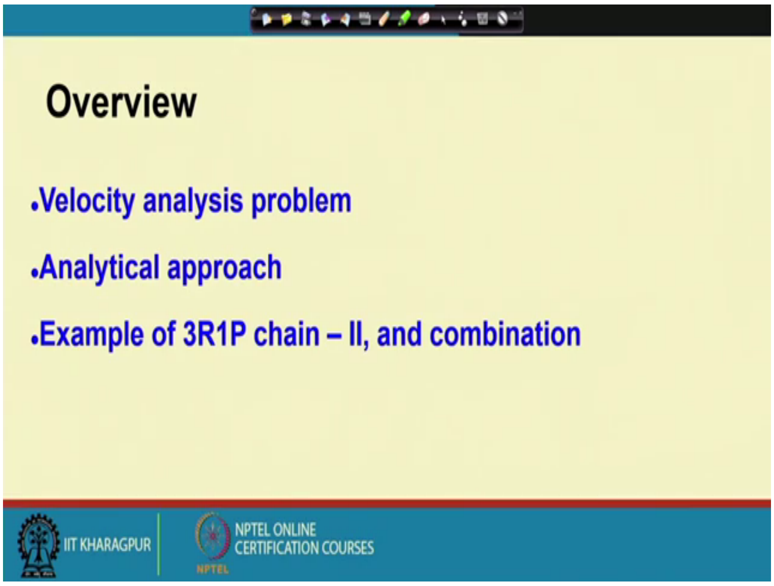
Kinematics of Mechanisms and Machines
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 26
Analytical Velocity Analysis –III

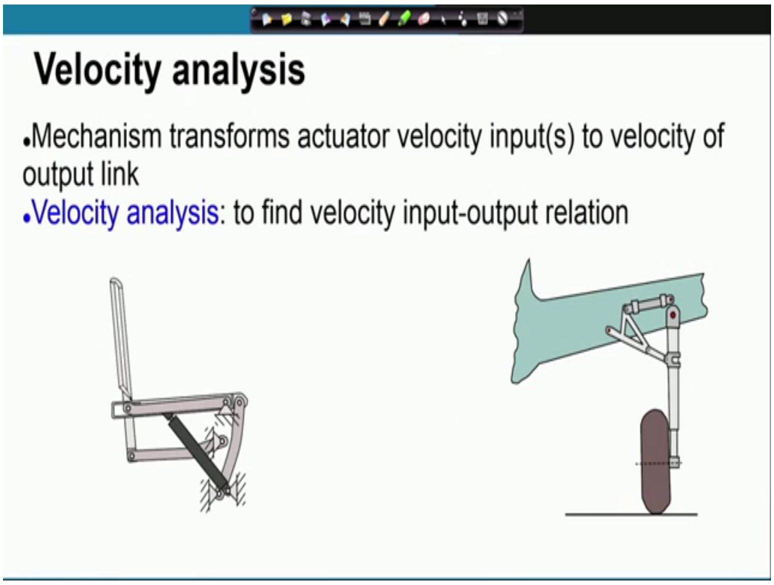
Let us continue our discussion on Velocity Analysis problem using the analytical approach. we are going to look at another example of 3R1P chain of type II in this lecture. So, I will give you the overview of this lecture.

(Refer Slide Time: 00:33)



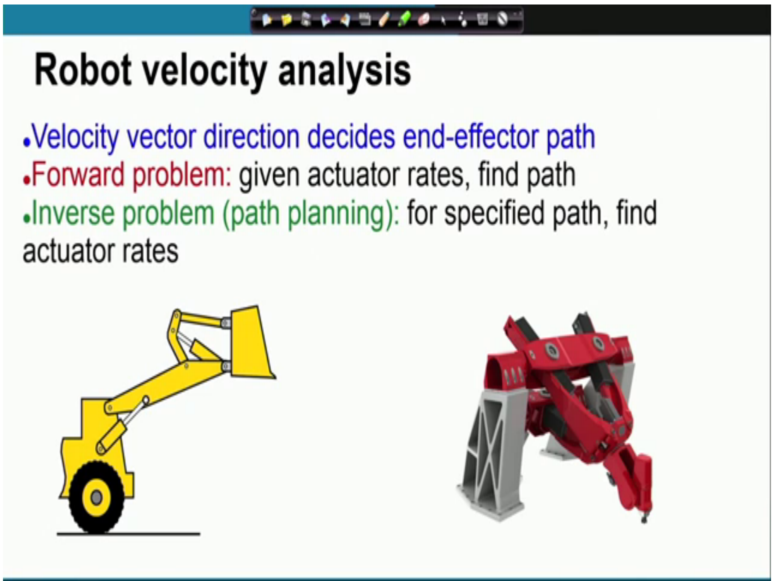
We are going to look at this example of 3R1P chain of type II. And we are also going to see briefly how in more complicated chains these ideas can be extended very very easily. Of course, the calculations might be little more involved, but then these ideas that we are discussing of velocity analysis can easily be applied to more complicated chains.

(Refer Slide Time: 01:09)



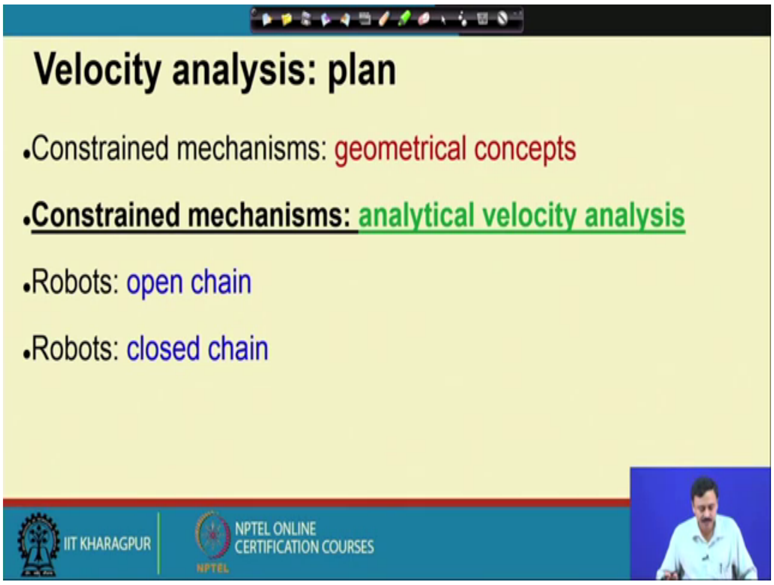
So, the velocity analysis problem has been defined as the as finding out the input output velocity relation. That means, from the actuator to the output we have these problems for the constraint mechanisms.

(Refer Slide Time: 01:31)



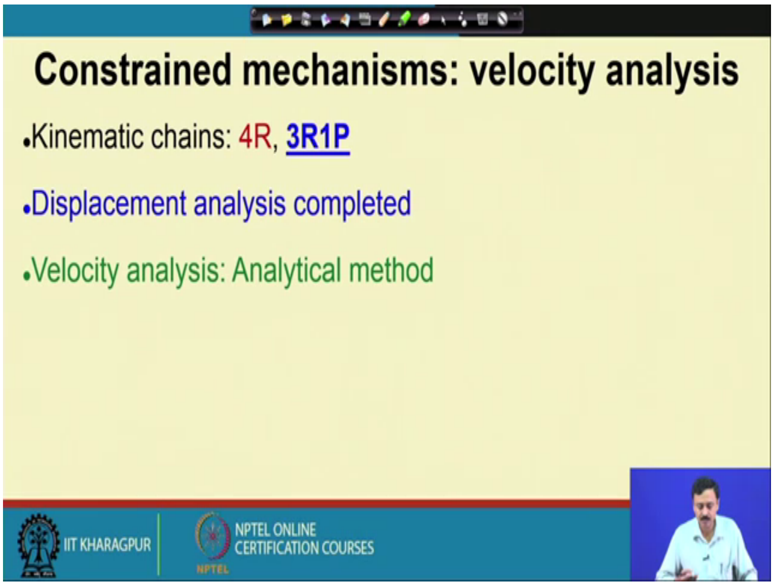
For robots in which the path planning problem or path generation problem is of importance; and which very intimately depends on the velocity analysis problem for robots; which we are going to discuss.

(Refer Slide Time: 01:49)



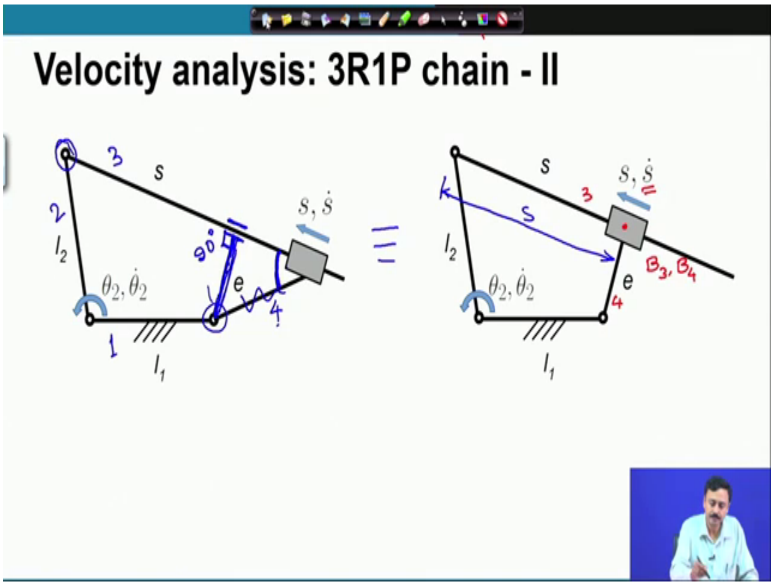
We will continue with the analytical velocity analysis approach for constraint mechanisms.

(Refer Slide Time: 01:53)



And we are going to look at the 3R1P chain. We will consider that this displacement analysis is completed which means that; we know the configuration of the mechanism. As you know in constraint mechanisms specifying one of the input one input will specify the configuration on the complete mechanism. So, I can find out all other angles and angles and of course, the throw of the prismatic pair. So, here in the 3R1P we have a prismatic pair. So, we can calculate everything given an input. So, we are now going to discuss the velocity analysis problem using the analytical method.

(Refer Slide Time: 02:43)

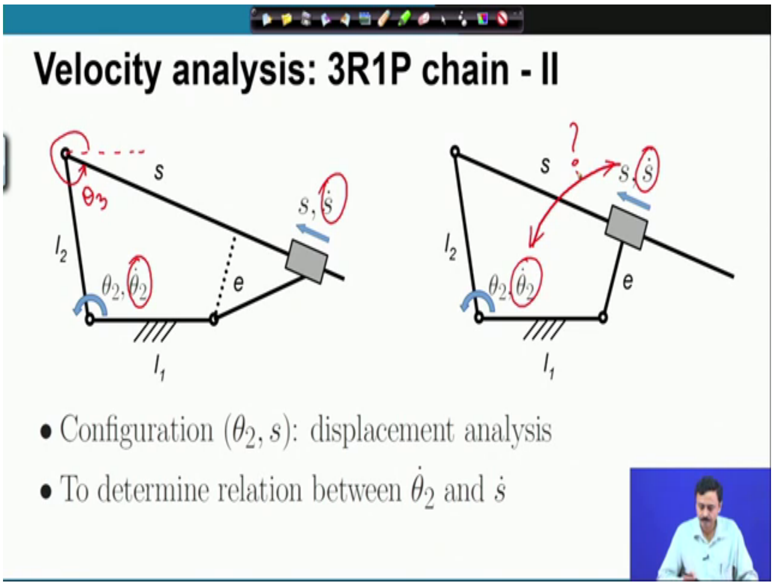


Now, in the 3R1P chain of type II we have a prismatic pair which is changing the direction of the sliding as you can see here. As we have discussed before you can see that this angle is going to remain fixed. When the prismatic pair expands or contracts so to number the links the ground is 1, this is 2, this is 3, this is 4. So, between lengths 3 and 4 we have the prismatic pair and this angle between lengths 3 and 4 is going to remain fixed.

So therefore, I can consider that this is same as having a prismatic pair like this, this is going to simplify my expressions and the analysis so this is e the offset. So, offset is measured perpendicular to the direction of sliding so this is 90 degree. So, this angle is 90 degree. So, e is measured perpendicular to the direction of sliding between the two revolute pairs at the end of the links containing the prismatic pair. So, 3 and 4 are connected by the prismatic pair. And these two revolute pairs are at the ends of links 4 and links 3 and link 3. So, this perpendicular distance is the offset. So therefore, rather than having this I can join these two with at 90 degree. And hence, obtain a chain like this which is equivalent these two are equivalent.

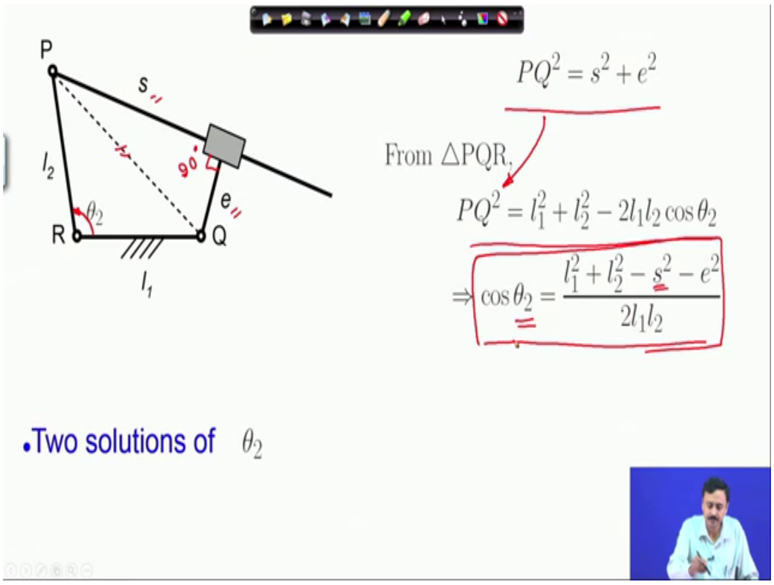
Here the distance s so this is the throw of the actuator we will consider this to be the throw of the actuator. So, one more important point here s is this length and s dot is the velocity of a point belonging to link 3. Suppose we have a point coincident point belonging to link 3 called B 3. And another point at the same location which means B 3 and B 4 are coincident. So, B 3 belongs to link 3 and B 4 belongs to link 4. So, the relative velocity between B 3 and B 4 is a start. So, that is the expansion rate of the prismatic actuator.

(Refer Slide Time: 06:31)



So, the configuration of the mechanism is known to us which means theta 2 s and also this angle theta 3 of course, these are all known to us. Theta 2 s, theta 3 these are all known to us from displacement analysis. We have to find out the relation between theta 2 dot and s dot. So, we have to find out this relation.

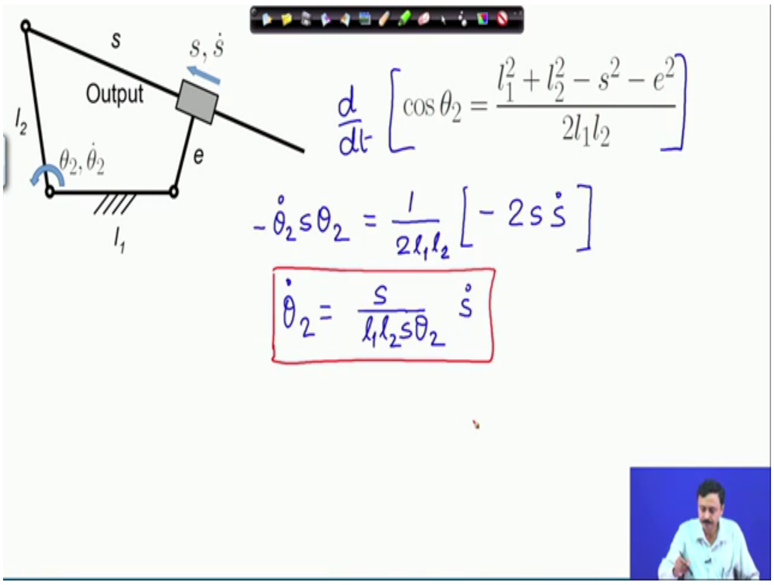
(Refer Slide Time: 07:19)



Now, we have looked at the displacement analysis problem for this kinematic chain. So, we are going to follow through PQ which is this distance. This distance can be related to s and e the offset by this relation because this angle is 90 degree. Therefore I can write PQ square is s square plus e square. Now from triangle PQR this PQ square using the cosine rule I can write in terms of the angle theta 2 so this angle is theta 2. So, I can relate PQ also from the other side using theta 2 and this is what we have.

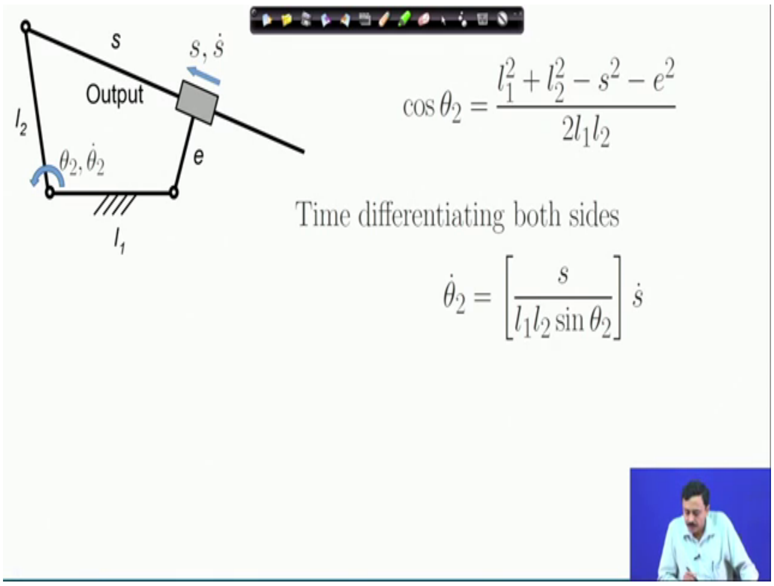
So, I have put this PQ square expression of PQ square in this to finally, obtain this relation. Now this equation this is the displacement relation relates theta 2 and s. Now, there are two solutions of theta 2 as you know you can find them. So, these are the two configurations corresponding to a specified value of s or a specified value of theta 2 there are two solutions.

(Refer Slide Time: 09:11)



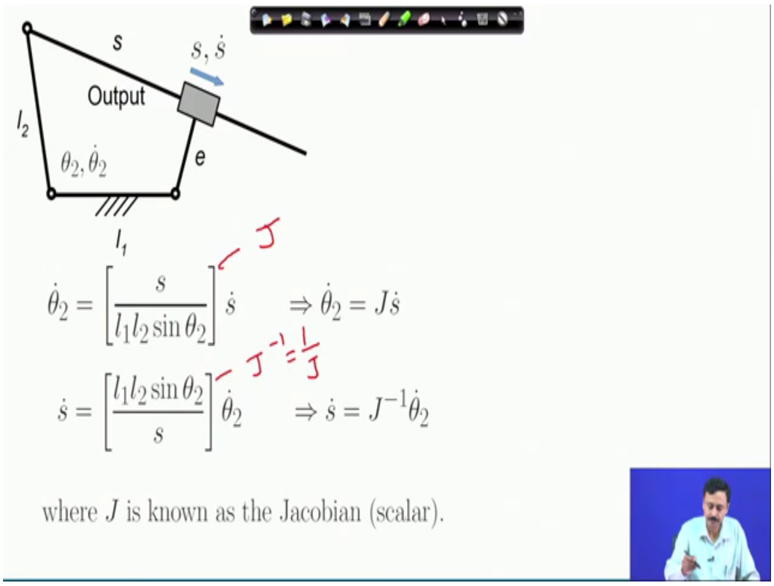
So, this is what we have this is the displacement relation. If I differentiate this with respect to time, as we have been doing so this is the displacement relation. So, what do we have minus of theta 2 dot into sin theta 2 is equal to 1 by 2 of l 1 l 2 times. Now these are constants l 1 is constant l 2 is constant the only thing variable is s e is a constant so this is as simple as that. So, therefore, I can relate let us say theta 2 dot as like this. So, theta 2 dot is s divided by l 1 l 2 sine theta 2 into s dot. And I can invert also I can express s dot in terms of theta 2 dot. So, let me formalize this.

(Refer Slide Time: 11:07)



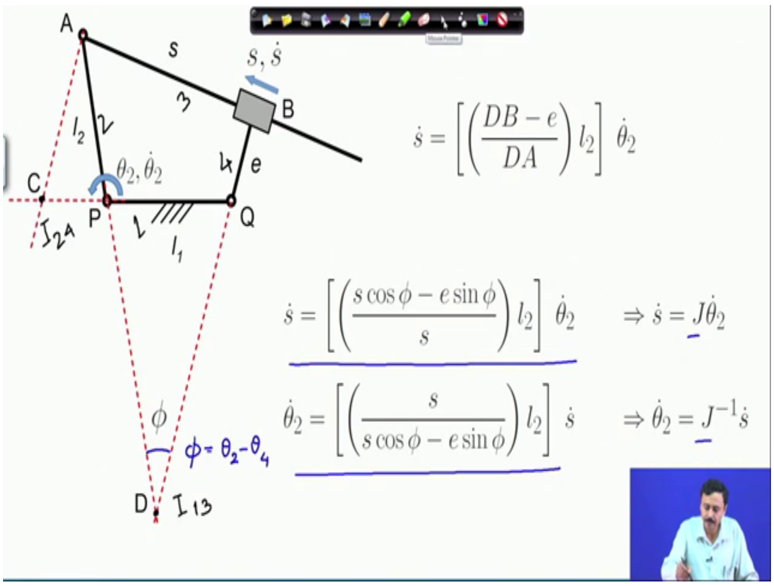
So, we have this expression which we just now derived relating s dot and theta 2 dot.

(Refer Slide Time: 11:17)



So, here I am defining this as the Jacobian which is again a scalar. So, Jacobian is in terms of s and theta 2. And if you invert the relation then you get it in terms of Jacobian inverse which is 1 over J it is a scalar. So, you have the Jacobian. So, these input output velocity relations are linear and related through the Jacobian.

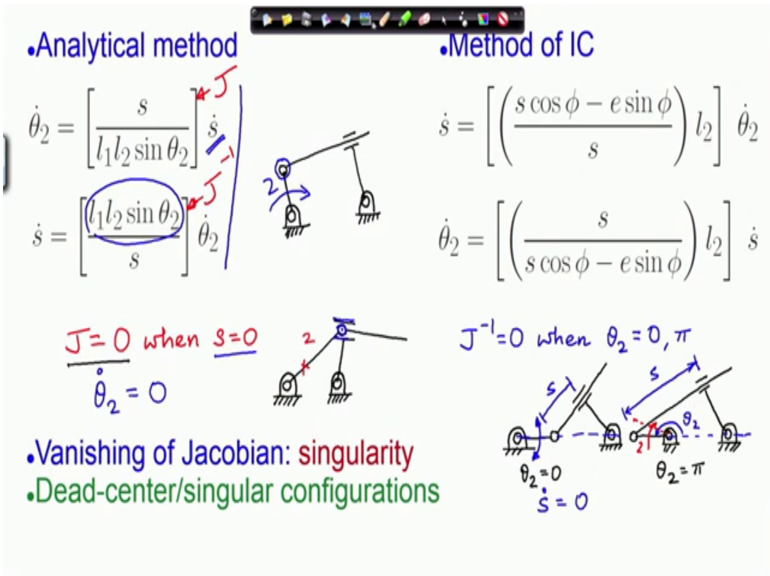
(Refer Slide Time: 12:15)



Previously we had also discussed the input output velocity relations using the method of I C which we just reiterate here. So, we had used these 2 instantaneous centers of rotation. So, this point D was I 1 3. So, this is 1 this is 2 this is link 3 and this is link 4. So, this is I 1 3 and this is I 2 4 indicated by point C. And using these instantaneous centers we had determined the input output velocity relations or the velocities velocity relations between the expansion of the actuator the expansion rate of the actuator and the angular rate of link 2. So, I have written that out here.

Here DB and DA are these distances which you can show we can you can see on the figure. And using this angle phi where phi is nothing, but theta 2 minus theta 4. Using this angle we had expressed these velocity relations as I have shown here. So, these expressions of the Jacobian as defined from as defined in this slide looks very different or more complicated so to say in when you compare it with that obtained from the analytical method. So, let us compare them.

(Refer Slide Time: 14:35)



So, these are the expressions of the velocity relations obtained from the analytical method. And on the right I have written out the velocity relations obtained from the method of IC’s. So, definitely these expressions on the left are more simple because here you have to determine phi. Now vanishing of this Jacobian gives us the singularities of the kinematic chain. And these singularities are nothing, but the dead center configurations of the chain.

Now specifying these or determining this dead center configurations was easier when we looked at the expressions obtained from the method of IC’s. But these are also now quite easy if you look at the expressions obtain from the analytical method. So, let us look at the configurations where the Jacobian or its inverse vanishes which means they become singular or non-invertible. So, if I look at the Jacobian so this is the Jacobian this is the Jacobian inverse.

So, when will the Jacobian vanish? The Jacobian will vanish when s equal to 0. So, J equal to 0 when s equal to 0. So, this was the configuration like this. So, our chain so from here if I start moving of course, the link lengths look a little different. So, when I start rotating the link 2 in the clockwise direction; there will come situation where this revolute pair will start going into the prismatic pair.

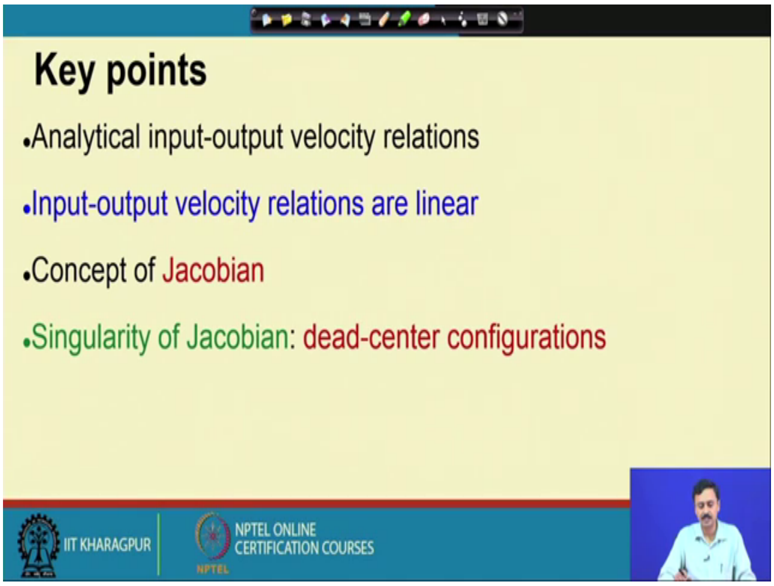
So, forgetting about the physical problems or the constructional problems we can say that when this revolute pair gets into the prismatic pair if it can then; s equal to 0 in that case that is a singular configuration of the mechanism. So, in this singular configuration irrespective of the value of s dot theta 2 dot goes to 0. So, at this configuration theta 2 dot must be 0 you cannot move the link 2 you cannot rotate it further. It must only move it can only reverse direction; that means, it can go only in the counterclockwise direction it cannot go any further in the clockwise direction.

Whatever be the value of s so not s dot it has to reverse direction. The other singular configuration is for the Jacobian is when. So, here I have the Jacobian inverse. So, this s equal to 0 is the singularity of the Jacobian. When you look at Jacobian inverse Jacobian inverse is equal to 0. So, when theta 2 is equal to 0 or pi radian. You can very easily see because the numerator of Jacobian inverse has this sine theta 2. So, that will vanish whenever theta 2 is 0 or pi and what are the configurations at which this will happen.

So, when theta 2 equal to 0; so, this is one configuration here theta 2 equal to 0 and theta 2 is equal to pi gives us this configuration. So, theta 2 equal to pi radian gives us this configuration. So, what happens here, irrespective of the value of theta 2 dot s dot is 0. So, which means this actuator now cannot expand any further it must reverse. So, it reaches a 0 velocity and it must now reverse back.

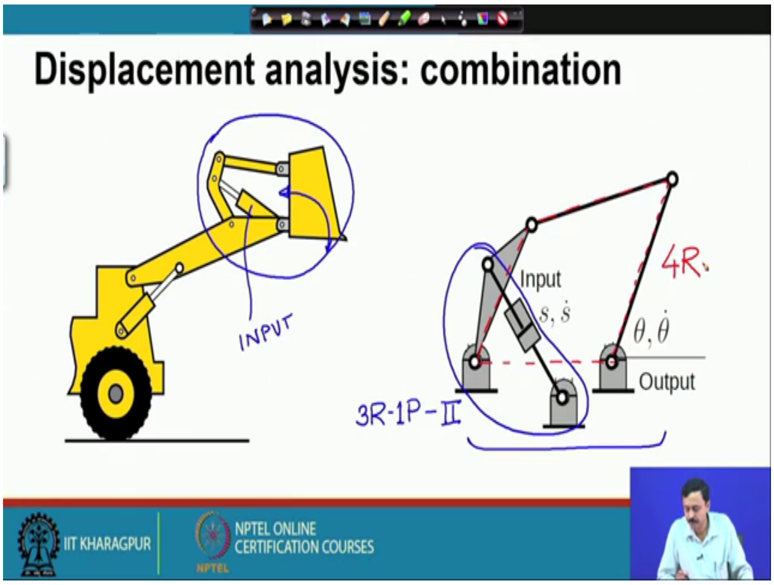
So, in other words here this while this link can rotate this distance the distance s the rate of change of distance s is 0. For example, if I move the link 2 in the clockwise direction then s at this configuration s dot is 0, but as you keep moving as the link two comes to such a position. So, s will contract so s dot will be negative. If you move in the other direction there also s dot must contact. So, exactly at this configuration s dot is 0. So, it has to reverse direction s has to reverse its direction.

(Refer Slide Time: 23:15)



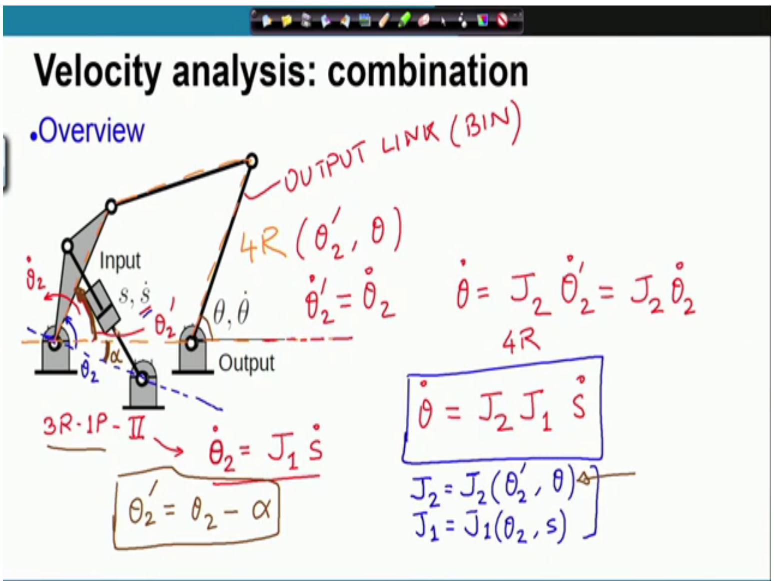
So, we have looked at the input output velocity relations for the 3R1P chain of type II and obtained these linear velocity relations between the angular rate and the rate of expansion of the prismatic actuator. We have introduced or looked at this concept of Jacobian we found the expression of Jacobian following the analytical approach. And we have compared it with Jacobian obtained from the method of IC's or instantaneous center of rotation. And we have also discussed about the singularity of the Jacobian and the corresponding dead center configurations.

(Refer Slide Time: 24:03)



So, this approaches that we have been discussing, now can be combined to find out the velocity relations for more complicated chains. So, here I have this example of an excavator you can see here that this if you focus on this part of the mechanism it is again to this kinetic chain; where the input is the prismatic actuator. And the output is the angular motion of the bin. So, you we have here you have here this 3R1P chain of type II and if you look at this mechanism this is a 4R chain. So, a 3R1P chain type II is driving a 4R chain.

(Refer Slide Time: 25:55)



So, I will just briefly give you the idea of how we can analyze such mechanisms. Here the ground hinge of the 3R1P chain is this ground link. So, this is the line of frame this was our theta 2 for the 3R1P chain. Now given the input in terms of s dot I can find out theta 2 dot, that we have seen. I can relate s dot with theta 2 dot. For this 3R1P chain of type 2 so this we have derived. So, I have this relation theta 2 dot is equal to let me call it J 1 s dot this we have derived this is for the 3R1P chain.

Now this same theta 2 also acts as the input for the 4 R chain. The same theta 2 dot is the input to the 4 R chain the only difference is the line of frame of this 4 R chain is different. So, for the 4 R chain the line of frame is different, but theta 2 dot remains the same and the definition of theta 2 for our chain is also somewhat different let me show you. So, this is the angle theta 2 let me call it theta 2 prime let me call it theta 2 prime this theta 2 prime is the angle input angle for the 4 R chain. So, the input is theta 2 prime and the output is theta and of course, theta dot.

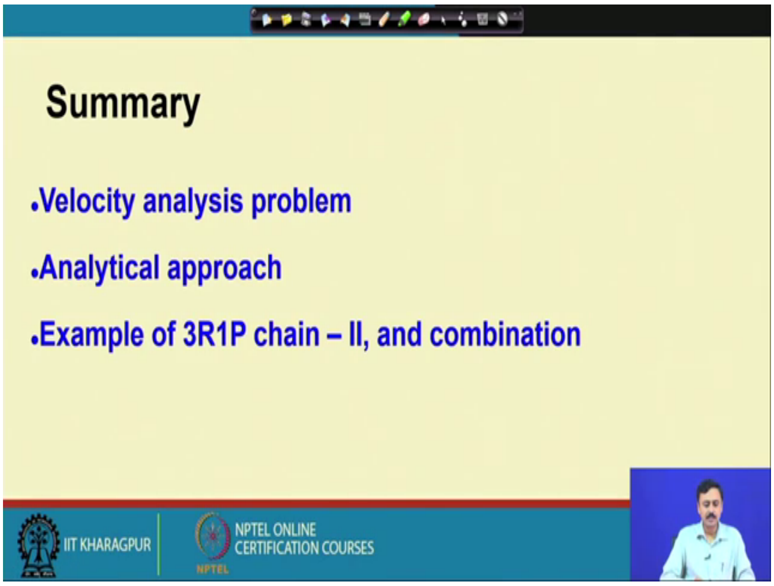
Now theta 2 prime dot is nothing, but theta 2 dot. As you can see theta 2 prime dot because the line of frame of the four R chain is a fixed line is a fixed reference line. So, therefore, theta 2 prime dot is also equal to theta 2 dot. Now given theta 2 dot prime finding out theta dot is the velocity relation for the 4 R chain. So, that we already know. So, theta dot is equals to J 2 which is for the 4 R chain times theta 2 prime dot so this is for the 4 R chain. And that is also therefore, equal to J 2 theta 2 dot and theta 2 dot we have already found in terms of s dot.

Therefore, theta dot which is the velocity of the output link. So, this is our output which is the bin actually of the excavator. So, theta dot therefore, becomes J 2 J 1 into s dot. So, we have related the angular speed of the bin with the expansion rate of the prismatic actuator. Only thing to remember here is that this J 2 is a function of theta 2 and theta 2 prime. So, J 2 will be a function of theta 2 prime and theta. And J 1 will be a function of theta 2 and s. So, this is the additional thing that has to be remembered. You have to derive this which we have derived.

Now, the relation between theta 2 and theta 2 prime is also very simple if you look here. Let us call this angle as some alpha then you can very easily see that theta 2 prime is equal to theta 2 minus alpha. Theta 2 prime which is this angle and theta 2 which is this blue angle they are related as theta 2 prime is equal to theta 2 minus this fixed angle alpha. Alpha is a fixed angle this is the angle between the lines of frame of the 3R1P and 4R chains. So therefore, we have relation between theta 2 prime and theta 2 which we can use in this expression and get everything in terms of theta 2 and s and theta, theta theta 2 and s.

Now, since this is a constraint mechanism given anyone let us say given s I should be able to find out everything. Or, given theta which is the output angle I should be able to find out all the other angles and the value of s which is the throw of the prismatic actuator. So therefore, the analysis though a little extended now, is otherwise straightforward. It will involve lot of algebra, but otherwise the ideas are absolutely clear. So, this is just an extension of the analysis of these individual kinematic chains which we will we have used to analyze more complicated kinematic chain.

(Refer Slide Time: 34:03)



So, let me summarize this lecture. We have considered the velocity analysis problem we have taken the analytical approach to find out the input output velocity relations we have analyzed the 3R1P chain of type II. And finally, I have shown you how the analysis of these fundamental chains the basic chains can help you in the analysis of more complicated chains which can be broken down into these fundamental chains.

So, we have followed the analytical velocity analysis approach. Of course, for the combination you can also use the geometric approach the final results will remain the same. So, I have shown you how to combine these various chains to have or get the velocity relations input output velocity relations for more complicated chains.

So, with that I will conclude this lecture.