Kinematics of Mechanisms and Machines
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Lecture – 20
Geometric Velocity Analysis – III

We will continue our discussion on the Velocity Analysis problem. We have been looking at the geometric problem in velocity analysis, we have looked at certain concepts of some of the certain geometry concepts like instantaneous center rotation and relative instantaneous center rotation, and based on that we have discussed an important theorem which is known as the Arnold Kennedy theorem.

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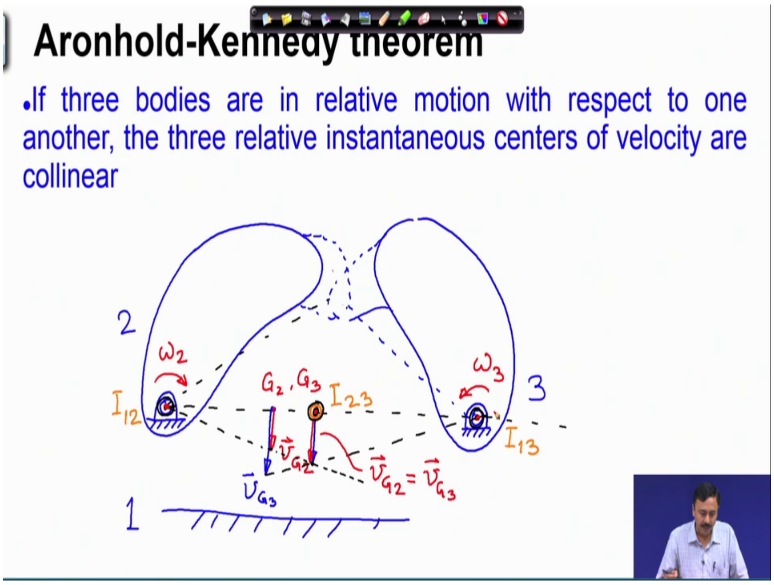

So, we are going to start by just reviewing the Arnold Kennedy theorem which states that if three bodies are in relative motion with respect to one another the three relative instantaneous centers of velocity are collinear. So, just to explain this once again let me consider three bodies. So, two and the third one I will define like this that, suppose this is the instantaneous center of rotation of let me call it body 2.

So, this point is the relative instantaneous center of body 2 with the ground which I call body 1 and let me assume that this is the relative instantaneous center of body 3 with respect to the ground. In other words these two points are hinged to the ground. These two points are hinged to the ground at this instant of time. I can consider that these two points are hinged to the ground at this point of time.

So, as per our nomenclature we can say that this is I 1 2 this is I 1 3. So, relative instantaneous centers of rotation of body two with respect to the ground is I 1 2 relative instantaneous center of rotation of body 3 with respect to the ground is I 1 3. Let me consider this, line joining I 1 2 and 1 3. Now I ask the question what is the relative instantaneous center of rotation of body 2 and 3.

Let me also make one more consideration that this is omega 2, this is omega 3 I have taken arbitrarily. Now I ask this question where lies the relative instantaneous center of rotation of body two relative to body 3. Now as we have seen before this point may lie outside the domain of the physical domain of the body, this relative instantaneous center of rotation I 2 3 may lie outside the physical domain of the bodies.

Let us consider a test point; we will begin with a test point. So, if I consider a test point let us say G then I know that velocity of G 2 because this point remember, this point I can always bring into extensions of body 2 and body 3. So, G 2 and G 3 are corresponding the points on body 2 and body 3.

So, velocity of point G 2 if I mark it with red, it must be perpendicular to this line joining I 2 3 and G, similarly the velocity of point G 3 must be perpendicular to this line. So, this is v G 3, this is v G 2, now they do not match. So, definitely G cannot be the relative I c between of 2 and 3, then we had seen that in order to have a point a feasible point which can be the relative I c it must be a point on the line joining I 1 2 and I 1 3, consider this is our test point now.

So, here we have again G 2 and G 3, they are coincident points on the extensions of bodies 2 and 3. Now once again if I mark by red the velocity of point G 2 it must be perpendicular to this black line. So, v G 2 and velocity of G 3 must also be perpendicular to this black line, now their directions match, but their magnitudes do not. Now how to determine then the exact point, what we do is since the red vector belongs to body 2. So, from the center of instantaneous center of body 2 I draw this black dashed line and instantaneous center of body 3, I draw this black line wherever they intersect you can very easily realize you must have velocities as same. These two velocities must be equal.

So, here I have v G 2 equal to v G 3. So, this must be the relative I c, I 2 3. So, this is I 2 3, here is I 13 and here is I 1 2 and you can see that I 1 2 I 2 3 and I 1 3 lie on a single straight line. Now here I have fixed the ground, but you can also make the ground move, the situation will not change the situation will not change even if I let the ground move because remember I 1 2 is the relative instantaneous center of body two with respect to the ground, that will remain the same. I 1 3 is the relative instantaneous center of body three with respect to the ground that also will remain same even if I let the ground move, but keeping the relative motions the same.

So, nothing is going to change. So therefore, we have this theorem known as the Arnhold Kennedy theorem which states that if three bodies on relative motion with respect to one another, the three relative instantaneous centers of velocity are collinear. Now we are going to look at the implications of this theorem after looking at some examples.

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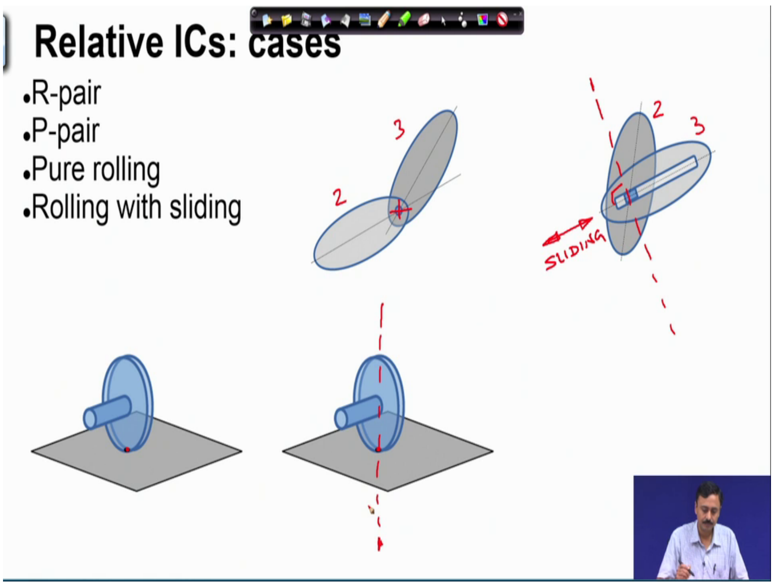

Which we have seen before so this is the relative IC between bodies 2 and 3, the revolute pair for the R pair the revolute pair itself is the relative IC between 23 in the case of a prismatic pair the relative IC between the bodies 2 and 3 lie on this line which is perpendicular to the direction of sliding.

So, this is the sliding direction and the relative IC lies on this line perpendicular to the sliding direction; so I 2 3 lies on this red dashed line somewhere at infinity. So, it lies at infinity on this red dashed line, in the case of purely rolling desk as you know the velocity of at the point of contact is 0 for pure rolling case. Therefore, this is the relative IC between the ground and the wheel, in the case of rolling with sliding we cannot exactly locate, but it must lie on this line, why? Because when it is purely rolling it must be at the point of contact when it is purely sliding when its only translating and sliding it must be at infinity either in the upward or downward direction. So, in between if it is rolling and sliding it will lie on this line somewhere.

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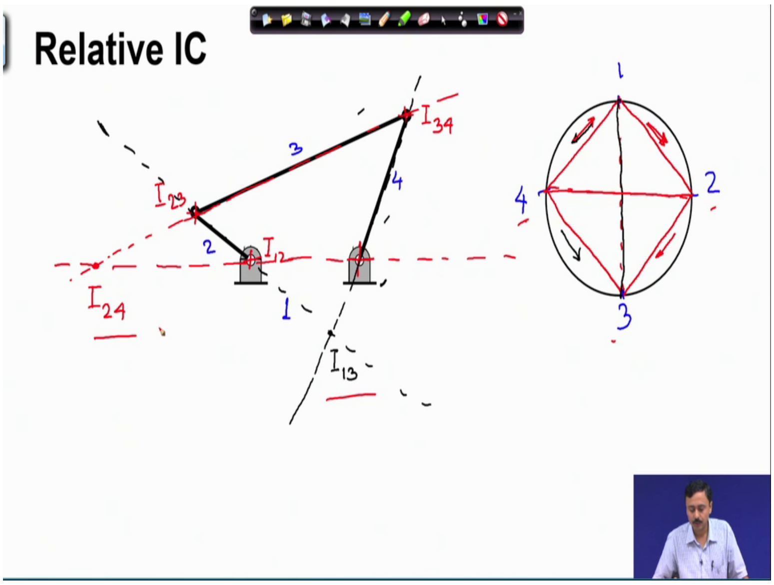

Now, let us apply these concepts in mechanisms. So, we will try to locate the relative IC's for this mechanism, let me number the links first so ground is 1, this is 2 3 and 4. Now to locate all the relative IC's we have what is known as the circle diagram. So, I will mark the number of links so 1, 2, 3, 4; so 1, 2, 3, 4. So, we have 4 links. So, I will mark 4 points on the circle, I will join the 2 2 points if I exactly know the relative IC between those two bodies.

So, on the circle as you realize I have marked out exactly the number of links I have and the relative IC's between two links if it is known, if the relative IC between two links is known then I will mark that on that circle diagram I will join. So, for example, I 1 2 is this point the relative instantaneous center of rotation between 1 and 2 is this ground hinge. So, I will join one with two, I also know the relative instantaneous center of rotation this is I 1 2 this I 2 3 I also know that so I know I 2 3 this is I 34 the relative instantaneous centers center of rotation of links 3 and 4.

So, I know I 3 4. So, I will join 3 4 and I also know the relative IC of four link four with respect to link 1; so I will join 1 and 4. Now what I do not know that the circle diagram immediately tells me for example, I do not know what is the relative IC between 1 and 3, this has not come out as yet, but can we find it the answer is yes we need to find out two independent paths starting from 1 or between 1 and 3 between 1 and 3 we need two independent paths. Why we need it? We will get clear very soon. So, one path is I 1 2 and I 2 3.

So, I 12 and I 23; so I 13 must lie on this line as per the Aronhold Kennedy theorem, I 13 must lie somewhere on this line, because I 12, I 23 and I 13 these are the three relative IC's of three bodies 1, 2, 3. So, three bodies 1, 2, 3 are in relative motion. So, therefore, their relative IC's must lie on a line. Now of the three relative IC's I exactly know 2 2 of them, I 12 and I 1 I 2 3 these are known I 1 2 and I 2 3 are known.

So, I 1 3 must lie on this line so this is the black line dashed line. Similarly 1 4 and 3, 1 is the ground, 4 and 3 these are 3 bodies again which are in relative motion. So, their relative IC's must lie on the line of which I 14 is known and I 14 is known, I 14 is here on the ground hinge and I 3 4 is here. So, I 1 3 must also lie on that line.

Now, when a point has to lie on both these lines is the intersection point this which must be I 13. So, I have located now I 1 3. So, I will join it by a solid line next is I 24 which is as yet unknown, but then I have two independent paths I 14 and I 1 2. So, I 2 4 must lie on that line, why? Because, 2 1 and 4 the lengths 2, 1 and 4 they are 3 bodies in relative motion. So, the 3 relative IC's must lie on that line of which I 1 4 and I 1 2 are known to me.

So, therefore, I 2 4 must lie on this red dashed line, similarly 2, 3, 4 2, 3, 4 are 3 digit bodies are in relative motion. So, therefore, the three relative IC's must lie on that line of which I 2 3 and I 4 3; I 2 3 and I 4 3 are known to me. So, therefore, I 2 4 must also lie on this line. So, the intersection point gives me I 2 4. So, I have found this as well and that completes all the relative IC's.

So, I have found I 1 3 and I have found I 2 4 which were unknown, using the Kennedy Aronold theorem and the circle diagram, circle diagram helps you to keep track of the bodies of the lengths and finding out searching for paths which connect two points or two links. So, let us proceed further.

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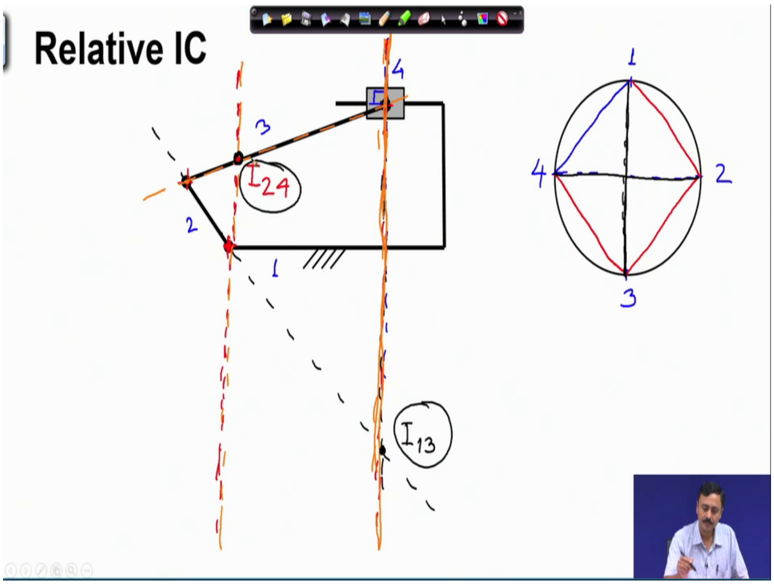

So, here we have a 3R 1 P chain, ground is 1, 2, 3 the slider is 4. So, again 4 bodies; now what do we know? We know I 1 2 we know, I 2 3, we know I 3 4, now I 1 4, this is a prismatic pair prismatic pair between lengths 1 and 4. So, I 1 4 must lie at infinity along this direction which is perpendicular to the direction of sliding.

So, this direction is perpendicular to the direction of sliding. So, I 1 4 must lie at infinity. So, I know this it is at infinity; now let me ask the question whether I can find I let us say 1 3. So, I have 2 paths I 1 2, I 1 2 is known and I 2 3 is known. So, therefore, I 1 3 must lie on this line, I 1 3 must lie on this line, the other path I 3 4 is known and I 1 4 is at infinity somewhere. So, therefore, I 1 3 must also lie on this line the intersection is this. So, this must be I 1 3. So, now, I also know I 1 3 next I ask the question whether I can find out I 24.

So, I 2 4 must lie on the line of I 2 4 and I 1 2; I 1 2 is at infinity and I 1 2 is this ground hinge and I 1 4 is at infinity. So, through this point through this ground hinge I must draw line to infinity. So, this line meets this dashed line this line at infinity. So, these two lines are parallel. So, what I have drawn is a line through I 1 2 to meet I 1 4; now I 1 4 is at infinity and as I have said I 1 4 lies on this I 1 4 lies on this line I 14 lies at infinity on this line therefore, I must draw a line parallel to that line through I 1 2 to meet at infinity to meet I 1 4 at infinity.

So, that is one, the other is other path is I 2 3 which is here I 2 3 and I 3 4 which is here. So, which means it is this line. So, I 2 4 must lie on this line as well. So, this point as you can see this point is I 2 4. So, this intersection point is I 2 4. So, now, I also know I 2 4. So, I have located all instantaneous centers of rotation.

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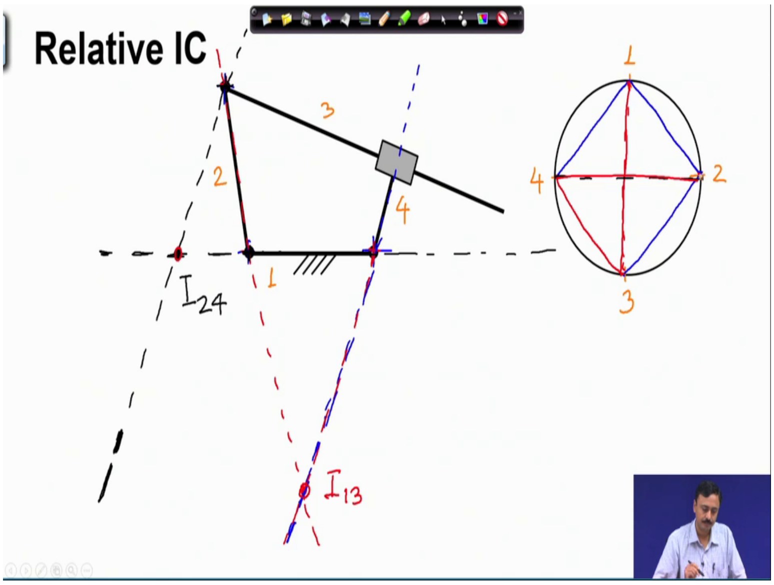

Let us move to the next mechanism. So, let me number 1, 2, 3 and 4. So, there are 4 links. So, four points on our circle diagram I know the IC between 1 and 2. So, I 1 2 is known I 2 3 is also known, I 1 4 is also known. Now I 3 4 must lie on this line which is perpendicular to the sliding direction.

I 14 I 3 4 sorry I 3 4 must lie on this line at infinity because this is a prismatic pair. So, this also I know is it that it is at infinity I 34 is at infinity on this blue dashed line; now I ask this question whether I can find out I 1 3. So, I 1 3 lies on the line of I 1 2, this is I 1 2 and I 2 3. So, which means this line I 1 3 lies on this red dashed line and I 1 3 also lies on this I 1 4 which is here and I 4 3 which is at infinity on this line. So, this point, this intersection point must be I 1 3. So, I know I 1 3 next I ask the question whether I can locate, I 2 4. So, I 2 4 must lie on I 1 2 and I 1 4; I 1 2 and I I 1 4 and I 1 2 so on this line.

So, I 1 2 and I 1 4 so, I 2 4 must lie on this black dashed line and I 2 4 must also lie on I 2 3 and I 3 4 I 2 3 I 2 3 is this and I 3 4 is at infinity along this blue dashed line remember. So, therefore, I must draw a line through I 2 3 which is parallel to this blue dashed line, so as to meet I 3 4 at infinity. So, this black dashed line and this blue dashed line they are parallel. So, where do these two lines meet? They meet here. So, this must be I 2 4.

So, we have now located both I 2 4 and I 1 3. So, all the relative IC's are now known.

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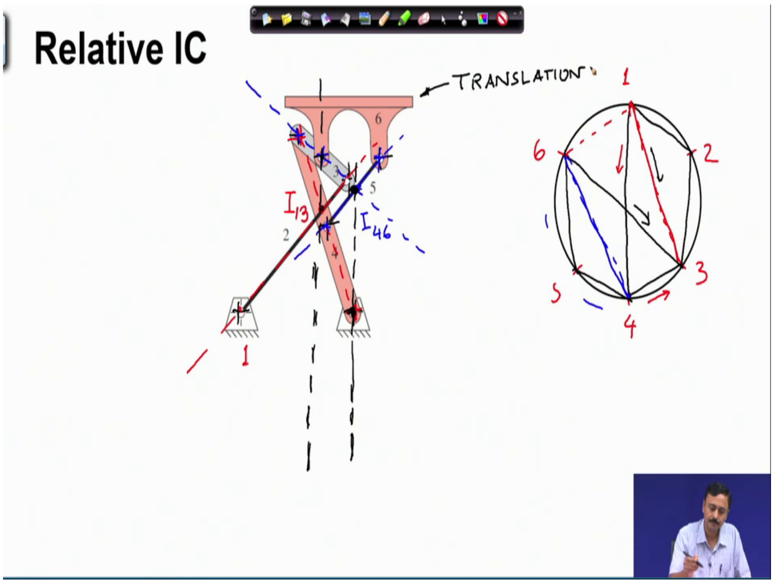

Let us look at this mechanism, now this has got 6 links 1, 2, 3, 4 5, 6. So, 6 links now we are going to again see what we know. So, I 1 2 is known, I 2 3 is known, it is here I 3 4 is known, I 4 1 is known, I 4 5 is known and I 56 is also known, I also know I 3 6 suppose I want to know I 1 6. So, I am asking this question whether I can find out I 1 6, now you will find that there is there are no two paths between two independent paths between 1 and 6. So, I must have a path, I define a path as one in which there are three rigid bodies because the then only I can apply Aronhold Kennedy theorem.

So, a path with three rigid bodies and I have none, but then if you see if I can find out I 1 3 then I have one path involving three rigid bodies, one will be 1 3 and 3 6. So, let us try to find out 1 3 I 1 3. Now for I 1 3 I have two independent paths involving two rigid bodies three rigid bodies. So, one is 1 2 I 1 2 and I 2 3, 1 2 and 2 3. So, this must be one line and the other path is 14 and 43, I 14 and 43. So, this is the other path.

So, this point intersection of this is I 1 3. So, I have now located I 1 3 let me now ask the question whether I can locate I 6 4 in that case I will have two independent paths involving three bodies each on these paths between 1 and 6 ok. So, I have this question of I 4 6, now I 4 6 must lie on I 3 6. So, 3 6 is here and 3 4 3 4 is here. So, on this line must lie I 6 4, similarly I 6 4 must also lie on I 56 which is this and I 5 4, I 5 4 is this. So, therefore so this intersection point is I 4 6. So, I have located I 4 6.

Now, I can ask the question whether I can find out I 1 6, now I have two independent paths involving three rigid bodies; so I 4 6 and I 4 1 or 1 4, I 4 6 and I 1 4. So, we use this. So, here I have this as my path, I passing through I 4 6 which is this and I 1 4 which is this so this line. So, I 1 6 must lie on this line and I 16 must also lie on 1 I 1 3 and I 3 6, I 1 3 I this is I 1 3 and I 3 6, I 3 6 is this. So therefore, it must also lie on this line now I 1 6 therefore, must lie on the intersection of these two black dashed line, now these two lines are actually parallel. Therefore, they meet at infinity, what it means is that body six is in pure translation because the instantaneous center of rotation I 1 6; that means, 6 with respect to ground is at infinity.

So, this body 6 is in translation, body 6 is in translation with respect to the ground and this mechanism is indeed a parallel transfer device where body 6 is parallelly transferred. So, it is in pure translation.

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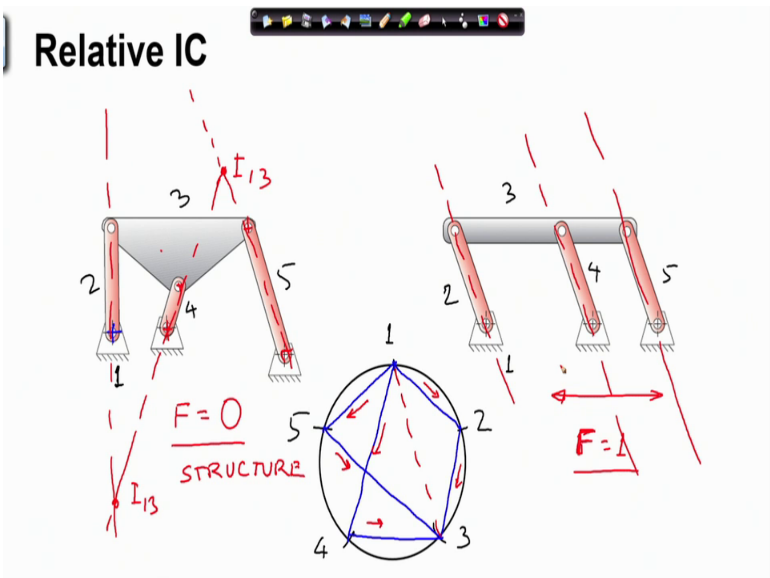

Let us look at this case of again a six link mechanism so 3 4. So, this is this is a five link mechanism. So, one is ground 2 3 4 and 5 on the right also I have another 5 link mechanism which is only different because of the dimensions; now let us look at the relative IC's. So, I know I 1 2, I know I 2 3, I know I 3 4 I know also 3 5, I know 41 and I know 5 1.

Suppose I want to find out what is I 1 3. Now there are two sets of independent path as you can see, I 1 3 must lie on I 1 2 and I 2 3, I 1 2 and I 2 3 which is this line and this must also lie on I 1 4 and 4 3, 1 4 is this and 4 3 is this so it must lie on this. So, this is the intersection, but then it also must lie on 1 5 and 5 3, I 1 5 and I 5 3. So, it meets somewhere, it also meets this now these are two independent paths as well I 1 4, 4 3 and I 1 5 and 5 3. So, this also is a possible possibility for I 1 3 this is also I 1 3 and there will be another I 1 3 that I will obtain using I 2 3 and I 1 2.

Now, a single body three cannot have three relative instantaneous centers of rotation, then it cannot move and if you remember the degree of freedom what we had calculated for this mechanism was 0. So, this is not a mechanism it is actually a structure, because there are three possibilities for relative instantaneous centers of rotation of body 3 with respect to 1. But then the mechanism on the right is actually a mechanism it has got 1 degree of freedom, on the right the kinematic chain on the right has 1 degree of freedom because of various special dimensions. And if, you again calculate or find out the relative IC's I 1 3 they must lie on this three lines as we had done here and all these lines meet at infinity, all these three lines meet at infinity.

So, three is in translation with respect to the ground. So, in the case of the mechanism on the right you have motion because the relative IC of 3 whatever way you calculate comes at infinity. So therefore, this can move.

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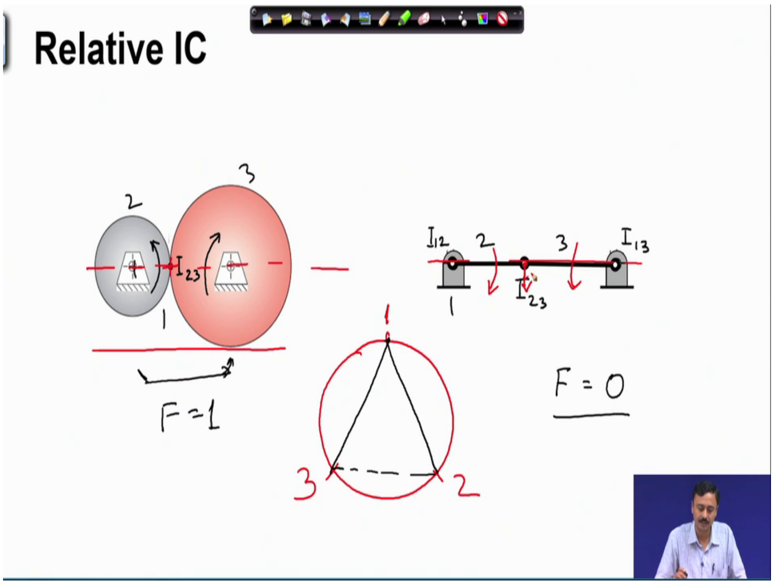

Here is the final example, here I have a chain with three links now I. So, this is two this is three ground is one I know I 1 2 I know I 1 3 and in both cases I also know I 2 3. So, this is in pure rolling. So, this is the IC of two related to 3. So, this is in the case here it is I 1 2 this is I 1 3 and this is I 2 3, but then you know this mechanism in which you have two friction disk they can continuously roll they this has one degree of freedom.

Whereas, the chain on the right is a structure with 0 degrees of freedom the reason is very simple the IC 2 3 on the on this gear always remains at this point when these two move. Whereas, when these two links if you try to move this I 2 3 is going to shift out of this line and once it shifts out of this line Aronhold Kennedy theorem does not hold, but it must hold therefore, this cannot move any further whereas, here it the Aronhold Kennedy theorem is always satisfied.

The Aronhold Kennedy theorem is always satisfied in the case of this two friction disks rolling because I 2 3 always lies on this line. Whereas, for the kinematic chain on the right this has a tendency to move out and that breaks the Aronhold Kennedy theorem which must be satisfied, it is a theorem, it must be satisfied if it does not satisfy then the thing cannot move. So, that makes the kinematic chain on the right as a structure.

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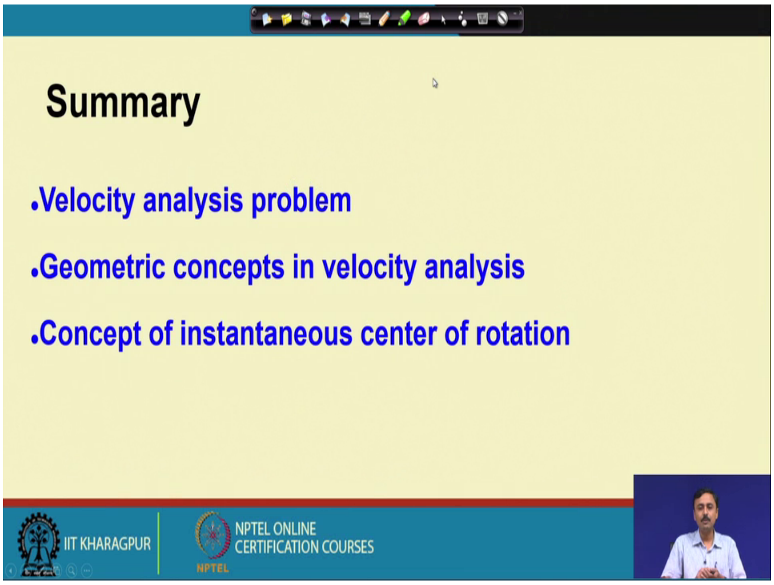

I will leave you with the summary of what we have discussed today.