Kinematics of Mechanisms and Machines
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Lecture – 21
Velocity Analysis: Method of IC – I

So, we will be discussing the Velocity Analysis problem in this lecture. We started our discussions on velocity analysis with certain geometric concepts, and we are going to look into the application of these geometric concepts in velocity analysis of mechanisms.

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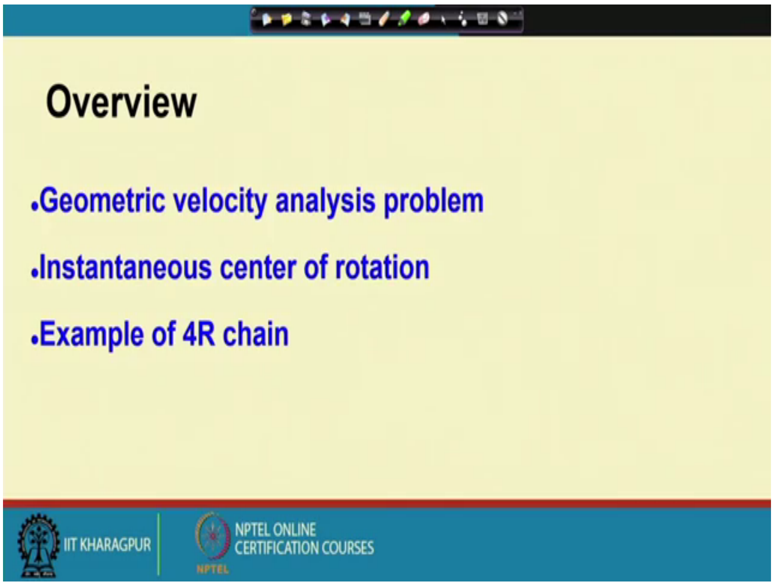

So, to give you an overview of what we are going to discuss in this lecture; we are going to discuss the geometric velocity analysis for mechanisms, using the concept of instantaneous center of rotation. And in this lecture we are going to look at the 4R chain.

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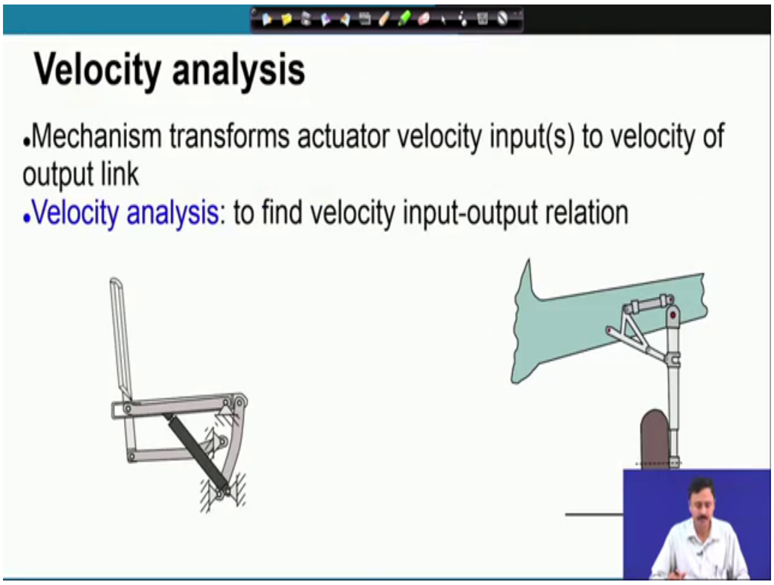

So, we have already discussed let me just review what we discussed about velocity analysis. So, mechanisms transform the actuator inputs there can be multiple actuator inputs velocity inputs to the velocity of the output link. Now this output link could be having a translator motion or a rotary motion or it may be motion in space on the plane or in space, it may also have orientation etcetera. So, the problem of velocity analysis is to find the velocity input output relation; input output means from the actuator to the output and the reverse.

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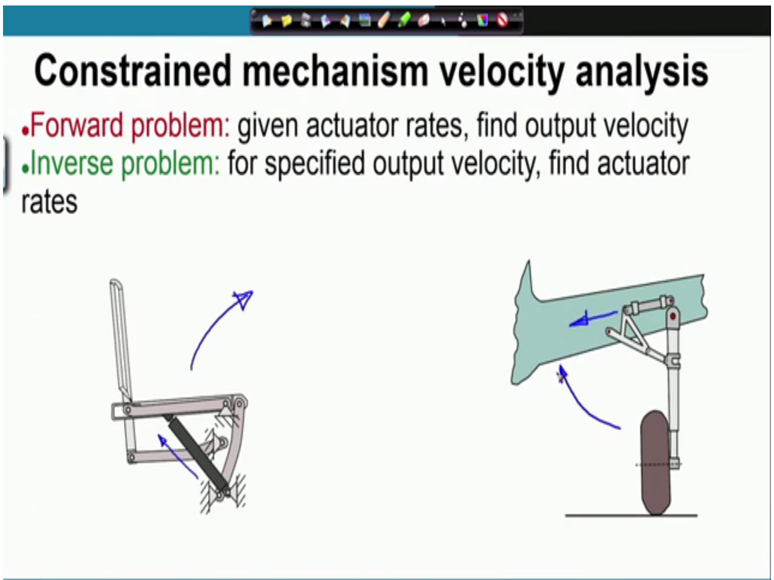

So, in constrain mechanism velocity analysis as you know we have defined this forward and inverse problems the forward problem in the forward problem we are given the actuator rates. And we want to find out the velocity of the output link and is just the reverse for the inverse problem where the output velocity or the desired output velocity is given to us. And we want to find out how the actuators should move what should be the actuator rates so that I can meet the output velocity specification.

So, here are 2 examples that we had considered. This is the transfer device where I need to find out the hydraulic or the pneumatic actuator expansion rate in order to provide a certain output rate for making a taking a person from the sitting position to the standing position. So, that relation we need to find out or for in this case of the landing gear we may have to find out what should be the actuator expansion rate so that I can fold the wheel at a certain desired rate.

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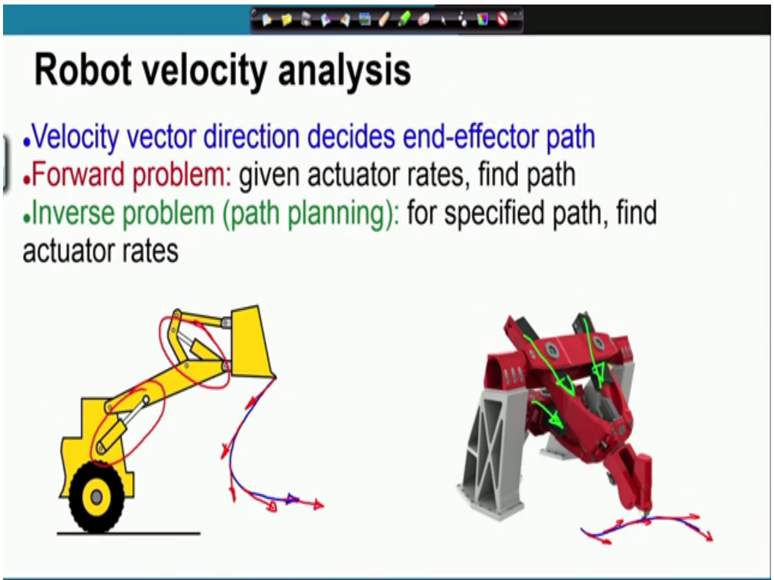

In the case of robot velocity analysis the problem is little more complicated. The reason is now we are moving a body in space we may have a certain path in which this body needs to be moved. And therefore, I would like to find out how I should move so that this path can get reversed. One way is to specify the velocities at various points on this path. So, I should be able to specify the velocity at each point on the path and corresponding to that I should be able to find out the expansion rate of the actuators.

The same case in this second example of robot manipulator: if I am able to specify the velocity along the path and determine what should be the actuator expansion rate corresponding to that velocity data desired at the output, then I should be able to follow the path.

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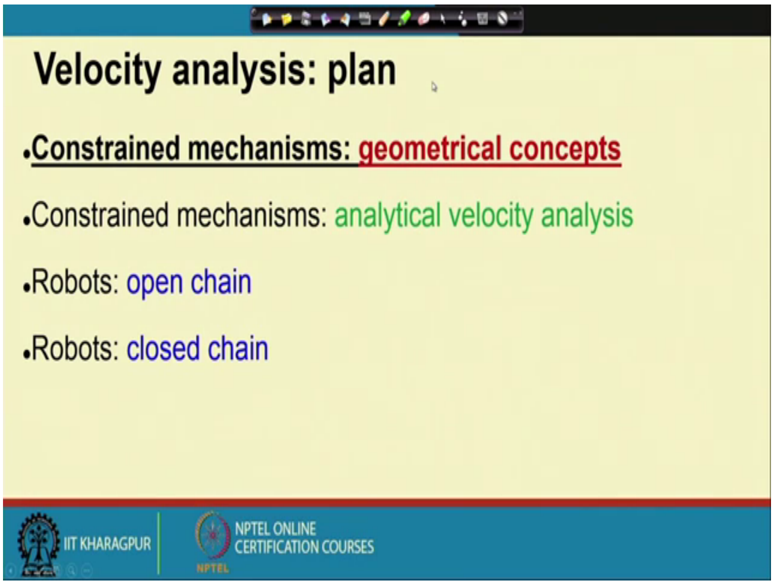

Now, our plan to study velocity analysis, we are continuing with our constrained mechanisms and geometric concepts. Later on we will move to analytical velocity analysis and subsequently to robots.

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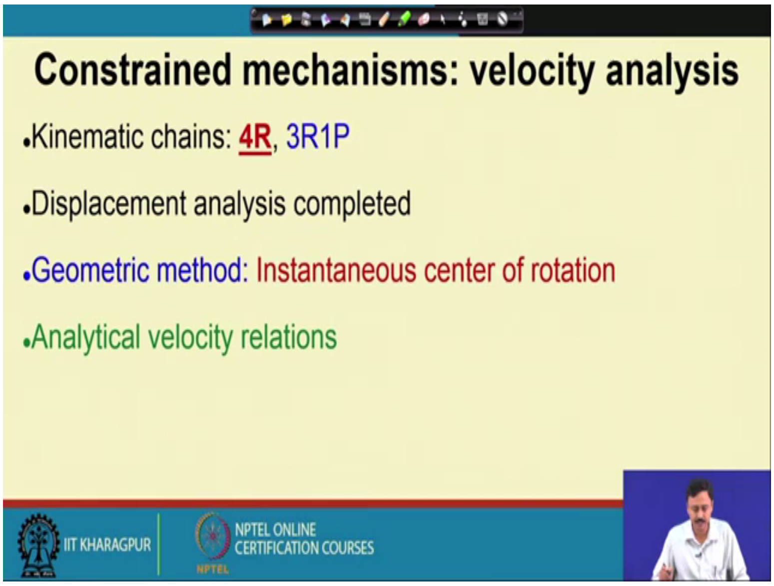

So, in this lecture we are going to look at the 4R kinematic chain. We will assume that the displacement analysis is completed. In other words I know the configuration of the mechanism this is a very important point. So, before I embark upon the velocity analysis problem I must have the displacement analysis problem completely solved. In other words I should be able to find out the configuration of the mechanism or I should know the configuration of the mechanism.

We are going to pursue the geometric method based on the instantaneous center of rotation and derive analytical velocity relations. So, this is the plan for the current lecture.

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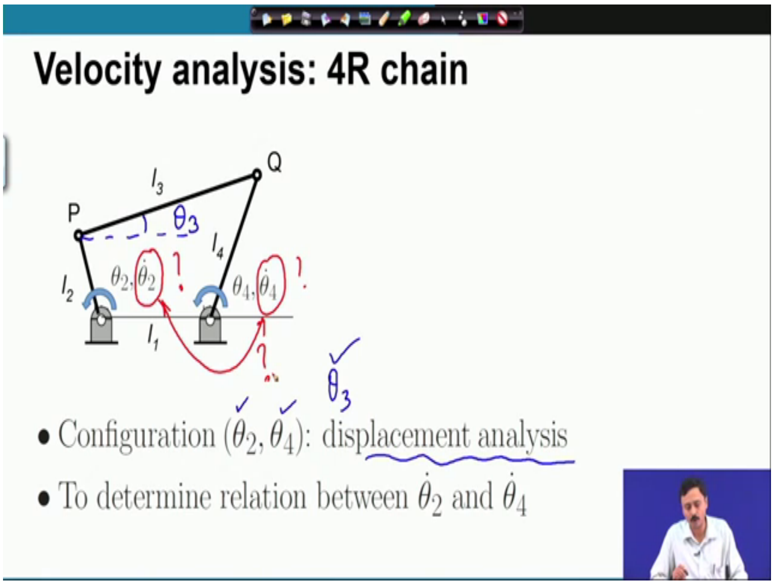

So, let us look at the problem of velocity analysis for a 4R chain. So, configuration is specified here I have written out theta 2 and theta 4. Of course, theta 3 is also to be included. So, this is theta 3. So, if I am given theta 2 I will definitely know theta 4 and I will also know theta 3.

So, that is the displacement analysis problem which I have assumed is completely solved. So, all these angles are completely known to me configuration of mechanism is completely known to me. I want to find out the relation between this theta 2 dot and theta 4 dot; so, the relation between the two. So, this relation is what I want. So, that is the velocity analysis problem.

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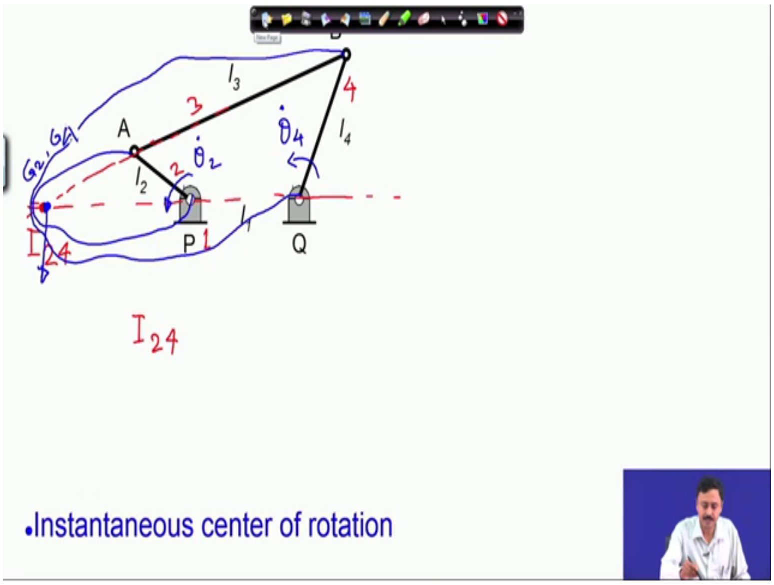

Now, we have discussed location of the instantaneous center of location. So, let me just tell you how we located I 2 4. So, the ground is one this is 2 this is 3 and this is 4.

These are the link numbers and how we located I 2 4? It lies on the line of I 1 2 and I 1 4 and it also lies on the line I 2 3 and I 3 4. So, this is the line. So, here we have I 2 4. Now this is what we need now why do we need this. Let me just briefly explain we have already gone through this. So, if I have an angular speed theta 2 dot for link 2 and theta 4 dot for link 4 remember that this is the point this I 2 4 is the point where the velocity of an extension of link 2 and an extension of link 4 has the same velocity.

So, velocity of a point G 2 and G 3, G 2 belongs to link 2 and G 3 belongs to not G 3, but G 4. In this case G 4 belongs to link 4 or an extension of link four. So, these 2 points will have the same velocity same magnitude same direction. So, they have the same velocity, G 2 and G 4 have the same velocity. These are the coincident points one belonging to link 2 the other belonging to link 4. So, that is I 2 4 that is the significance of I 2 4. So now, let us see how this is this information is used to find out the relation between theta 2 dot and theta 4 dot.

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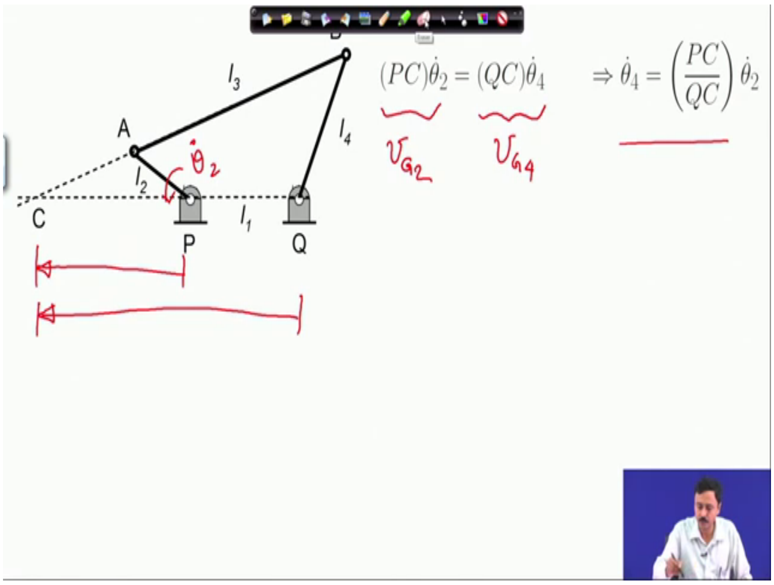

So, I have located for you I 2 4 which is the point C. Now you can very easily from geometry write this expression that PC which is this distance PC times theta 2 dot. What is this? This is nothing but velocity of G 2 PC times theta 2 dot. So, this is theta 2 dot must be equal to QC times theta 4 dot this is the condition. So, this is V G 4; now this is the condition that first we satisfied at I 2 4.

So therefore, theta 4 dot must be equal to PC divided by QC times theta 2 dot this is straight forward now we are need to determine this PC and QC. So, how do we do that?

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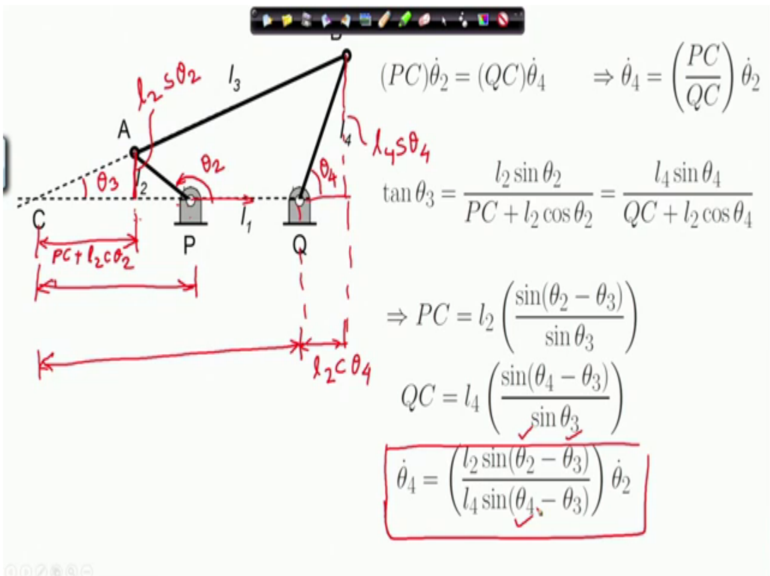

So, you can very easily write that tangent theta 3. So, this is our angle theta 3 this angle is theta 3 theta 3 tangent of theta 3 is nothing but l 2 sin theta 2 this is l 2 sin theta 2 remember that this is theta 2 I am writing s theta 2 in place of sin theta 2. So, if this is angle theta 2 then this vertical projection of the link is l 2 sin theta 2 divided by. So, tangent theta 2 is l 2 sin theta 2 divided by PC which is this distance plus l 2 cosine theta 2, now that is the projection of l 2 on the x axis.

Now, it happens in this case to be negative because if theta 2 is greater than 90 automatically this takes care of the sin. So, in this particular case it will be PC minus this distance, it is PC minus this distance. So, I will actually have this distance and you know that this l 2 sin theta 2 divided by this distance is the tangent of theta 3. So, this distance is PC plus l 2 cosine theta 2. So, I have written t theta 2 in place of cos theta 2 and that is also equal to l 4 sin theta 4 which is this distance l 4 sin theta 4 this is theta 4 divided by QC this is QC plus this distance this is l 2 cos theta 4. So, that all both of these ratios are tangent theta 3.

So, from here I can very easily solve to determine PC and QC and which I have done out I have derived it for you. So, PC is l 2 sin theta 2 minus theta 3 divided by sin theta 3 and QC is l 4 sin theta 4 minus theta 3 by sin theta 3. And hence I have finally, this relation between theta 2 dot and theta 4 dot in terms of you can see theta 2 theta 3 theta 4. Now as I have said that displacement analysis is completed; so, I know each of them I know theta 2 I know theta 3 I know theta 4.

Of course one of them will be specified. So, if theta 2 is specified I will be able to solve for theta 4 and theta 3. If theta 4 is specified then I will be able to solve for theta 2 and theta 3. So, in whatever way I know theta 2 theta 3 theta 4 in that case I have found the relation between theta 2 dot and theta 4 dot.

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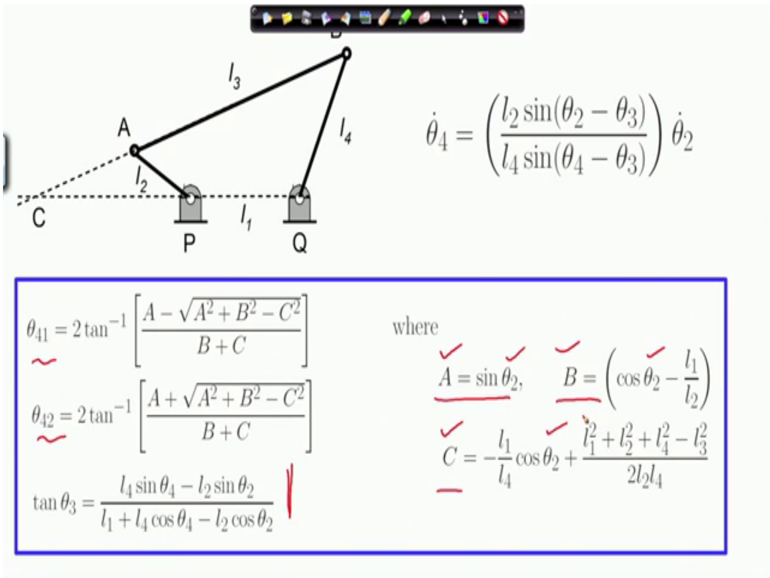

Thus we have this relation theta 4 dot as this ratio times theta 2 dot and just to recapitulate our displacement analysis problem. Just now I was mentioning that I can solve for theta 4 given theta 2 I can solve for theta 4 there are 2 solutions of theta 4, and corresponding to those 2 solutions of theta 4 I can find out the solutions for theta 3.

Here, A, B and C are completely known because theta 2 I am assuming theta 2 is specified. So, this is completely known. So, once I know A B C I can find out the 2 solutions of theta 4 from where I can find out also theta 3.

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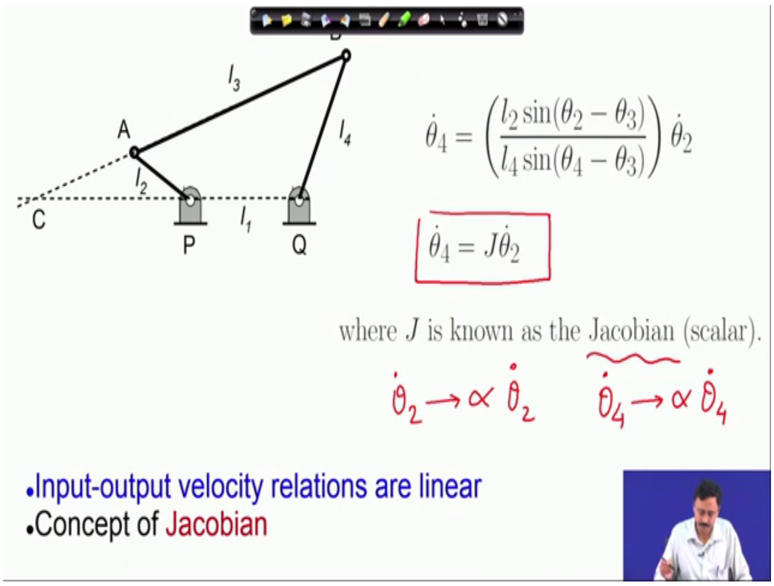

Now, this relation between theta 4 dot and theta 2 dot I will write in a compact form as theta 4 dot is equal to J times theta 2 dot this J is known as the Jacobean here this is a scalar quantity. That means, it is a number given theta 2 or having known theta 2 theta 3 theta 4 I can exactly calculate this number which is J this is called the Jacobean.

So, the Jacobean relates the input output velocities of the 4R chain. It is interesting to note that the velocity relations are linear between theta 2 dot and theta 4 dot it is linear. So, if I scale theta 2 dot by a certain quantity theta 4 dot gets scaled by the same quantity. In other words what I mean is if I take theta 2 dot to alpha times theta 2 times then theta 4 dot goes to alpha times theta 4 dot.

So, this relation is linear, but remember that in displacement analysis the relation between theta 2; and theta 4 was highly non-linear just through those trigonometric functions, but the velocity is linearly related. The input output velocities are linearly related and we have introduced this concept of Jacobean which will be very useful and we will continue discussing this concept and this is very useful in the case of robotics as well.

You will come across Jacobean in robot kinematics which we will discuss in the subsequent lectures.

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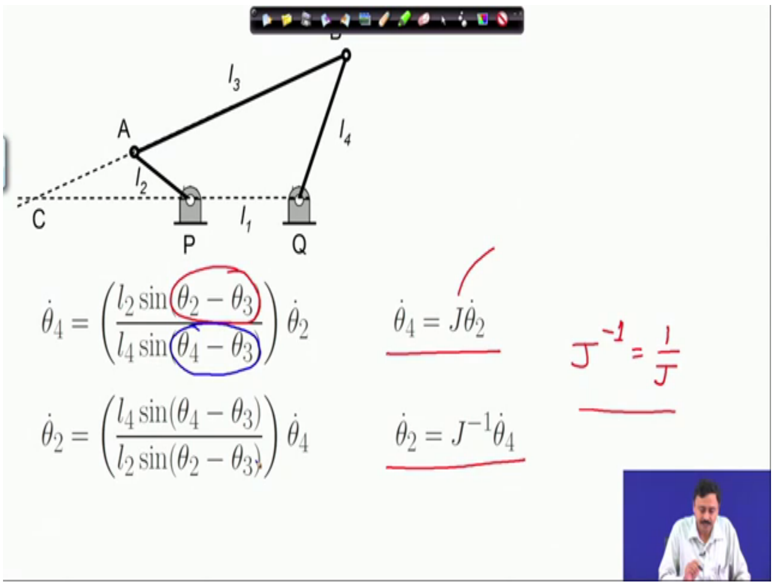

So, let me write down the relations ones again. Now what I have done here is; I have written it in 2 ways I have kind of inverted you see I have written out theta 4 dot in terms of theta 2 dot. And here I have written theta 2 dot in terms of theta 4 dot by the Jacobean inverse which is 1 by J essentially Jacobean inverse since it is a scalar Jacobean inverse is 1 over j.

Now, this brings in some issues as you can very well realize that for certain relations of theta 2 and theta 3 the sin of theta 2 minus theta 3 can go to 0. If it goes to 0 then I cannot invert the Jacobean because, Jacobean has gone to 0 I cannot find out 1 by 0 it can also happen the other way if it. So, happens that theta 4 and theta 3 are such that sin of theta 4 minus theta 3 is 0 the Jacobean is infinity or the Jacobean inverse is 0 then it is another problem.

So, what are these problems? Problematic configurations as you can very well realize that these are certain configurations at which this will happen.

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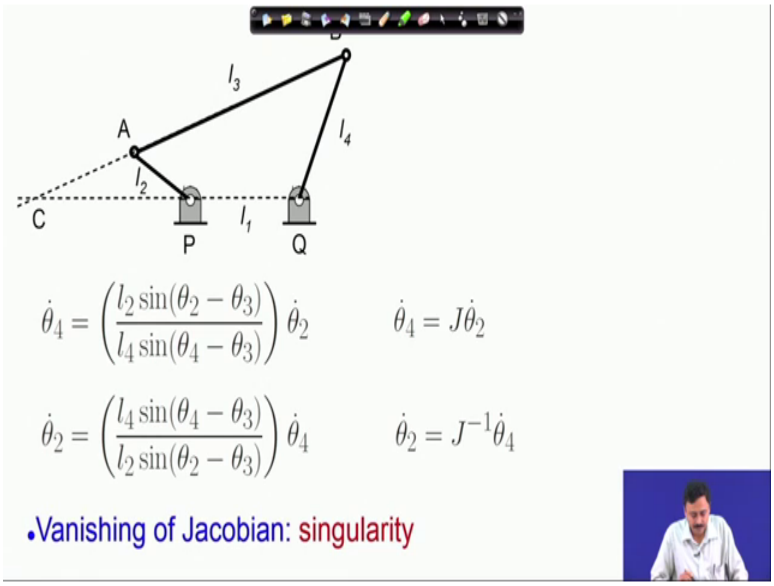

So, there are very special configurations at which such conditions are met. So, vanishing of the Jacobean is called the singularity of the kinematic chain or the mechanism. So, we will spend a minute some minutes discussing this vanishing, because this is very important. And I will relate this to one of the previous concepts that we have used which is the dead center configuration.

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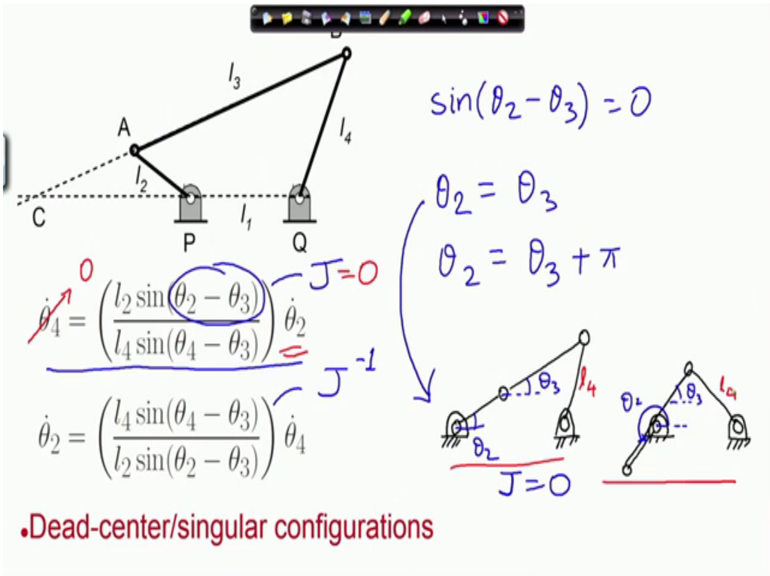

If you remember in the previous lectures, we have discussed dead center configuration there I wrote singular configurations as well. So, the reason is this that at these configurations the Jacobean or it is inverse can vanish for a let us consider this situation where the Jacobean. So, this is Jacobean and this is the Jacobean inverse. So, when can Jacobean vanish whenever sin theta 2 minus theta 3 is equal to 0 this can happen in 2 ways theta 2 equal to theta 3 and theta 2 equal to theta 3 plus pi.

So, in both these cases Jacobean will vanish. Now what are these configurations? They are very special configurations for examples as you can easily see. Theta 2 equal to theta 3 let me try to draw it out. This is the configuration where theta 2 equals theta 3 this is theta 2 and this is theta 3 as you can see theta 2 equal to theta 3. And this we have discussed is a dead center configuration. So, this is for this case, let us look at the other case theta 2 is theta 3 plus pi. So, this is the folded configuration this also dead center configuration and what is happening when sin theta 2 minus theta 3 equals to 0?

So, let me mark this out. So, this is theta 2 and this is theta 3. So, theta 2 is theta 3 plus pi both are dead center configuration and what is happening at this dead center configuration the Jacobean is going to 0 J equal to 0 and what does it mean if I look at this relation? It means that whatever be theta 2 dot it does not matter whatever be theta dot theta 4 dot is 0. So, if J goes to 0 theta 4 goes to 0 whatever be the value of theta 2 dot and indeed these are the configurations where the output link which is l 4, the fourth link goes to 0 velocity.

Now, let us discuss the other case which is the Jacobean inverse case.

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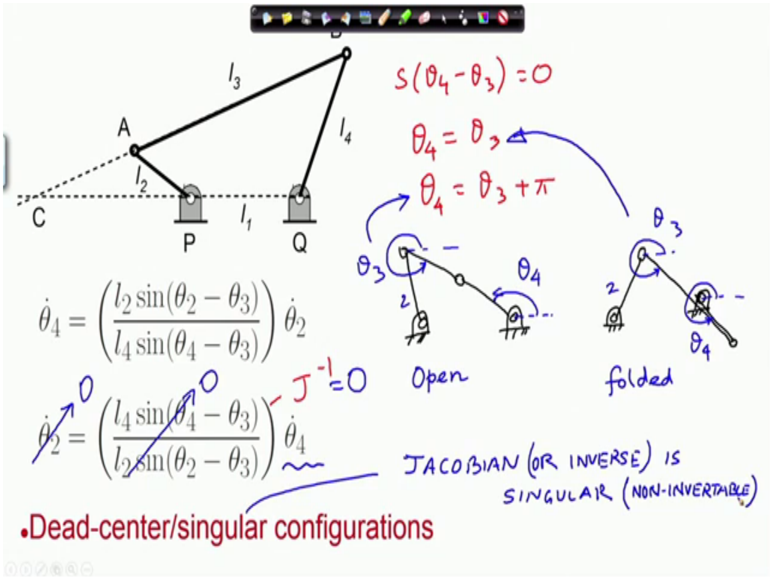

So, this is J inverse. Now when will J inverse go to 0; when sin theta 4 minus theta 3 equal to 0. So, the situation is absolutely same. So, these are the 2 situations theta 4 is equals to theta 3 or theta 4 is theta 3 plus pi. So, the configurations are also very similar. Only thing it occurs on the other side. So, this is the case this is theta 4, and remember this is theta 3. So, this is a situation in this case theta 4 is theta 3 plus pi. So, this is the open configuration. There is another configuration corresponding with the first case. So, here this is theta 4 and this is theta 3 and you can see that both are equal. So, this corresponds to the first solution theta 4 equals to theta 3. So, this is known as the folded configuration.

Now, what happens when these 2 configurations occur? J inverse goes to 0. So, this goes to 0. So, whatever be theta 4 dot theta 2 dot goes to 0. So, there is no motion of link 2. Link 2 must come to a stop it cannot move whatever be theta 4 dot. So, these are the dead center configuration which we have discussed before, and these are also singular configurations for the reason now you realize that these are the singularities of the Jacobean. So, singular configurations otherwise imply that these are the singularities of the Jacobeans or it is inverse. So, singularity sometimes we define in terms of non invertiblilty; so non invertible.

So, singular in the sense that it is not invertible; so these are the dead center or singular configuration dead center configurations or singular configurations where the Jacobean or it is inverse is singular which means they are not invertible.

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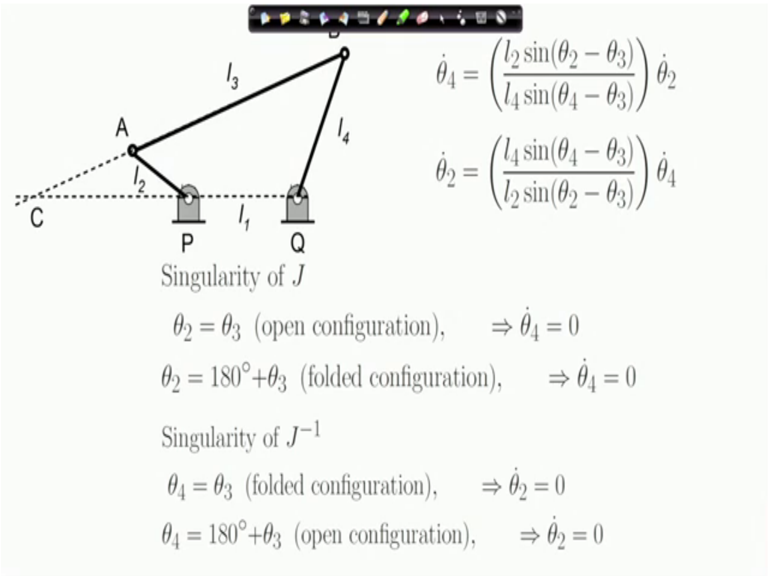

So, this is what I have discussed; singularity of J when theta 2 equal to theta 3 then J is singular theta 4 dot goes to 0 this is the open configuration and the other solution theta 2 is 180 degree plus theta 3 is a folded configuration where also theta 4 dot goes to 0. And the other situation where singularity of J inverse; that means, J inverse becomes 0 or non invertible, where theta 4 is equal to theta 3 which is folded and theta 4 is 180 degree plus theta 3 is open both cases theta 2 dot is 0.

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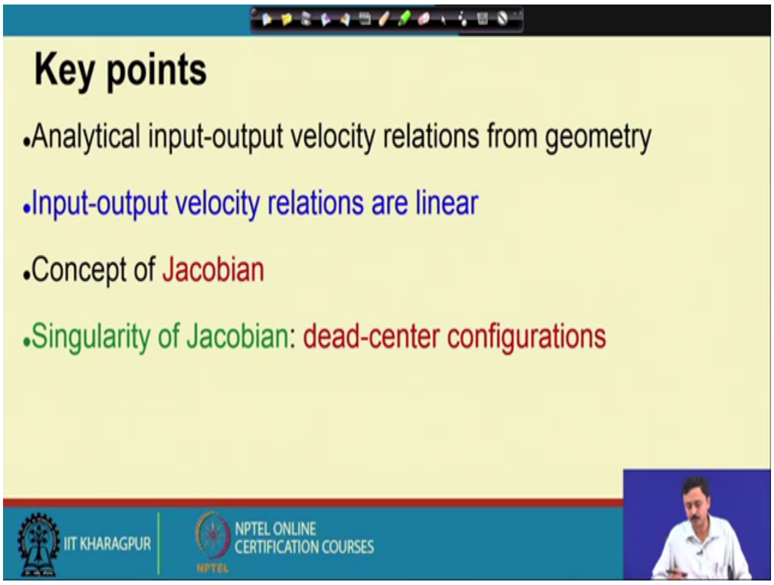

So, let us review the key points that we have discussed in this lecture. We have derived analytical input output velocity relations from geometry purely from geometric considerations. We have found that the input output velocity relations are linear; we have introduced the concept of Jacobean and the singularity of Jacobean. That means, non inevitability of Jacobean or the inverse of the Jacobean they correspond to the dead center configuration or the singular configurations of the mechanism.

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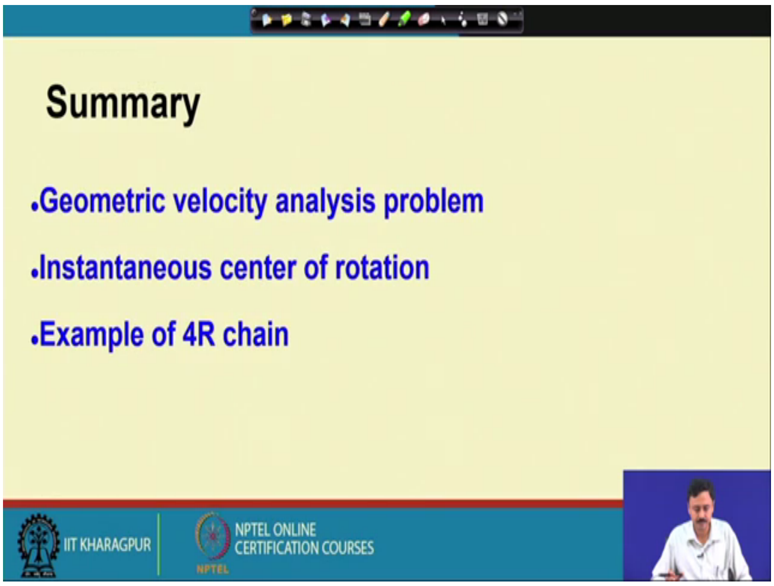

So, to summarize we have looked at the geometric velocity analysis problem based on the instantaneous center of rotation. We have derived the analytical relations between the input output velocity of 4R chain.

So with, that I will conclude this lecture.