Kinematics of Mechanisms and Machines
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Lecture – 24
Analytical Velocity Analysis – I

We have been looking at the Velocity Analysis Problem and in the previous couple of lectures, we have used the concept of the instantaneous center of rotation to determine the velocity relations; based on certain geometric relations that the velocity of a rigid body satisfies; geometric conditions that a rigid body satisfies when it moves in a plane. So, based on that geometric approach we have determined the input output velocity relations.

Now, this approach this geometric approach based on the instantaneous center of rotation has certain limitations. These limitations will come when we go to acceleration analysis. You can very well imagine that the velocity relations were determined using this concept of instantaneous center of rotation. Now, this instantaneous center of rotation is configuration specific. So, at a particular configuration you have a particular instantaneous center of rotation instantaneous.

Now, when we go to acceleration analysis we need velocities at two infinitesimally separated positions. Now, during that and when you go from one position to another position which is infinitesimally separated, this instantaneous center of rotation can also suffer acceleration. So, in general we will not use we mean we will not be able to use the concept of instantaneous center of rotation for acceleration analysis.

So, what we have to do is we have to determine based on analytical relations or the geometry of the chain we have to relate the velocities of input and output which will remain unaffected under acceleration. So, we are not defining any new point which is configuration dependent etcetera. So, we are going to look at in today’s lecture how we relate the input output velocities based on the displacement analysis relation relations, displacement relations of the chain.

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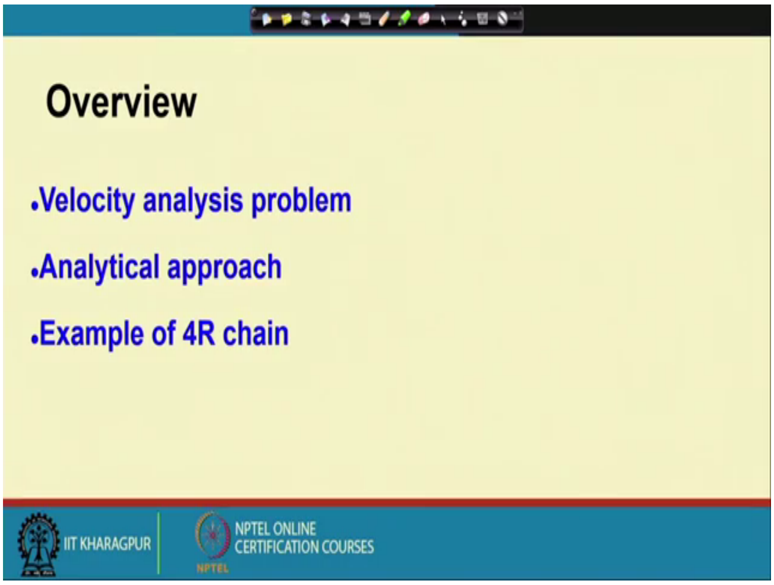

So, to give you an overview of today's lecture we are going to start with the analytical approach and I will show you this approach using the example of a 4R chain.

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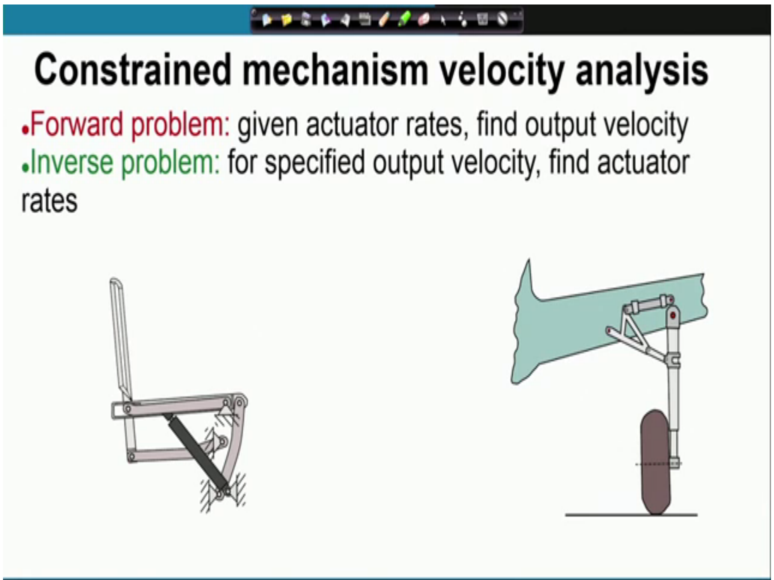

So, we are aware of the velocity analysis problem which is to find out the input output velocity relations; that means, between the actuators and the outputs which is the end effector or the output link. We have these two problems; forward and inverse problems given the actuator rates, finding out the output velocity is the forward problem and given the output velocity desired output velocity, finding out the actuator rates is the inverse problem.

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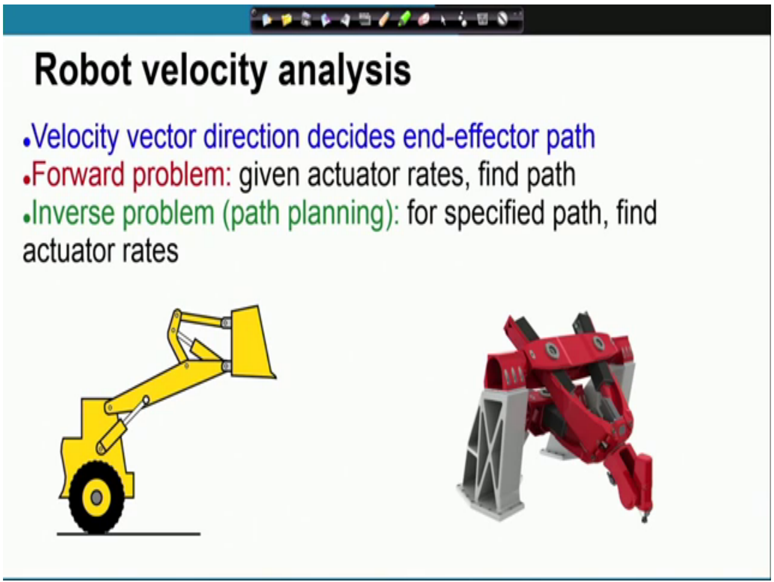

So, in the case of robots we have looked at this we have defined the problem of velocity analysis and its complexity. So, this is intimately related with the path planning or path generation problem in case of robots which we are going to look at in subsequent lectures.

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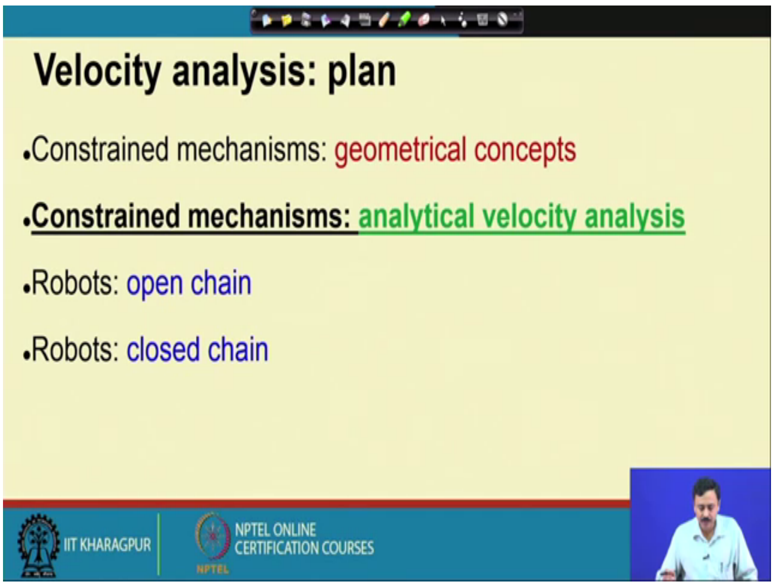

So, we have already discussed the geometric approach to velocity analysis for constraint mechanisms. So, in this lecture we are going to start with the analytical velocity analysis. Subsequently we are going to look at robots.

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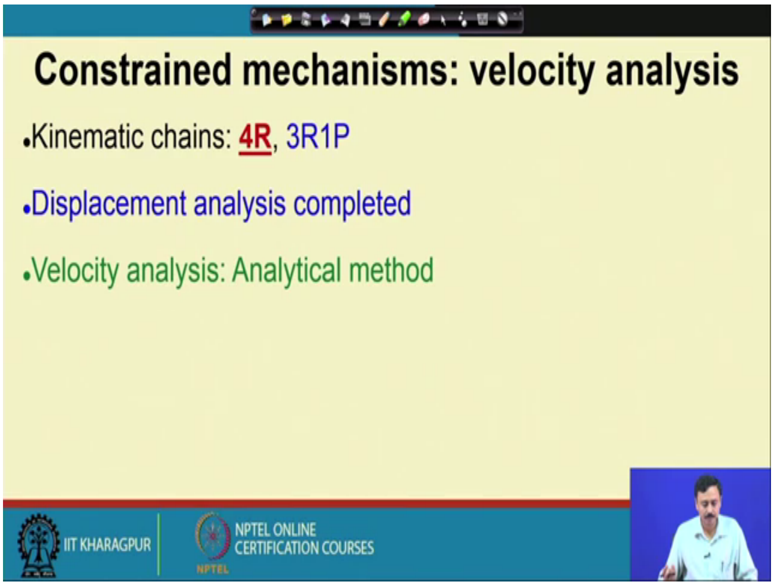

So, in today's lecture I am going to take up the 4R kinematic chain. I will assume that the displacement analysis is complete; that means, we know the configuration of the mechanism. Now, these are constraint mechanisms. So, given let us say the input angle everything of the mechanism is known. Since this has 1 degree of freedom, there is a constraint mechanisms have 1 degree of freedom. So, given one input you can find out everything about the mechanism, all the configuration. So, that is completed.

Next, we are going to embark upon the velocity analysis problem using the analytical method.

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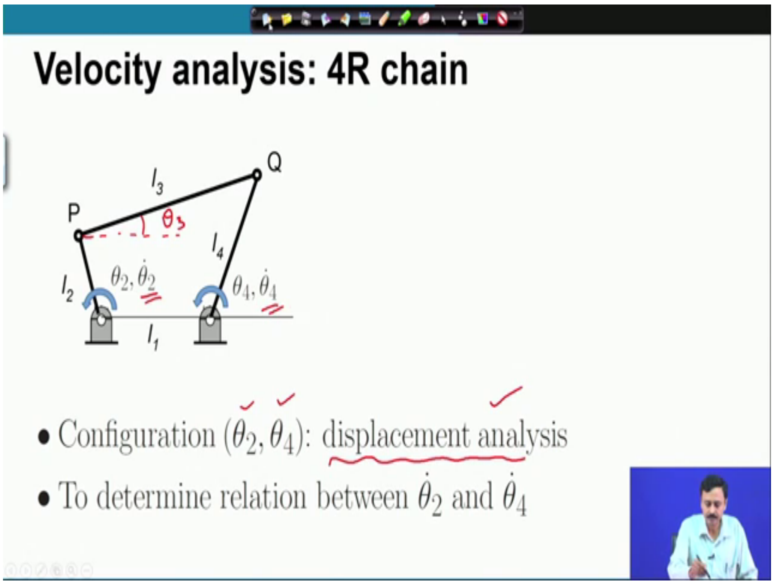

So, as I mentioned that the displacement analysis is completed this is completed. So, which means I know theta 2, I know theta 4 and I also know theta 3, this is displacement analysis. Finding out all these angles given one of them; so, if theta 2 is given I can find out theta 4 and theta 3 or if the output angle theta 4 is given I can find out theta 2 and theta 3. The problem now is to determine the relation between theta 2 dot and theta 4 dot. So, that is the velocity analysis problem.

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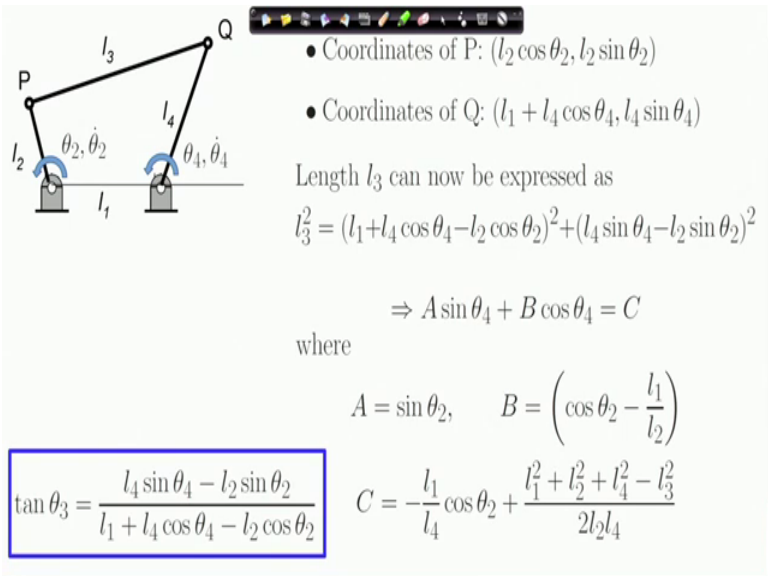

So, we will begin with what we have already discussed before the coordinates of point P I can relate the coordinates of point P, as I have written out for you. So, l 2 cos theta 2, l 2 sine theta 2, so, this is point P. So, coordinates of point P our coordinate frame is of course, like this which we have already discussed in displacement analysis and what we are following actually is from the displacement analysis problem.

So, the length of link 3 can be expressed in this form and we have seen all these steps which can. So, this expression can be rewritten in this form A sine theta 4 plus B cosine theta 4 equal to C where A, B, C are completely known because theta 2 is known. So, this was the displacement analysis problem of finding theta 4 given, theta 2. Now, for velocity analysis what are we going to do; here I have also written out the expression of theta 3 in terms of the coordinates of point P and Q which we have seen before.

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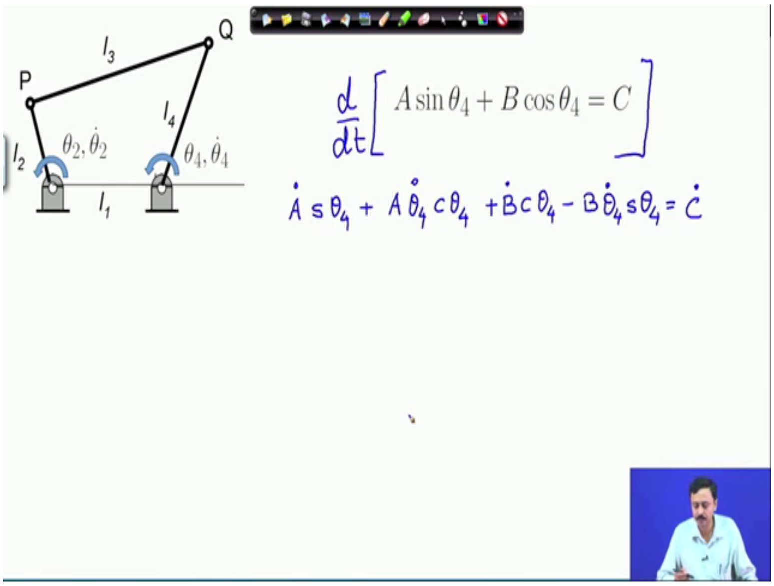

So, let us proceed further. So, this is our expression that relates theta 4 with theta 2. So, A, B, C has theta 2 as we have just now seen. Now, if we derivate this with respect to time. So, let me do that. So, if I differentiate this whole expression with respect to time. So, I have remember A is a function of theta 2. So, A also is a variable quantity. So, A dot is d dt of A s theta 4 stands for sine theta 4 plus A theta 4 dot C theta 4 stands for cosine theta 4 plus B dot cosine theta 4 minus B theta 4 dot sine theta 4 is equal to C dot. So, this is what we will get by differentiating with respect to time this expression.

Now, we know the expressions of A, B and C in terms of theta 2.

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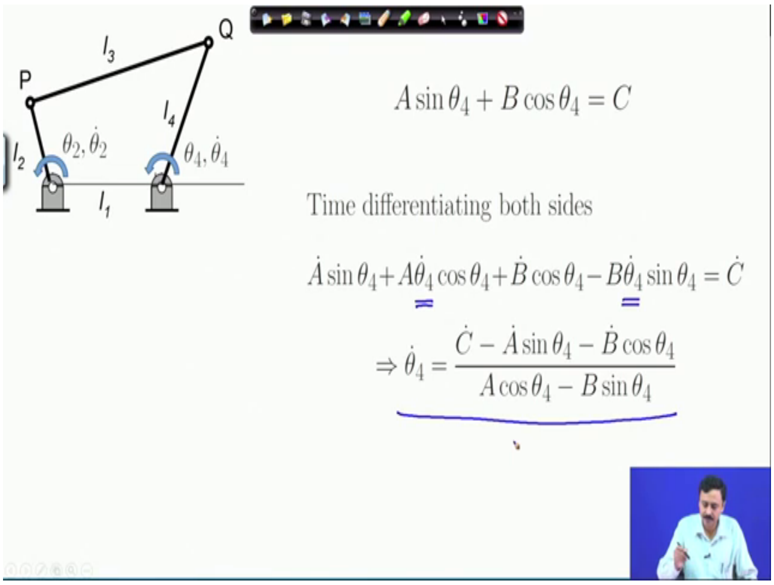

So, let me show you. So, here what I have done extra is I have collected terms of theta 4 dot and I have expressed theta 4 dot in terms of the other quantities. So, I have taken out. I have extracted theta 4 dot out. This is a very easy step. Now, I need to look at what are the expressions of A, B and C and take their time derivatives. So, this I will do next.

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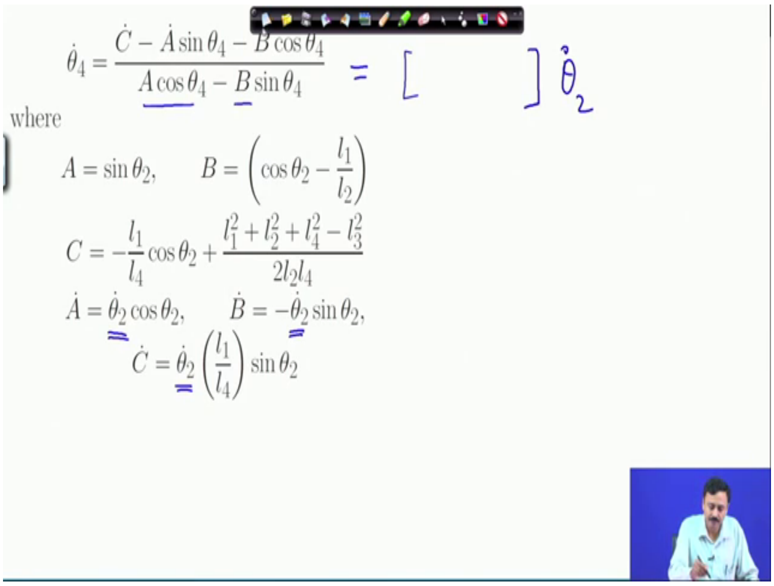

So, here are the expressions. So, I have written out this relation that we just now derived again here a is sine theta 2 therefore, a dot is theta dot theta 2 dot cosine theta 2. Similarly, you can derivate B to obtain this expression and similarly you can derivate C to obtain this expression. So, I know the expressions of A dot B dot C dot and A, B, C are of course, known.

So, if I now substitute these expressions then what do you find? All these quantities A dot B dot C dot they have theta 2 dot multiplying them. They are linear in theta 2 dot. So, therefore, this expression can be written as something into theta 2 dot because that all the dot terms say A dot B dot C dot are appear in the numerator only, in the denominator we have only A and B. All the derivatives of A, B, C; so, A dot B dot C dot they appear in the numerator and with A dot B dot C dot they are linear in theta 2 dot.

So, therefore, theta 2 dot can come out and we have a little more complicated expression which actually we can simplify.

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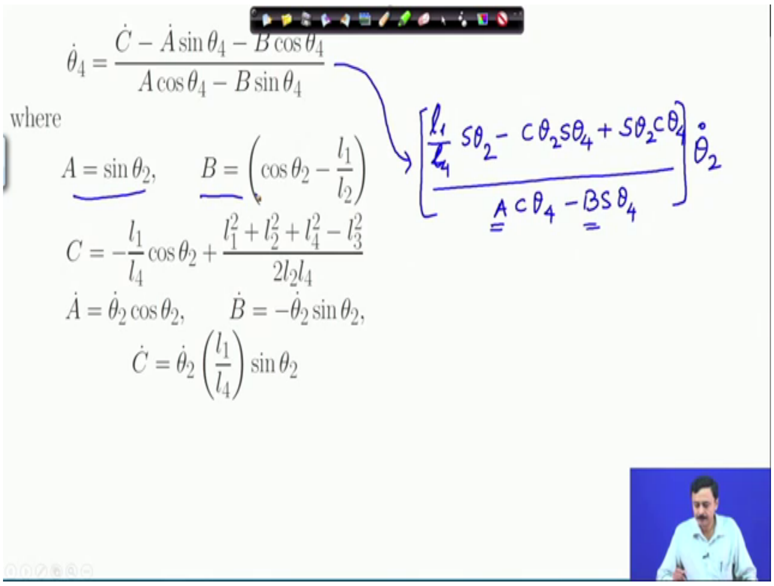

So, if you just substitute in here, so, C dot is l 1 by l 4 sine theta 2; I will take out theta 2 dot and A dot is cosine theta 2 sine theta 4 and this is plus sine theta cosine theta 4 this whole thing getting multiplied by theta 2 dot.

So, it follows from here and I can replace this A and B once again from these expressions.

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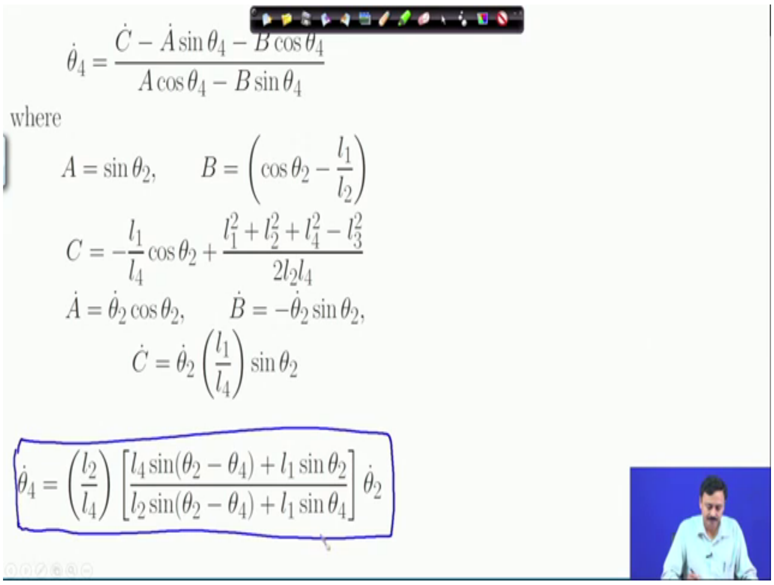

So, once I do that and I simplify I have this final expression which relates theta 4 dot and theta 2 dot, in terms of theta 2 and theta 4. And, you can see once again that this is a linear relation between theta 2 dot and theta 4 dot, there is a multiplier.

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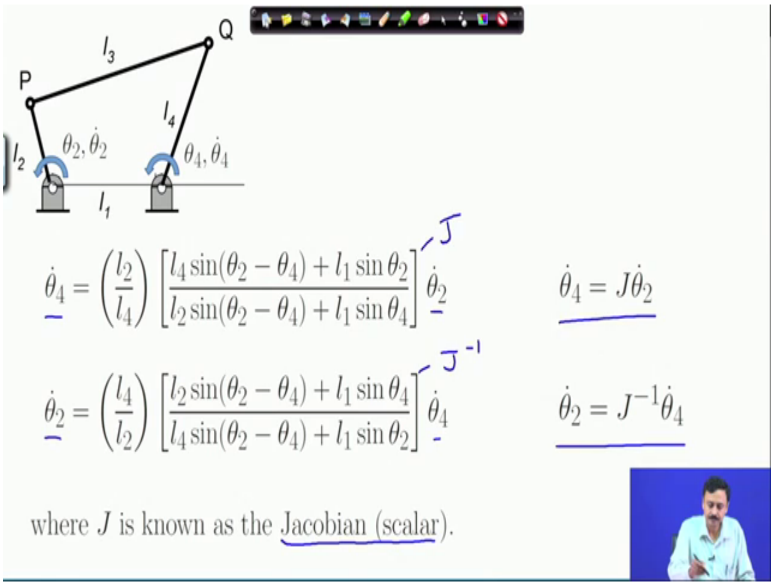

So, this multiplier is the Jacobian that we had been talking about. So, here I have defined this is the Jacobian. So, that so, the Jacobian relates theta 2 as input and gives theta 4 theta 2 dot as input and gives us theta 4 dot; whereas, this Jacobian inverse takes in theta 4 dot and gives us theta 2 dot. So, these are the expressions compactly written in terms of the Jacobian. Now, here again the Jacobian is a scalar; that means, the number. Given the configuration now you can very easily find out the Jacobian.

Now, in the in the previous discussions, we had found this Jacobian in terms of another set of angles theta 2 and theta 3. When we use the method of instantaneous center of rotation, so, from the from geometry when we derived this relation between theta 2 dot and theta 4 dot we had the Jacobian in terms of theta 2 and theta 4 theta 2 and theta 3. So, previously, when using the method of instantaneous center of rotation the Jacobian was determined in terms of theta 2 and theta 3.

Here we have determined the Jacobian in terms of theta 2 and theta four, but that hardly matters because we are armed with the displacement analysis. We know how to relate theta 4, theta 3, theta 2 etcetera. So, we can always find out the Jacobian and the Jacobian value will remain the same. It has to remain the same whether you do by method of instantaneous interpretation or through this analytical displacement analysis differentiating the displacement analysis displacement relations.

So, the Jacobian value is going to remain the same at a given configuration.

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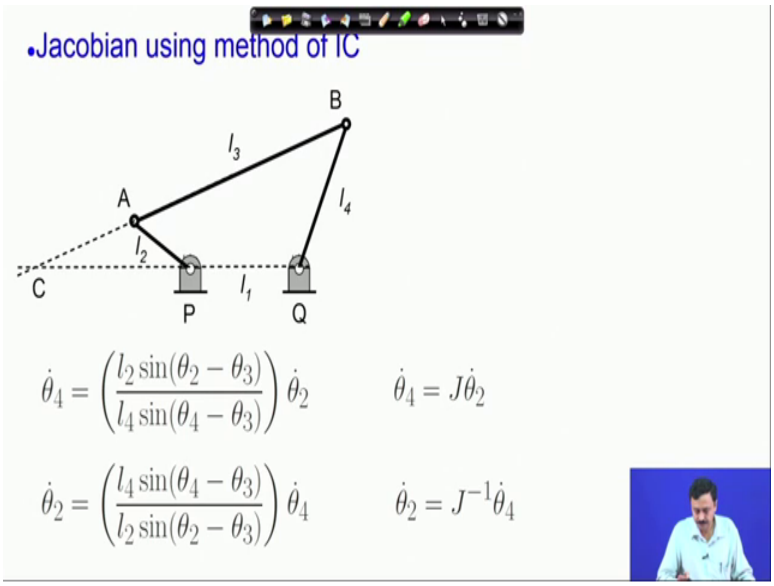

So, let us compare the Jacobian obtained using the method of instantaneous center of rotation. So, you remember that theta 4 was theta 4 dot was related to theta 2 dot using the angles theta 2 and theta 3 and the inverse relation is also given here.

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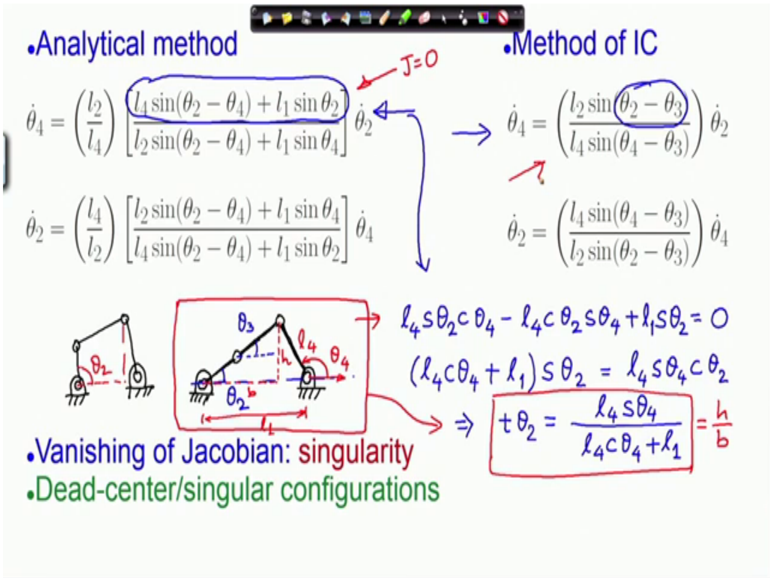

So, let me put them side by side. So, this is the analytical method relation we obtain from the analytical method. On the right I have put the expressions obtained from the method of IC. The value at a particular configuration will remain the same. The expressions may look very different, but they represent the same relation, the relation between theta 2 dot and theta 4 dot. We have already discussed this vanishing of the Jacobian indicates a singularity or a dead center configuration of the mechanism.

So, these are also called singular configurations because they correspond to the singularity of the Jacobian or the singularity of the Jacobian inverse. So, just to reiterate we have looked at these issues before. So, when you go to when you use the method of ICs we had found that this is one of the singular configurations. This was one of the singular configurations because theta 2 equals theta 3. So, therefore, the numerator if you look at this expression if theta 2 equal to theta 3 then the numerator vanishes.

Now, here also on the corresponding expression on the left, this numerator will vanish and if you do a simplification of this you will get a relation between theta 2 and theta 4. As you can see if you make the numerator these whole thing vanish, then you will get a relation between theta 2 and theta 4. Now, you can very easily find that relation by equating that to 0. So, if you do that. So, from here by equating the numerator to 0, I obtain this expression. I am opening up this sine theta 2 minus theta 4 and plus l 1 sine theta 2. So, that I am putting to 0. So, therefore, collecting these expressions so, this is what I have is equal to l 4 sine theta 4 cos theta 2. So, that implies tangent of theta 2 is equal to l 4 sine theta 4 divided by l 4 cosine theta 4 plus l 1.

So, by equating the numerator of the Jacobian obtained from the analytical method to 0, so, I have equated the numerator of this numerator of this Jacobian to 0 and obtained this expression. And, upon simplification I find that tangent of theta 2 has this expression in terms of the angle theta 4. Let us interpret this tangent of theta 2. What is tangent of theta 2? I can write it as ratio of these two distances. So, h let us say and this is b. So, tan theta 2 is h over b.

Now, what is h? Nothing, but l 4 sine theta 4 this is l 4 this is theta 4 and what is b? b is l 1; l 1 is this distance and l 4 cosine theta 4 is the projection of l 4 along our x direction. Now, that in this configuration happens to be negative. So, therefore, this is nothing, but b. So, with this configuration only this relation will be satisfied you take any other configuration this relation is not going to be satisfied. For the mechanism if you take any other configuration for example, this you will never get this tangent of theta 2 as this relation.

So, it is only at this configuration we have this relation when theta 2 equal to theta 3. So, this was not very directly observable from the Jacobian obtained from the analytical method which was very directly observed from the Jacobian obtained from the method of ICs. So, we could locate the singular configuration very easily using the method of IC. From the method from the analytical method of course, it is going to give you the same configuration, but to understand that you will have to do a few more manipulations with the Jacobian. So, here I have demonstrated to you what you have to do to correlate these two Jacobians the singular configurations from these two Jacobians.

So, whatever it is these two Jacobians that you obtain by method of IC or the analytical method they will represent the same velocity relations at a given configuration.

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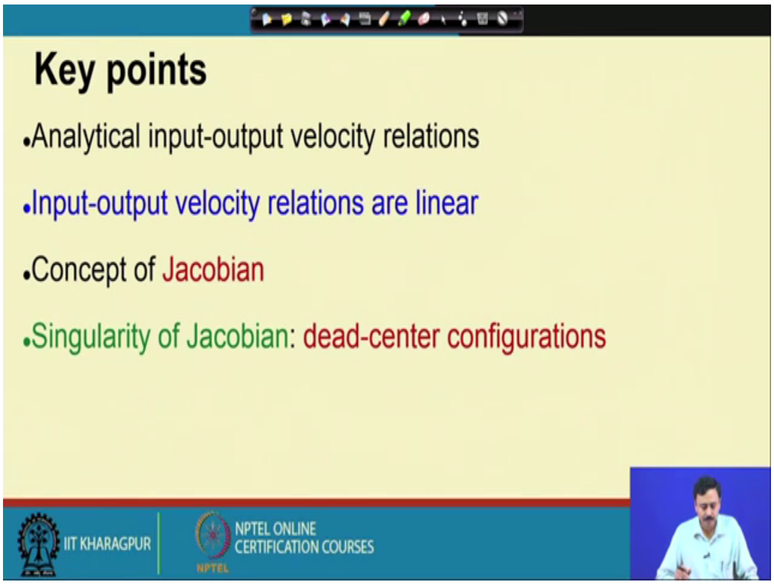

So, what we have observed is that the input output velocity relations are linear. We have used the concept of Jacobian, we have looked at the singularity singularities of the Jacobian which correspond to the dead center configurations. We have done all this for the 4R chain.

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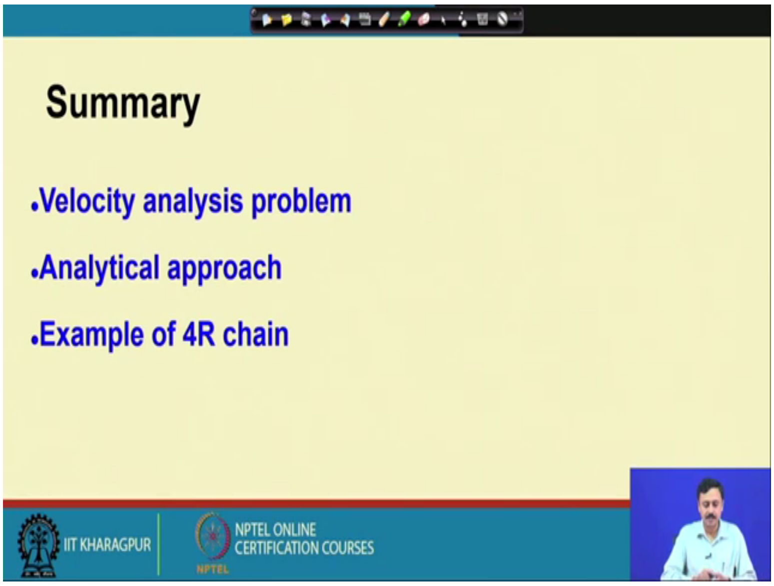

So, let me summarize we have looked at the velocity analysis problem and followed the analytical approach for the velocity analysis. We started off with the displacement relations, differentiated them to find out the velocity relations. The reason that we are following the analytical approach I have mentioned already that this is the approach, that we will take that will carry over to the acceleration analysis while the method of ICs has limitations, when you want to do acceleration analysis. So, we have looked at the 4R chain example in this lecture.

So, with that I will close this lecture.