Kinematics of Mechanisms and Machines
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Lecture – 22
Velocity Analysis: Method of IC – II

We will be discussing the Velocity Analysis problem for constraint mechanisms, which we have started. So, in this lecture we will consider the same problem, we will go through this the velocity analysis problem based on geometric methods. And we will look at different example today.

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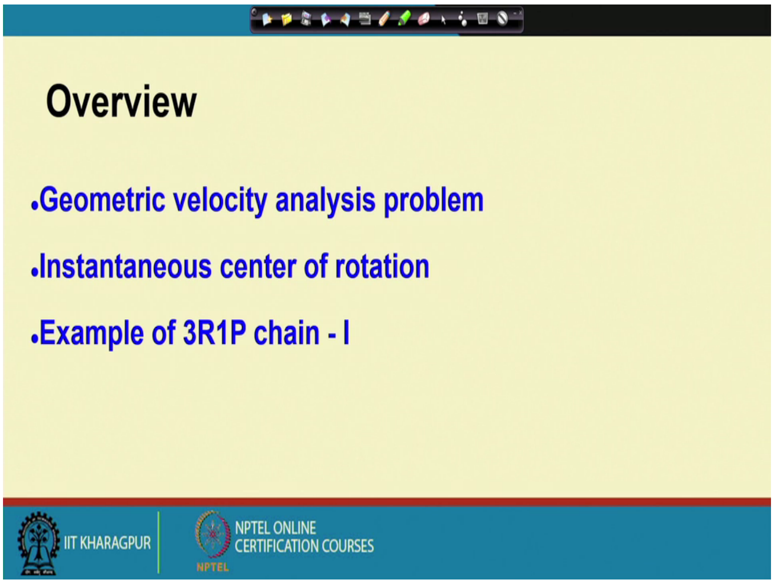

So to give you the overview, we are going to look at this velocity analysis problem using the instantaneous center of rotation concept for a 3R1P chain of type 1.

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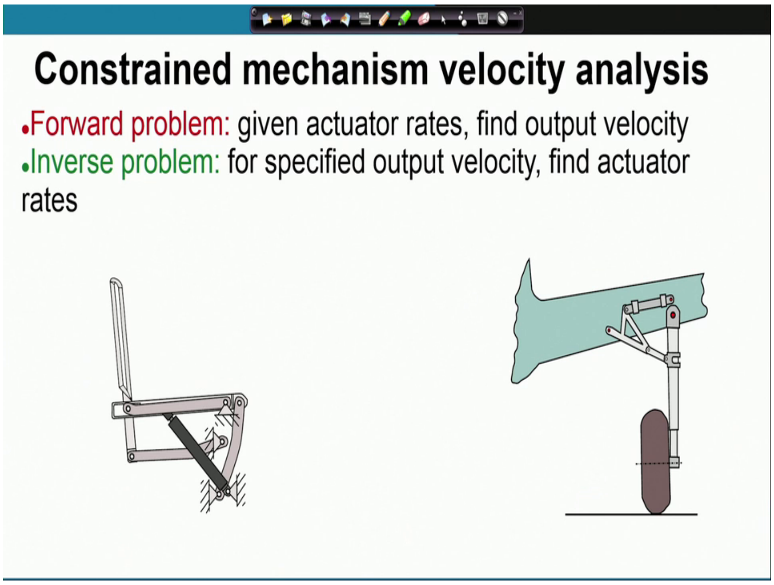

So, we have looked at what velocity analysis problem is about? It is about finding the input output velocity relations. And we have the 2 problem forward and inverse problems which are very easily found, if we have solved 1, the other is very easy to find.

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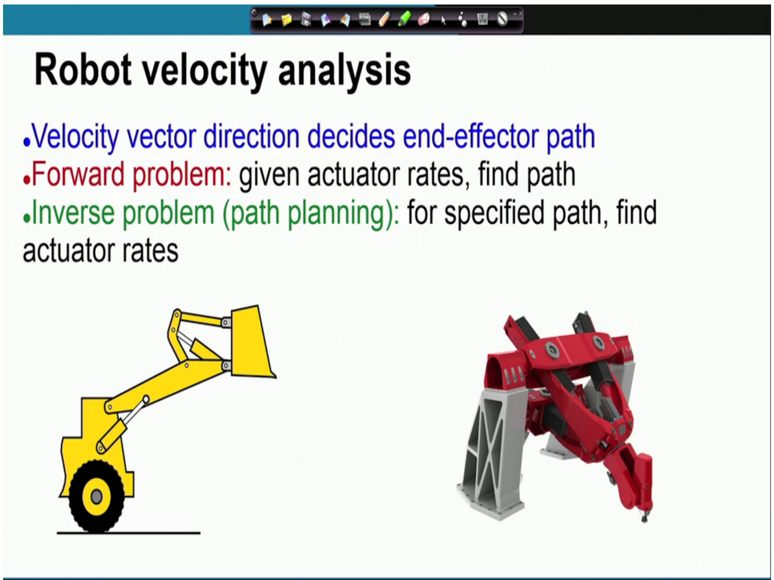

In the case of robots, we have seeing that velocity analysis problem is little more complicated, where we need to specify the velocity along a path, because the velocity vector will always be tangent to the path and corresponding to that velocity vector. We can find out the actuator rates through the velocity analysis by solving the velocity analysis problem.

So, this inverse problem in which the path is specified we have to find out the actuator rates, this is known as the path planning or path generation problem.

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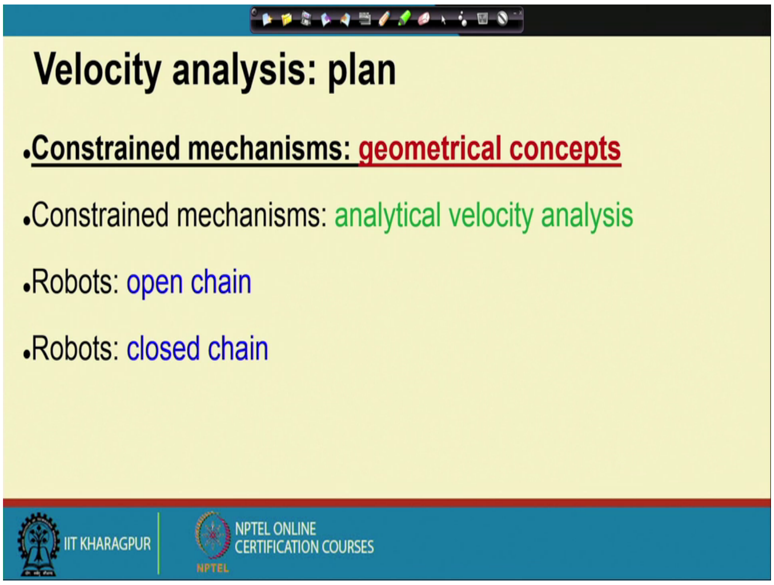

So, we are going to continue our discussions on constraint mechanisms velocity analysis through geometric concepts. Later on we will look at analytical velocity analysis and also for robots with open and closed chain configurations.

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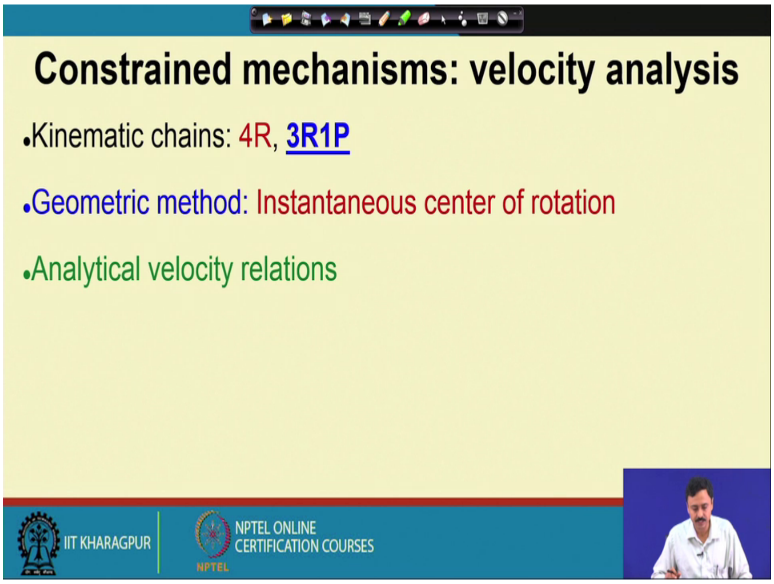

In this lecture we are going to discuss the 3R1P chain, using the geometric method of instantaneous center of rotation. And we are going to derive analytical relations for the velocity.

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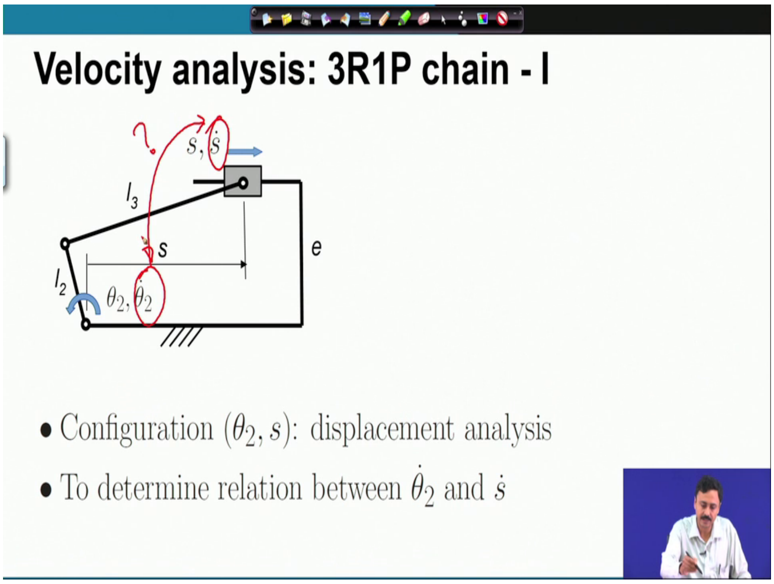

So, this is our 3R1P chain type 1. Here we have this angle theta 2 and the displacement of the slider S. Now the configuration of the mechanism is specified either by specifying theta 2 or by specifying S. And through the displacement analysis problem, by solving the displacement analysis problem I can solve the other. So, when I say configuration is given then, I mean that everything like theta 2 S and even this angle this angle theta 3 are known to me. So, when I say the configuration is known to me, I would mean that theta 2 S, theta 3 is there all known to me and this is ensured by the displacement analysis problem. If I solve the displacement analysis problem, I have solve for these relations. So, I know the configuration of the mechanism.

The problem is now to determine the relation between theta 2 dot and S dot. So, I want to find out the relation between these 2. Given the configuration I would like to find out the relation between theta 2 dot and S dot, that is the velocity analysis problem.

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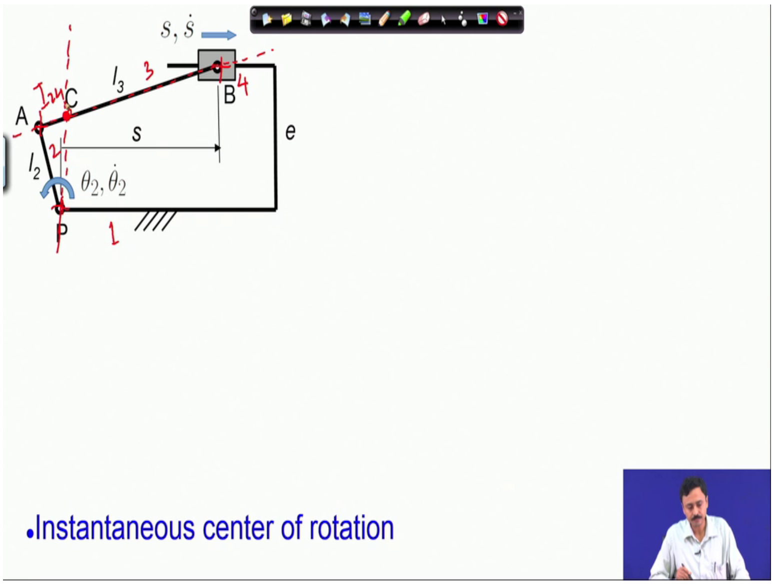

So, for this as I have mentioned, we are going to use the geometric concept of instantaneous center of rotation. So, the instantaneous center rotation that I will use for this analysis is I 2 4. So, this lies on this line passing through I 2 3 that mean mark out this is 1, this is 2, this is 3 and the slider is link 4. I want to find out I 2 4.

So, I 2 4 must lie on the line connecting I 2 3 to this point and I 3 4, it must also lie on the line connecting I 1 2, which is the ground hinge P and I 1 4, which is that infinity remember and this is obtain by drawing this line perpendicular to the direction of sliding through the point P. So, this point is I 2 4. So, formally i have done this for you, so C is the instantaneous center of rotation I 2 4. So, this is I 2 4.

What is the significance of I 2 4, let us recollect. It is the point on extensions of bodies 2 and 4 which have the same velocity at that point. So, coincident point belonging to link body four which is the slider and link 2 which is this link; so the coincident point belonging to links 2 and 4 at this point at point C has the same velocity and which must of course, something like this.

So, this velocity must also be the slider velocity. That means, velocity of body 4 and it also must be the velocity of a point this is on the extension of link 2; that is concept of I 2 4.

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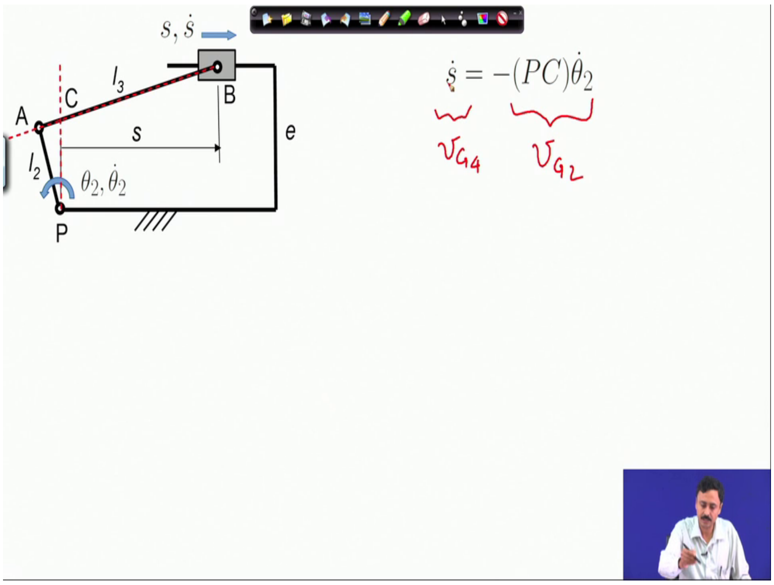

So, what do we do? With the sin convention that I have taken that theta 2 dot is counter clockwise positive while S dot is positive to the right, I must write S dot is equal to minus the distance P C times theta 2 dot has to be this is the relation, this is what the point C stands for. The velocity at that point belonging to the link 2 is same as the velocity of the slider. So, in other words, this is nothing, but velocity of G 2 and this is also equal to then, velocity of G4 which is also S dot. So, this is the idea.

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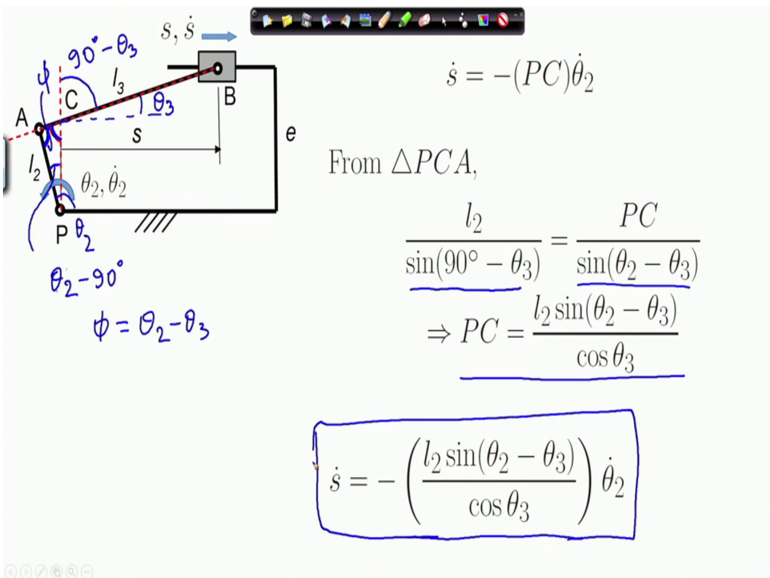

Now, we have to find out P C. To do that, we look at triangle PCA. So, you can that easily find out PCA and use the sin rule. You can see that l 2 divided by this angle. Now what is this angle? Remember that this is theta 3. So, that angle that link 3 makes with the x axis or the horizontal axis is theta 3. So therefore, this angle must be 90 degree minus theta 3.

And that should also be equal to then this angle. So, therefore, l2 divided by sin 90 degree minus theta 3, must be equal to PC, PC is this distance this arm of the triangle divided by the sin of the angle opposite to it. So, this is the angle, which you can very easily relate as theta 2 minus theta 3, how? This angle is theta 2, this angle is theta 2. So, therefore, this angle is 90 degree minus theta 2.

So, this angle is 90 degree, so theta 2 minus 90 degree. So, this angle will be theta 2 minus 90 degree, and if I call this angle as phi which I want to determine, then you can very easily see that phi must be equal to theta 2 minus theta 3. So that is what I have written out. So, from this, I can solve for PC is l 2 sin theta 2 minus theta 3 divided by cosine theta 3.

Now, there is 1 point to be noted here, I have drawn the mechanism as certain configuration and for that this is the calculation. If the mechanism is in a different configuration, there the procedure remains the same; the procedure will remain the same, the relations might change or the triangle might appear in a different manner and finding out these angles might be slightly different. So, this angles my turn out be slightly different, but the basic approach remains the same.

So, we are going to look out such a will going to find out such a triangle and we can relate, we can find out this distance PC geometrically, so that is our objective. So, we are finding out PC by geometry, the distance PC by geometry.

So therefore, if I substitute this expression; then S dot is equal to minus of this expression this PC times theta 2 dot. So, we have found the relation between S dot and theta 2 dot.

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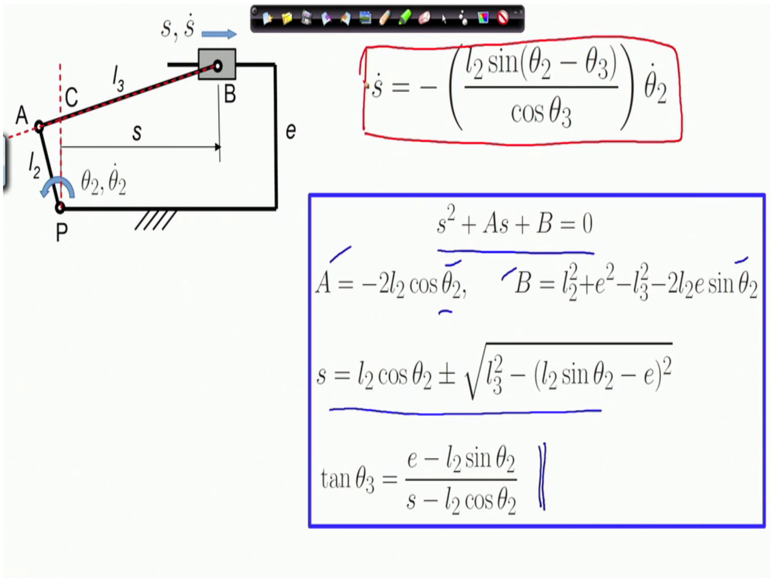

So, this is our relation. Now remember our displacement analysis. So, given theta 2, let us say if I am given theta 2, then through this displacement analysis, we had found S here, A B C, A B are completely known because, theta 2 is given, these are known; so AB unknown. Hence we could solve for S and we call we could also solve for theta 3.

So therefore, in this relation of velocities, I know everything.

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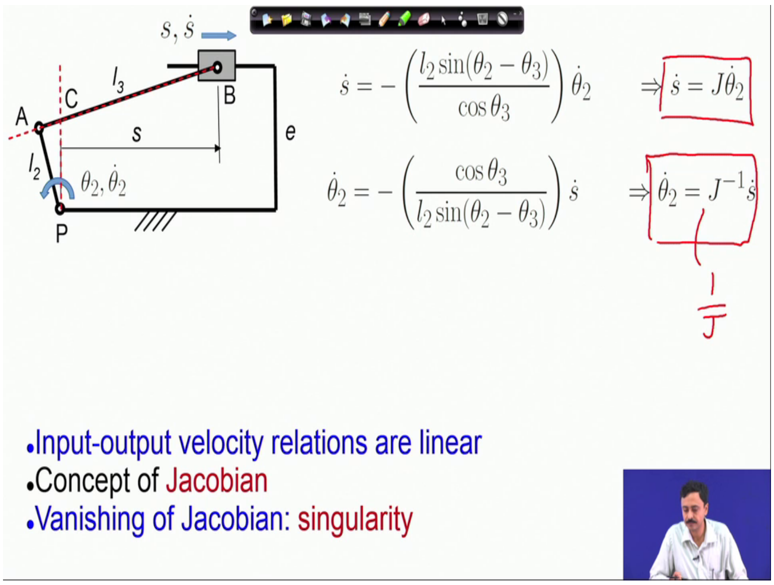

So, I will write this in a compact form as I have done here. So, s dot is the Jacobean times theta 2 dot and by inverting the Jacobean can write theta 2 dot is Jacobean inverse, which is again one over J, Jacobean inverse here it is a scalar. So, Jacobean inverse is 1 over J times S dot. Again we find that, the input output velocity relations are linear and we have introduce the concept of Jacobean and vanishing of the Jacobean implies singularity. This is what we are going to study next.

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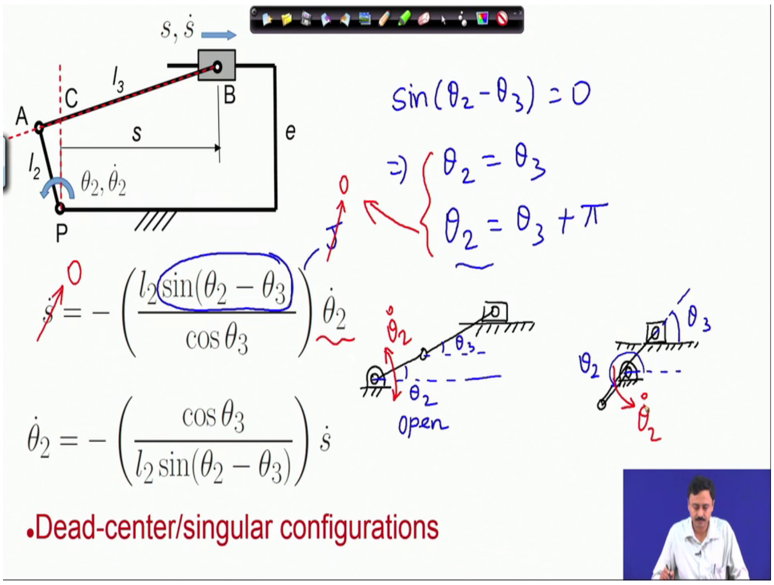

So, singularity is again take us to the dead center configurations. So, let us understand these singularities. So, this is our Jacobean. When is Jacobean singular; that means, non invertible when it becomes 0, Jacobean is singular or non invertible when it becomes 0. So, in other words sin of theta 2 minus theta 3 is 0, that implies either theta 2 equal to theta 3 or theta 2 is theta 3 plus pi radian. So, both these 2 conditions we have the singularity of Jacobian.

Now, what does that imply? Jacobian goes to 0, at such configurations the Jacobian goes to 0, which means that irrespective of value of theta 2 dot S dot goes to 0; that means the slider is not going to move. So, slider is in the dead center configuration, mechanism is that center or singular configuration. So, let us see these configurations. So, there are 2 solution, so corresponding these 2 solutions, we have the singular or dead center configurations. This is one where as you can see theta 2 is equal to theta 3. So, this is the open configuration.

There is another configuration corresponding to the second solution here, where we have the situation like this. So, theta 2 is this angle and theta 3 this angle, now you can very easily that theta 3 plus 180 is theta 2. So, this is the second dead center configuration and at these configurations irrespective of how theta 2 dot is specified or how theta 2 is changing irrespective of the value of the theta 2 dot; we will have the slider at the static position.

So, thus the slider velocity will go to 0. So, this is corresponding to the singularity of J. Now let us look at the other situation.

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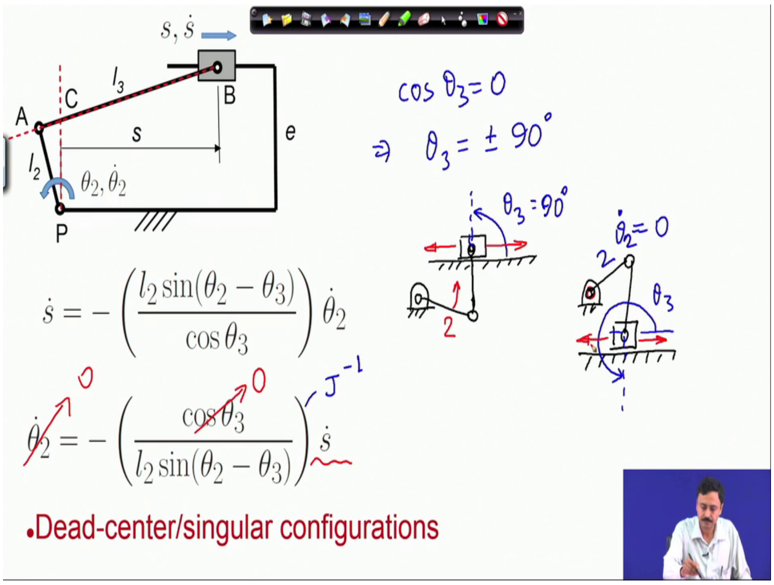

This is J inverse or 1 over J. Now when will this go to 0? So, this J inverse goes to 0, when cosinetheta 3 goes to 0; that implies theta 3 is equal to plus or minus 90 degree. Then cosine theta 3 is 0. So, the again there are 2 configurations. Let us look at this configuration first. If you can have a situation like this as in a mechanism in a 3R1P chain, where you see this angle, this angle is theta 3 is 90 degree. Then what this says is the numerator of the Jacobian inverse goes to 0.

So, therefore, irrespective of the value of a S dot theta 2 dot goes to 0. You can realize when I try to move the slider left or right at this configuration, this link 2 cannot rotate. If I move to the right if I move to the right, if I move slider to the right, this will move up, if I move to the left also this will move up, but exactly at this configuration it cannot move any further, it has to come to a stop. This is the 0 velocity configuration at the dead center configuration of the link 2.

In a similar manner I can have the other configuration, configuration of minus 90 degree in situation like this, remember that angles or being measured like this. So either you say that this is 270 degree or minus 90 degree. So, both are same. So, this is another singular configuration, where link 2 cannot move. So, theta 2 dot must necessarily be 0, has to be 0 irrespective of the motion of the slider. So, this is another configuration which is singular configuration or a dead center configuration.

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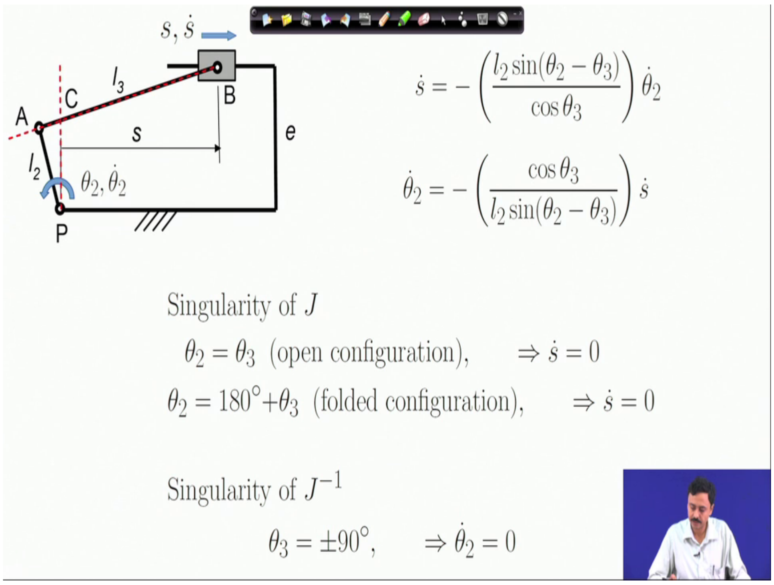

So, I have written out these velocity relations again. So, singularity the Jacobian occurs when theta 2 equals equal to theta theta 3. The singularity is an open configuration 1 where S dot 0 or you can also have singularity when theta 2 is 180 degree plus theta 3, in which we have the folded configuration, in which also S dot is 0.

If you look at the singularity of J inverse, then we have at theta 3 equal to plus or minus 90 degree and here theta 2 dot is 0. That means, the link 2 cannot move at exactly at that configuration, it must have 0 velocity irrespective of the velocity of the slider.

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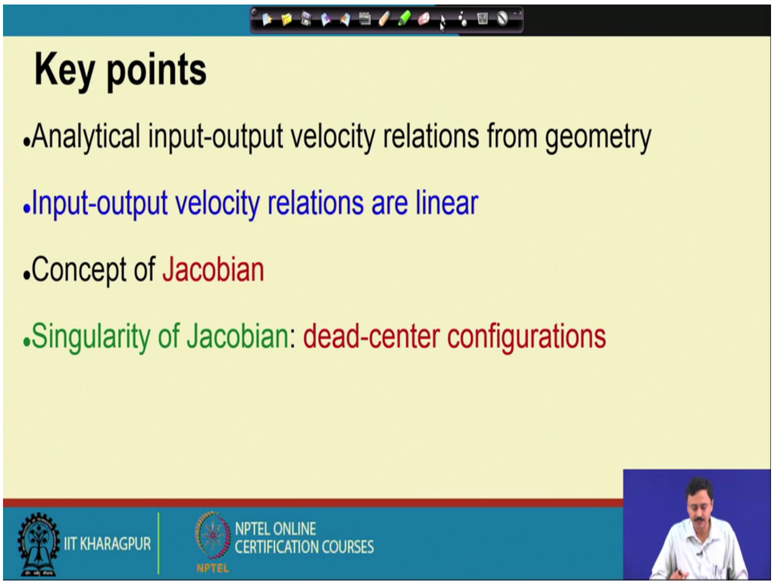

So, let us recapitulate the key points. We have found the analytical input output velocity relation from geometry. Now here I must mention again that we have use the concept of instantaneous center of rotation of the mechanism and then from geometric relations we have try to find out certain lengths.

Now if the configuration of the mechanism changes the procedure remains the same, but the triangle may look different, the triangle may look different, but we have to divides the methods of finding out these distances like here. We had this distance from the ground hinge to the instantaneous center. So, that we determine geometrically. So, that geometry one has to do, irrespective of the configuration mechanism and every time you will find that this can be done.

So, we have found the analytical input output velocity relations from geometry, which were found out to be linear if introduced again brought in the concept of Jacobian. And the singularity of the Jacobian gave as the dead center configurations of the singular configurations of the mechanism. So, this dead center configurations, singular configuration they are synonymous with the singularity of the Jacobian or the singularity of the inverse of the Jacobian.

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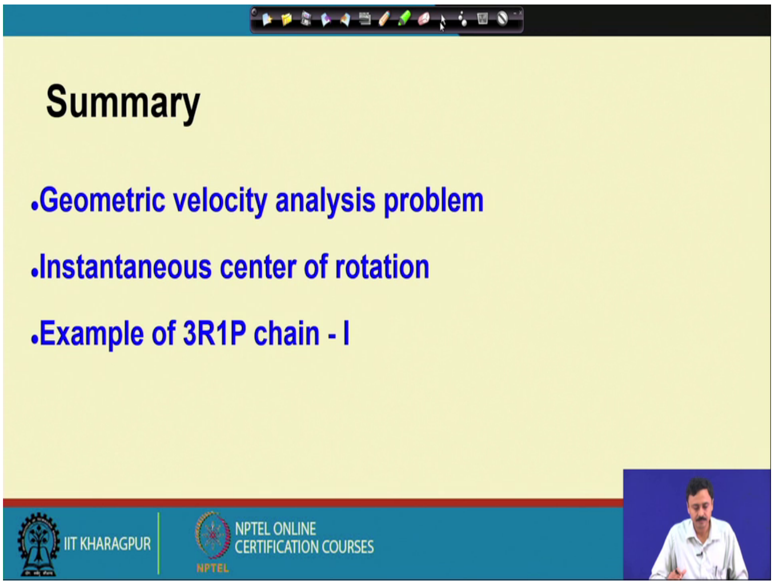

So let me summarize: we have discuss the geometric velocity analysis problem using the concept of instantaneous center of rotation and we have considered this example of 3R1P chain of type 1. We have looked at the singularity of the input output velocity relations and we have discussed about the dead center configuration of the mechanism.

With that I will conclude this lecture.