differentiate this with respect to time you get a complicated expression which is doable, but it is a little lengthy. In that you involve theta 3 dot which we want to find out, theta 2 dot and theta 4 dot and the joint angles theta 2 and theta 4. We have already found out theta 2 dot and theta 4 dot. Therefore, by just substituting these expressions and the joint angles which we already know we find out theta 3 dot.

(Refer Slide Time: 09:00)

Example 1



$$\dot{\theta}_3 = \cos^2 \theta_3 \left[\frac{l_2 \{l_2 - l_1 \cos \theta_2 - l_4 \cos(\theta_2 - \theta_4)\} \dot{\theta}_2}{(l_1 + l_4 \cos \theta_4 - l_2 \cos \theta_2)^2} + \frac{l_4 \{l_4 + l_1 \cos \theta_4 - l_2 \cos(\theta_2 - \theta_4)\} \dot{\theta}_4}{(l_1 + l_4 \cos \theta_4 - l_2 \cos \theta_2)^2} \right]$$

where

$$\dot{\theta}_4 = -0.658 \text{ rad/s}, \quad \dot{\theta}_2 = -0.316 \text{ rad/s}$$

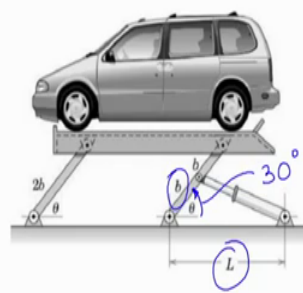
$$\Rightarrow \dot{\theta}_3 = -0.975 \text{ rad/s}$$

So, that completes this problem in which we had to find out the angular speeds of all the links and including the bin and this $\dot{\theta}_3$ is actually the angular speed of the bin. And negative sign indicates that its decreasing therefore, it is in the clockwise direction the rotation is in the clockwise direction.

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Example 2

In the car-lift mechanism shown $L = 2b = 2\text{m}$. The hydraulic actuator expands at a constant rate 1 m/s . Determine the upward speed of the platform when $\theta = 30^\circ$.

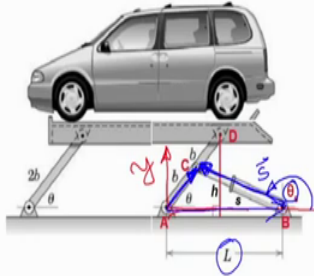


In the next example we are going to look at this car lift mechanism, in this mechanism is given that this length L is equal to 2 times this length b and this length is 2 meter.

So, the problem says the hydraulic actuator expands at a constant rate of 1 meter per second, determine the upward speed of the platform when theta equal to 30 degree, but this is the angle theta which is specified 30 degree. So, at theta equal to 30 degree; what is the upward velocity of the platform?

(Refer Slide Time: 10:22)

Example 2



Let

$$\vec{AB} = L, \vec{AC} = b, \vec{BC} = s$$

From $\triangle ABC$, $b = L + s$

Hence,

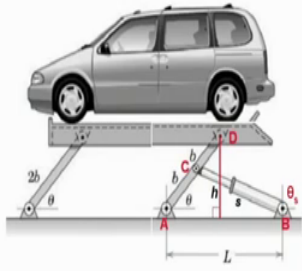
$$\begin{aligned} x \rightarrow b \cos \theta &= L + s \cos \theta_s \\ y \rightarrow b \sin \theta &= s \sin \theta_s \end{aligned}$$

We define these vectors in the coordinate system x y with origin at A. So, this vector is the L vector, AC is this vector b and this vector is vector s. Therefore, from the vector loop equation we can write b as vector L plus vector s.

Now, if you look at the components, if you look at the components of the vector B L and s from the x component we have the first equation and from the y we have the second equation, here we involve this angle theta s. So, this angle is theta s ok.

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Example 2



$$b \cos \theta = L + s \cos \theta_s$$

$$b \sin \theta = s \sin \theta_s$$

Eliminating θ_s

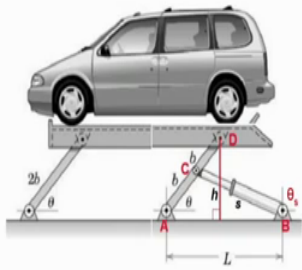
$$s^2 = (b \cos \theta - L)^2 + (b \sin \theta)^2$$

$$\Rightarrow s^2 = L^2 + b^2 - 2Lb \cos \theta$$

So, these are our relations, if you eliminate theta s then in order to eliminate theta s we take 1 on this side square and add and we have this expression of s in terms of theta.

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Example 2



$$s^2 = L^2 + b^2 - 2Lb \cos \theta$$

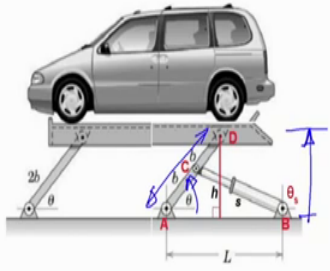
Differentiating w.r.t. time

$$\dot{s} = \frac{L}{s} b \dot{\theta} \sin \theta$$

So, this is our relation of s with theta, if you differentiate this with respect to time and simplify we involve s dot and theta dot, L and b are constants therefore, this involves s dot and theta dot. And it also involves s which we already know.

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Example 2



Further,

$$h = 2b \sin \theta$$

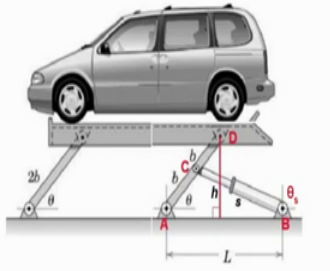
Differentiating w.r.t. time

$$\dot{h} = 2b\dot{\theta} \cos \theta$$


Now, here we have another displacement which actually gives us the motion of the platform which is h . So, h can be related in terms of this length AD and the angle θ and is given by $2b \sin \theta$ and if you differentiate this, this is going to relate $\dot{h}$ and $\dot{\theta}$.

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Example 2



Therefore,

$$\dot{s} = \frac{L}{s} b \dot{\theta} \sin \theta$$

$$\dot{h} = 2b\dot{\theta} \cos \theta$$

Eliminating $d\theta/dt$ and using $s^2 = L^2 + b^2 - 2Lb \cos \theta$

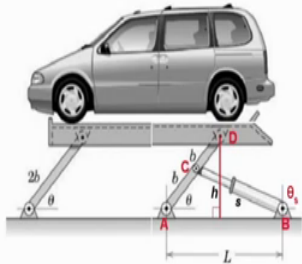
$$\dot{s} = \frac{\dot{h}L}{2s} \tan \theta = \frac{\dot{h}L \tan \theta}{2\sqrt{L^2 + b^2 - 2Lb \cos \theta}}$$

Now, we have two relations one is involving $\dot{s}$ and $\dot{\theta}$ the other relation involves $\dot{h}$ and $\dot{\theta}$. So, if you eliminate $\dot{\theta}$ and use the relation of s and

theta then you come up with this expression, now here we have $\dot{s}$ related to $\dot{h}$ in terms of theta. Now, from the displacement analysis we have already found out theta.

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Example 2



$$\dot{s} = \frac{\dot{h}L}{2s} \tan \theta = \frac{\dot{h}L \tan \theta}{2\sqrt{L^2 + b^2 - 2Lb \cos \theta}}$$

From given data

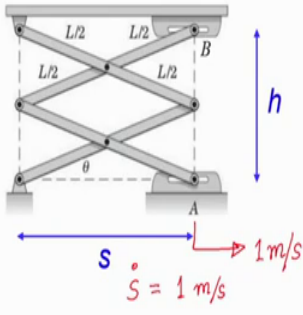
$\dot{s} = 1 \text{ m/s}$, $\theta = 30^\circ$ and $L = 2 \text{ m}$
 $\Rightarrow \dot{h} = 2.1466 \text{ m/s}$

So, we are given this theta as 30 degree. So, at theta equal to 30 degree we are given $\dot{s}$ as 1 meter per second other data the link length data are all given therefore, we can find out $\dot{h}$ which turns out to be 2.146 meter per second.

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Example 3

Determine the speed of the platform when pin A slides to the right at a speed of 1 m/s.



$\dot{S} = 1 \text{ m/s}$

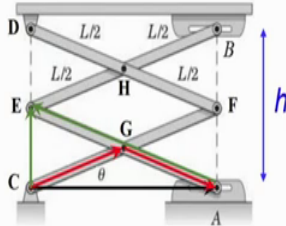
$\dot{h}?$

Next we look at the Nuremberg mechanism which you have discussed before in the context of displacement analysis, here it is given that the pin A slides to the right at 1

meter per second. In other words $\dot{s}$ is given as 1 meter per second we have to find out $\dot{h}$, basically the speed of the vertical travel speed of the platform.

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Example 3



From displacement analysis

$$s = L \cos \theta$$

$$h = 2L \sin \theta$$

Differentiating w.r.t. time

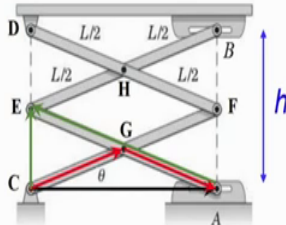
$$\dot{s} = -L\dot{\theta} \sin \theta$$

$$\dot{h} = 2L\dot{\theta} \cos \theta$$

Now, we have looked at the displacement analysis of this mechanism before and we are related s and h in terms of θ which are shown here. Now once again if you differentiate these two expressions you involve $\dot{s}$ and $\dot{\theta}$ and $\dot{h}$ and $\dot{\theta}$.

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Example 3



$$\dot{s} = -L\dot{\theta} \sin \theta$$

$$\dot{h} = 2L\dot{\theta} \cos \theta$$

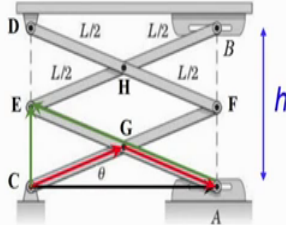
Eliminating $d\theta/dt$

$$\dot{h} = -2\dot{s} / \tan \theta$$

If you eliminate $\dot{\theta}$ you relate $\dot{s}$ and $\dot{h}$.

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Example 3


$$\dot{h} = -2\dot{s} / \tan \theta$$

From given data

$$\dot{s} = 1 \text{ m/s}, \theta = 30^\circ \Rightarrow \dot{h} = -2\sqrt{3} \text{ m/s}$$

Now, when $\dot{s}$ and θ are given; so, here it is given as 1 meter per second and θ is given as 30 degree you just substitute you will get the expression of $\dot{h}$.

So, that completes this example, with that I will close this lecture.