Kinematics of Mechanisms and Machines
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Lecture – 23
Velocity Analysis: Method of IC-III

In this lecture we are going to look at the Velocity Analysis problem using geometric methods that we were discussing. So, today we are going to look at another kinematic chain of the type 3R1P.

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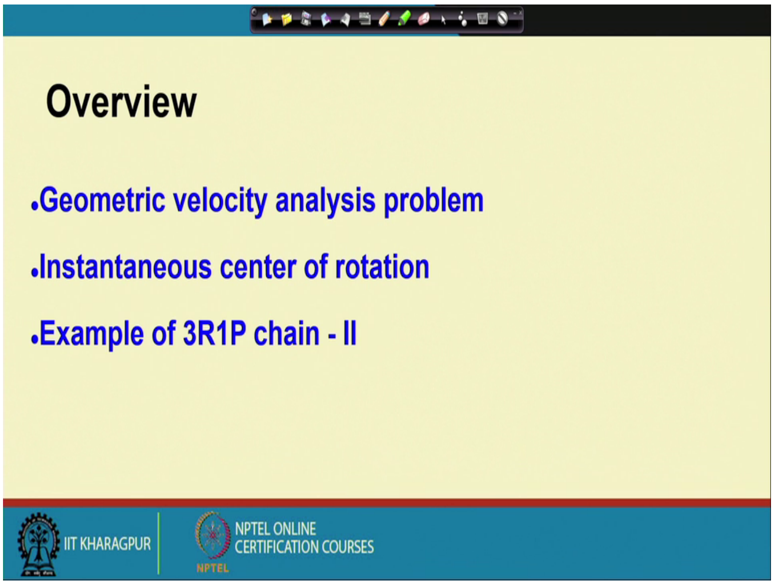

So, to give you an overview of what we are going to discuss today; we are going to continue with the geometric velocity analysis problem using the instantaneous center of rotation with the example of a 3R1P chain of type 2.

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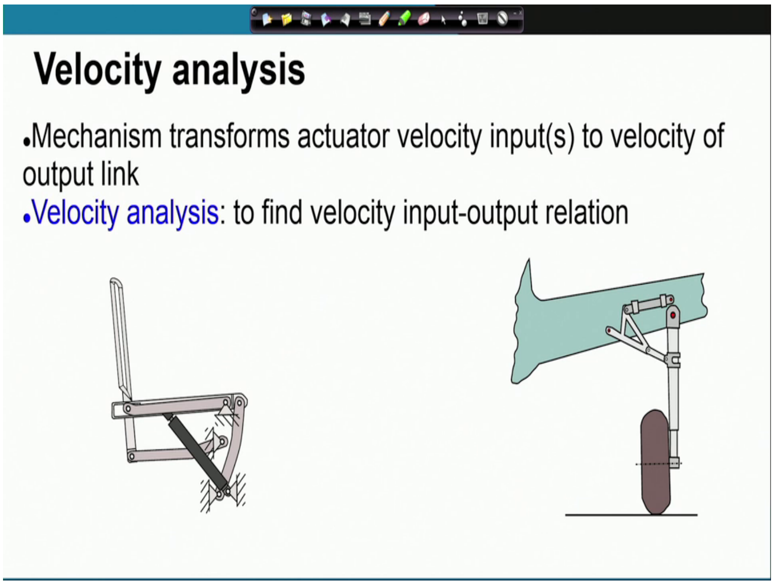

So, we have looked at the velocity analysis problem before. So, we will quickly go through that. So, essential goal of the velocity analysis problem is to find out the velocity input output relation, and the inputs are the actuators and the output is it can be a link or the end effector of robotic manipulator.

So, in the case of constraint mechanisms you have just 1 output link and corresponding to an input velocity where to find out the velocity of the output link. So, we have the 2 problems forward and inverse given the actuator rates finding out the output velocity is the forward problem. And for a specified output velocity finding out the actuator rates is the inverse problem.

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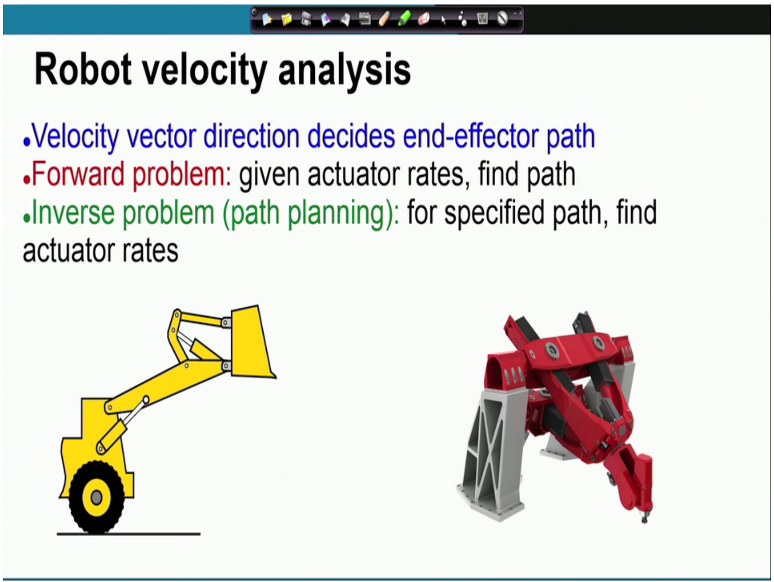

In the case of robots as we have seen before. Again the 2 problems are little more complicated than in the case of constraint mechanisms. The reason is the velocity vector is tangential to the path. So, you if you are given a path you can specify the tangent vectors and you can correspondingly find the actuator rates in order to produce that velocity vector at a certain point on that path.

The inverse problem is the path planning problem in which the path is specified and you are find out the actuator rates.

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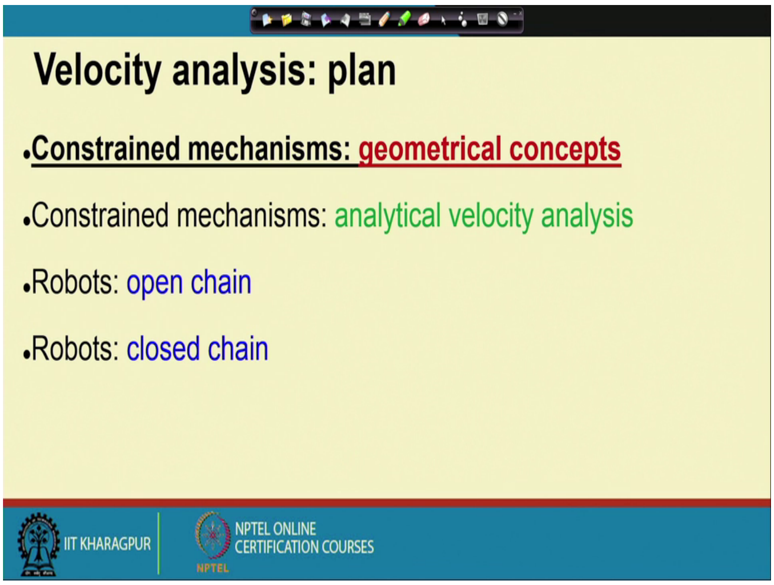

So, according to our plan, we are discussing the geometrical concepts of velocity analysis for constant mechanisms. We will look at the analytical velocity analysis in the subsequent lectures which will be followed by velocity analysis of robotic manipulators.

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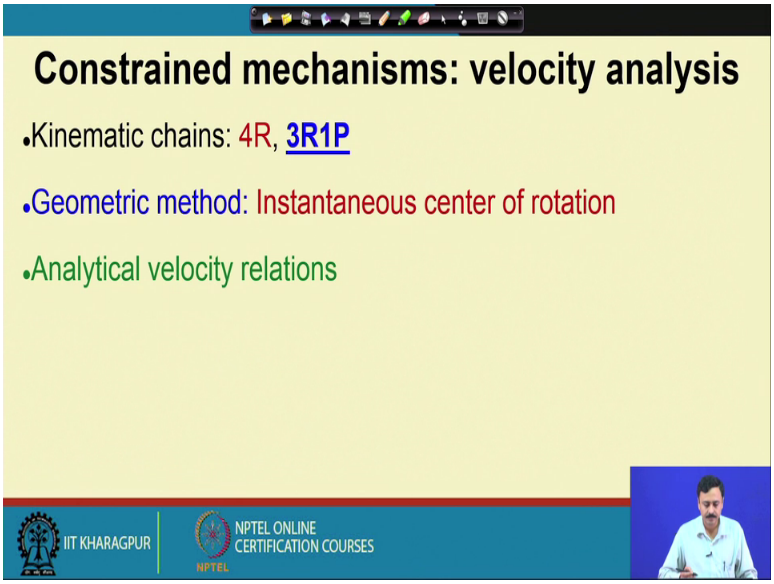

So, in the constraint mechanism examples today we are going to look at the 3 R 1 P chain of type 2 we are going to use the geometric method using the concept of instantaneous centre of rotation. And we will finally derive the analytical velocity relations based on this geometric method.

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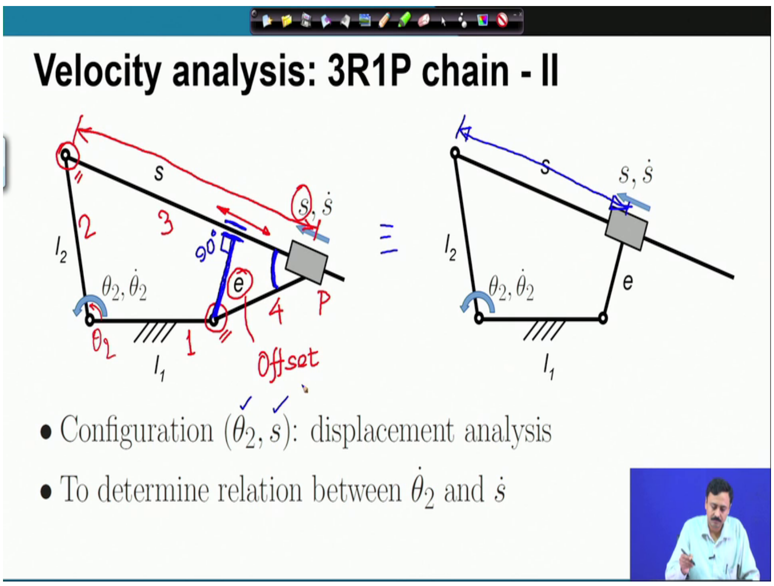

So, here is the 3R1P chain type 2. Here we have the angle theta 2 and s this variable s is this length this length is s. So, this is the expansion of the prismatic actuator. So, here we have the prismatic actuator. So, s is the length of expansion of the prismatic actuator. Previously we have defined this offset e which is measured in the direction perpendicular to the direction of the prismatic pair. So, this is the direction of expansion of the prismatic pair or motion of the prismatic pair s e is measured as the distance perpendicular to this direction of motion between the 2 hinges connecting the 2 links of the prismatic pair. So, the prismatic pair is between. So, if I number the links this is 1 the ground is 1 this link is 2 3 and this is 4.

So, this prismatic pair is between link 3 and 4 and at the other end of link 4, we have this revolute pair. And on the other end of the link 3 we have the other revolute pair. So, it is the distance between these 2 revolute pairs measured perpendicular to the direction of sliding. So, that is offset.

As you can see this angle is going to remain fixed this angle is going to remain fixed, because the prismatic period does not allow relative rotation between the links 3 and 4. Therefore, I can always shift this prismatic pair here. So, that this angle is 90 degree. So, length of link for then I mean this length becomes e exactly e this simplifies the analysis substantially as you will see. Therefore, what we are going to do is replace this pi the equivalent kinematic chain as shown on the right these 2 are absolutely equivalent and we redefine s as this distance that is s.

So, once we have analyzed the mechanism of the right we can always go back to the original mechanism, because these angles as I have mentioned between angles between 3 and 4 which remains fixed we can always find the corresponding motions of the mechanism shown on the left.

So, what is the velocity analysis problem? So, we are specified the configuration of the mechanism.

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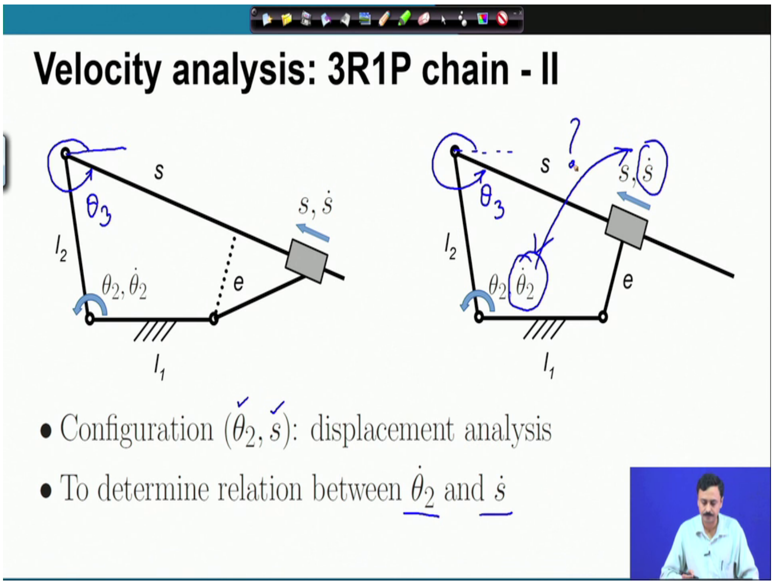

Now, since this is a constant mechanism specification of theta 2 will also specify s, because we know from displacement analysis that given theta 2 I can always find s. I can also find out this angle this is theta 3 or in this here. So, given figure 2 I can find out using displacement analysis as well as theta 3 or I might be specified s then also I can find out theta 2 and theta 3 this we have already discussed before. So, the problem of velocity analysis is determined the relation between theta 2 dot and s dot. So, we have a theta 2 dot s dot we want to find out the relation between these 2.

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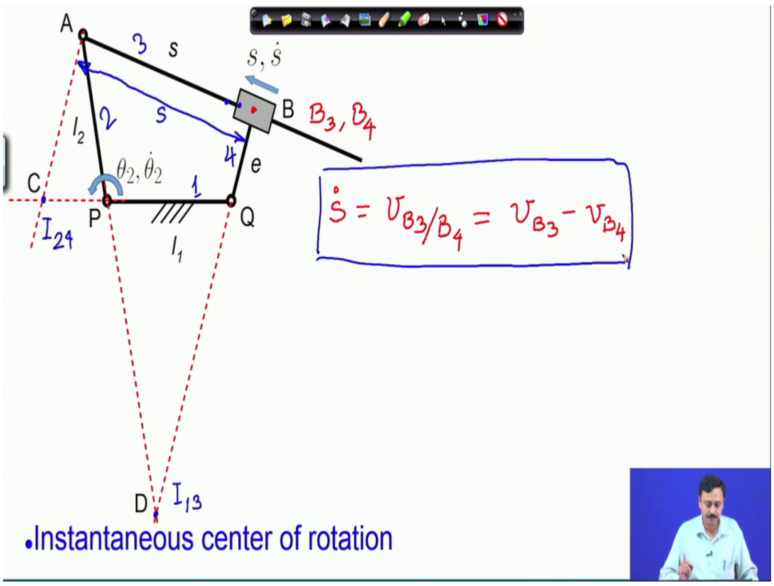

So, previously we had discussed about the instantaneous centre of rotation. We had seen that we can determine these 2 instantaneous centre of rotation. So, this instantaneous centre of rotation is I 13. So, let me again number. So this is 1, this is 2, this is link 3 and this is link 4. So, this is I 13 which is indicated by D and there is another instantaneous centre of rotation which is I 2 4 denoted by C. So, we are going to use these 2 instantaneous centers of rotation in this analysis. There is one fine point that must be remembered that me point that out this s of course is this distance.

So, s is this distance. Now s dot is the velocity of a point belonging to link 3 with respect to a point belonging to link 4. Since s is this expansion of the prismatic actuator. Therefore, s dot is the relative velocity between a point on link 3 and a corresponding point on link 4 to making more specific. Let me consider a point here. So, this point is denoted by B, now B can have 2 locations the coincident points B 3 and B 4. So B 3 B 3 belongs to link 3 and B 4 belongs to link 4. So, is the relative velocity between B 3 and B 4 that is s dot. So, s dot should remember is the velocity of B 3 relative to B 4.

In other words this is equal to v B 3 minus B 4. So, you have to remember this that s dot is the relative velocity between B 3 and B 4 which is given by vb 3 minus v B 4.

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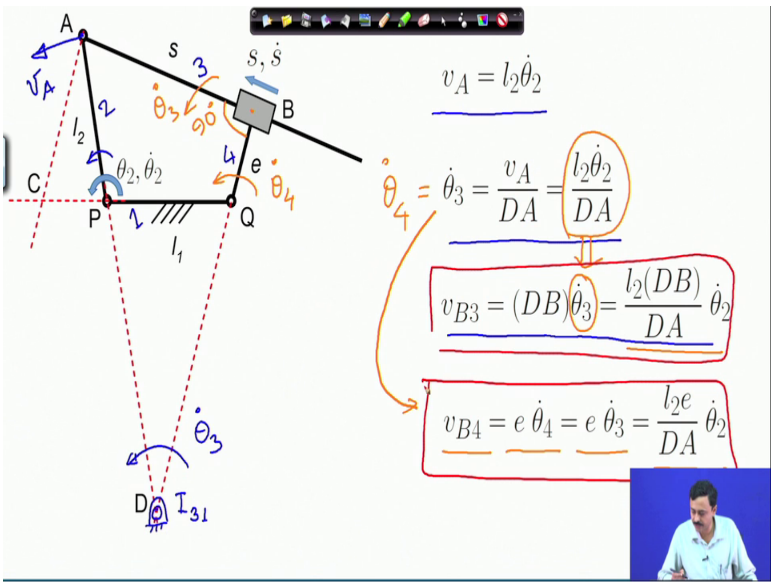

Now, here I have written out all the velocity relation let us go through 1 by 1 the first velocity relation says that v a is l 2 times theta 2 dot.

So, what is theta 2 dot? Theta 2 dot is the angular speed of link 2 times l 2 is a velocity of point A, this we know. So, this is v A then I can find out the angular velocity of link three. So, this is link 3 I can find out angular velocity of link 3 which I have written out as theta 3 dot is v a divided by the distance DA. So, velocity of point a divided by this distance DA because D is the instantaneous centre of rotation, remember for link 3. So, this is I 1 3 or 3 1. So, as if link 3 is hinged at this point. So, if I know velocity v A, I can find out the angular speed of link 3 using the angular speed of link 3 I can relate velocity of D 3.

So, what I have done here? I have multiplied theta 3 dot so, theta 3 dot. So, this you can consider that this is the angular speed of link 3 which is this whole triangle a DB so and D 3 a DB 3. So therefore, velocity of B 3 is DB this radial distance DB time theta 3 dot. Now here I just replaced this theta 3 dot in terms of theta 2 dot. This I have replaced to get this expression. Now this one more interesting observation this angle this is 90 degree we know.

And this angle remains fixed whatever be the relative motion of 3 with respect to 4. So, the sliding motion of 3 with respective 4; whatever happens this angle 90 degree is going to remain 90 degree. So therefore, the angular velocity of link 3 is same as the angular velocity of link 4. So, this is theta 3 dot will be same as theta 4 dot. Why? Suppose I have this link which maintains this angle. If my pump has angular velocity theta 3 dot let us say then you can see my tumbles. So, has the same angular velocity because this angle is remaining fixed.

So, for that same reason here theta 3 dot is also equal to theta 4 dot. Now I relate v B 4 which is a point on link 4 v B 4 is a well is B 4 is a point on link 4 and v B 4 is the corresponding velocity of that point on link 4 that must be e times theta 4 dot; e being the offset as you can see from the figure. Now theta 4 dot is equal to theta 3 dot; so, you can replace this

And since, I have that expression of theta 3 dot, I use it here and I get v B 4. So, I have expressions of v B 3 and I have expressions of v B 4. So now, it is just a matter of taking the difference to relate s dot which I am going to do now.

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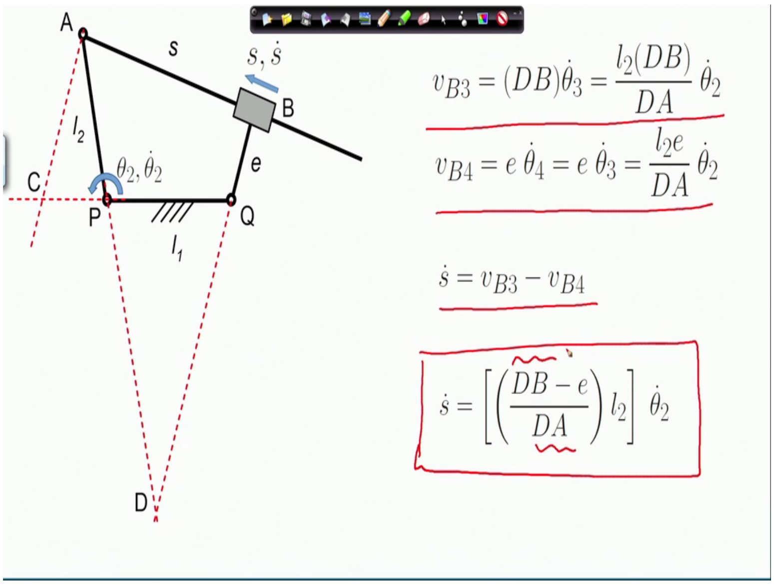

So, here I have rewritten these expressions of v B 3 and v B 4 and just few slides back I showed you that s dot is equal to v B 3 minus v B 4. So therefore, s dot I can write like this in terms of theta 2 dot.

Now, here we have these 2 as yet unknown lengths DB and DA which we are going to now determine from geometry.

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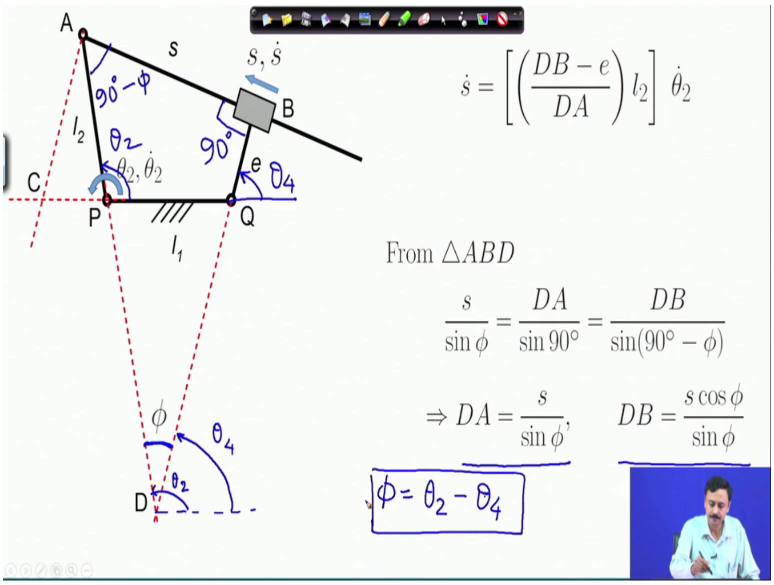

So, I have again written out that expression of s dot relating with theta 2 dot. As I mentioned DB and DA are as yet unknown let us look at this triangle ABD; so ABD. So, we are looking at this triangle ABD. So, using sine rule I can easily write s divided by this angle sign phi sign of this angle. So, s divided by sin of this angle phi is equal to DA divided by sin 90 degree this angle is 90 degree.

And also equal to DB divided by sin of 90 minus phi. So, if this is phi, this is 90. So, this must be 90 minus phi. So, that implies DA is equal to s divided by sin phi and db is s cosine phi divided by sin phi. Now what is phi? If you see that this angle is theta 2. So, this angle is theta 2 and this angle is theta 4. So, in other words this angle is theta 2 and this angle is theta 4.

So therefore, there is the relation between phi theta 2 and theta 4 in this case it turns out that phi is equal to theta 2 minus theta 4. So, you can relate phi using theta 2 and theta 4 which we can find from the displacement analysis.

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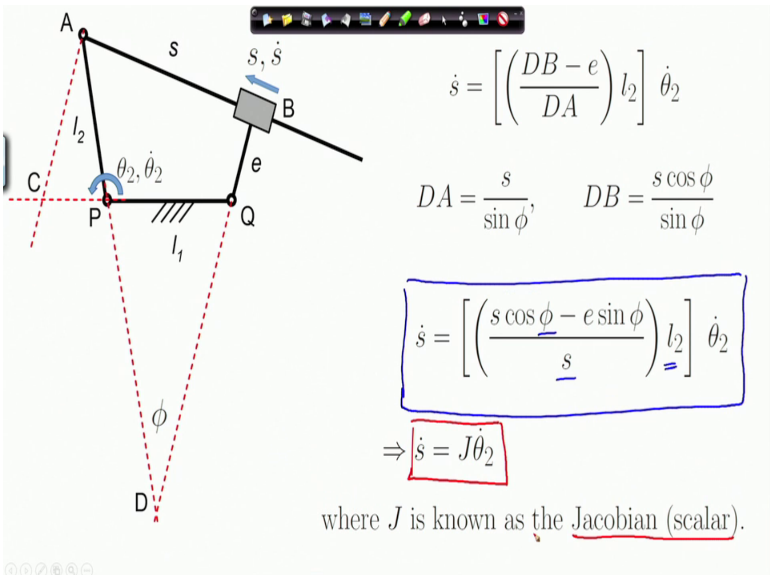

So, let us move on. So, we have these expressions of s dot DA and DB, we have just now derived. So, when we substitute and simplify, finally we have this expression which uses s phi and others are the link parameters; so, s l 2.

So, we have s phi l 2; l 2 is fixed s can be determined if we are given the configuration and similarly phi can also be determined from the configuration. Now we write this in a compact form this relation in a compact form using the Jacobean which we have introduced. Here the Jacobean is a scalar once again it is a number. So, given s and phi you can write down the Jacobean you can determine the value of the Jacobean.

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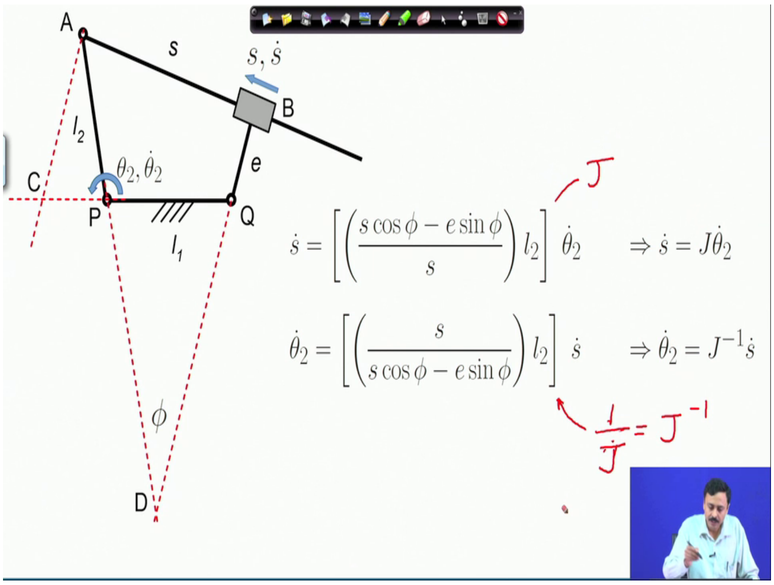

So therefore, we have these 2 relations. Now in the second relation I have inverted, so this is the Jacobean. And this is the Jacobean inverse this is 1 over the Jacobean so, which I am writing as Jacobean inverse.

So, once again we are face with the same question is the Jacobean invertible or is Jacobin inverse invertible whether they can go to 0.

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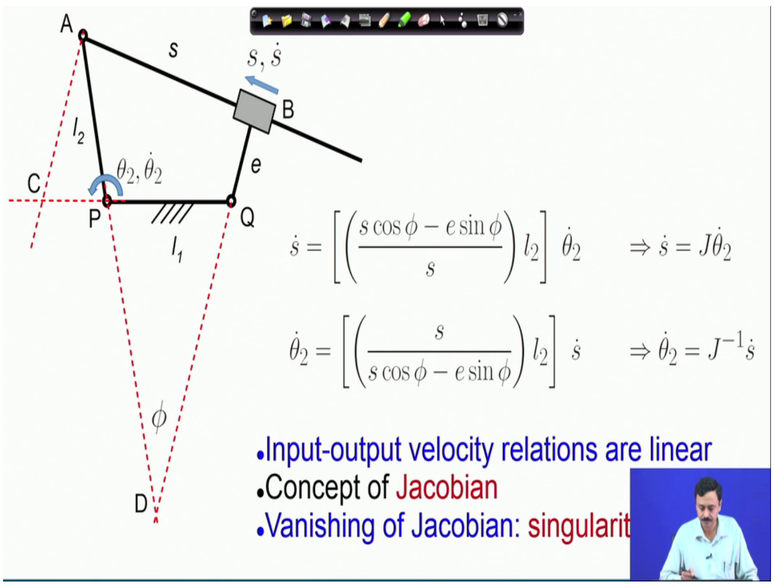

So, let us look at that issue. So, we have already observed that the input output velocity relations are linear and they are related through the Jacobean. So, you come to this question of vanishing of the Jacobean or its inverse which indicates singularity in the configuration in the mechanism.

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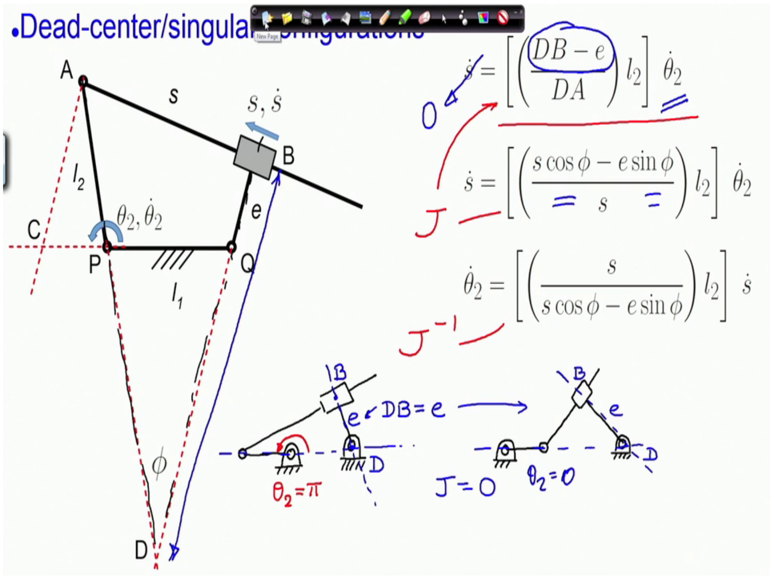

So, here I have written out the relations both direct and inverse. So, we can see very easily that. So, this is the Jacobean and this is the Jacobean inverse. So, when is so, this is also the Jacobean the question is when is the Jacobean 0 is very easy to see from the first expression that the Jacobean goes to 0 when the numerator vanishes which means DB is equal to e so, D b.

So, this distance becomes equal to e. Now how can this happen? This can happen in this way. Remember D is located by the intersection of a line through the point Q in the direction perpendicular to the sliding direction and through line through A and P. So, what happens in this case?

Let us see what happens in this case. So, the line through this is passing like this and the other line passing like this. So, where is D, the intersection? So, that is D. And where is B? This point is B. You can where easily see now the DB is equal to e. So, this is the configuration where the Jacobean vanishes. So therefore, theta 2 is equal to pi or 180 degree.

There can be another configuration. Let me draw that. This is a configuration where again DB is equal to e. So, this is also the configuration where the Jacobean vanishes. So, in both these 2 configurations the Jacobean vanishes. So, it is a non invertible. So, which means whatever be theta 2 dot whatever be theta 2 dot s dot 0 because the Jacobean is 0 irrespective value of theta 2 dot.

So, which means the prismatic actually cannot expand at this these 2 configurations. So, these are singular configuration. Here, theta 2 is equal to 0. You can find these relations in terms of phi as well using this expression. What is the other situation? The other situation is the other singularity is when the Jacobean inverse.

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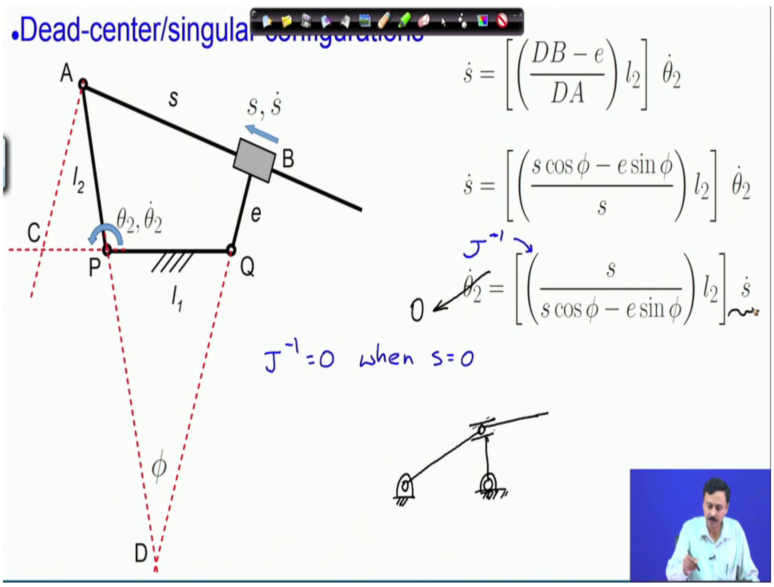

So, this is Jacobean inverse. So, Jacobean inverse vanishes when s equal to 0 and when can that happen? At this configuration, something like this. So, here s equal to 0 and that is a singularity where theta 2 vanishes irrespective of value of s dot.

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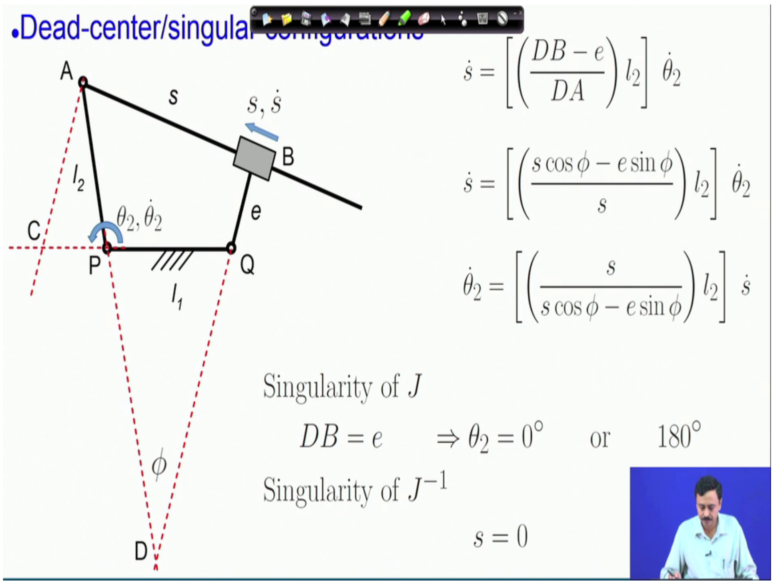

So, these are the 2 singular configurations which I have written out for you.

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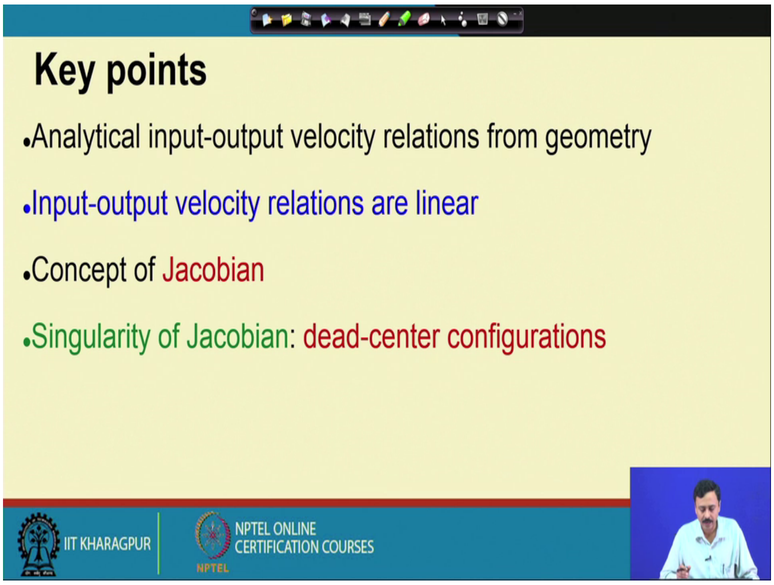

So, what are the key points? We have a look at the analytical input output velocity relation from geometry which are linear. The input output velocity relations are linear introduced the concept of Jacobean, and we have looked at the dead center configuration of the mechanism.

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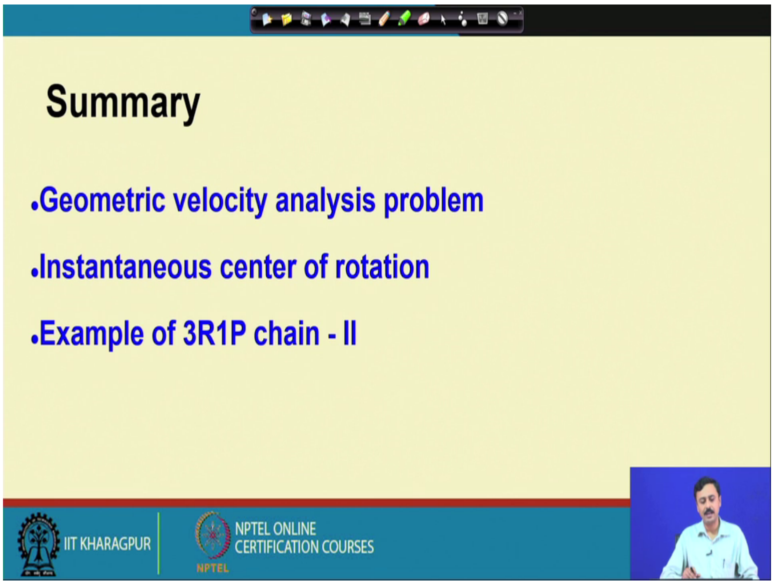

So, with that let me summarize. We have looked at the geometric velocity analysis problem for the 3R1P chain of type 2, and we have looked at the singularities of the kinematics chain.

So with that, let me close this lecture.