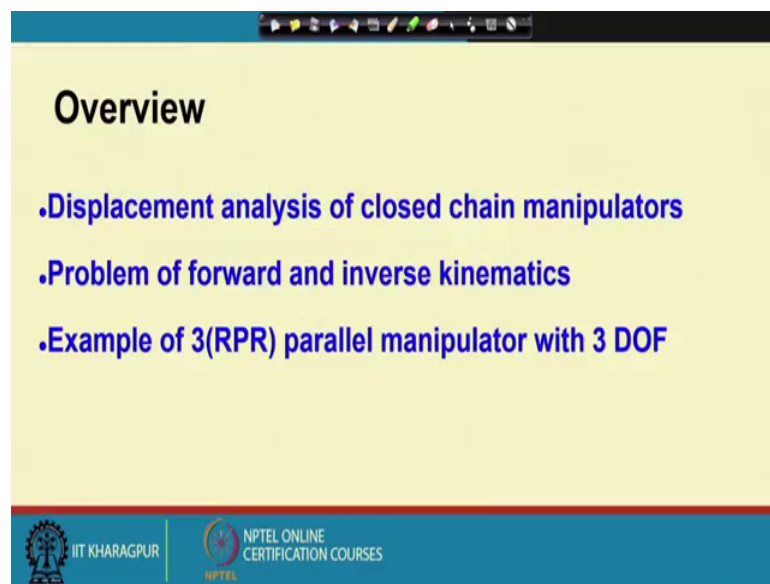


Kinematics of Mechanisms and Machines
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Lecture – 17
Displacement Analysis of Robots – III

We are going to continue our discussions on closed chain parallel manipulators so, closed chain manipulators.

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So, to give you the overview of what we are going to discuss in this lecture. So, the displacement analysis of closed chain manipulators we are going to look at the example of a 3 RPR parallel manipulator, which has 3 degree of freedom.

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Closed chain robots

- Parallel manipulators
- All actuators connected parallelly to the end-effector





Exechon's Parallel Kinematic Machine (PKM)
Source: www.twitter.com/ibonevi/status/339217235300208640

We have discussed what a closed chain robot is it is also known as a parallel manipulator because all the actuators are connected parallelly to the end effector, as this figure shows. So, this is one actuator this is a second actuator the third actuator they are all connected parallelly to the end effector. So, this is the end effector and all the actuators are directly connected through links to the end effector. So, this is a parallel manipulator.

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Robot displacement analysis: plan

- Planar robots: open chain
- Planar robots: closed chain




So, we are going to continue our discussions on closed chain planar robots. Let us look at some of the 3 degree of freedom closed chain planar robots.

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Closed chain planar robots: 3 DOF

- Kinematic chains: 3(3R), 3(RPR)

3R-3R-3R

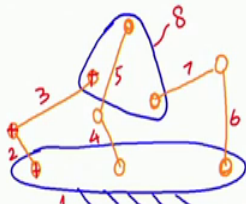


The first chain I have mentioned is the 3 3R. So, according to this nomenclature actually it is 3R dash 3R.


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Closed chain planar robots: 3 DOF

- Kinematic chains: 3(3R), 3(RPR)



$n_L = 8$ $n_J = 9$ $\sum f_i = 9$
 $F = 3(8-1) - 3(9) + 9 = 3$

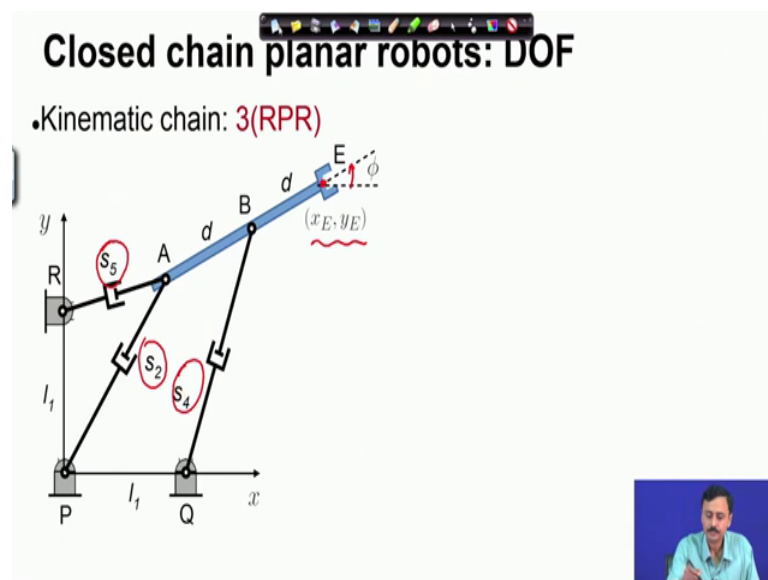


So, as per our previous nomenclature this should be 3R dash 3R dash 3R. So, written in a compact form as 3 times 3R. So, let us look at this kinematic chain, we have the ground link and the end effector link as I have shown. Now, we have three 3R legs, this is one 3R leg, this is the other 3R leg these are third 3R leg.

Let us calculate the degree of freedom of this, this ground is 1, 2, 3, 4 5, 6, 7 and the end effector is 8 link 8. So, number of links is 8, number of joints 1, 2, 3 plus 3, 6 plus 3 9. So, we have 9 joints 9 kinematic pairs, each having degree of freedom 1, since they are all revolute pairs. So, therefore, summation of degree of freedom is 9. So, therefore, degree of freedom is 3 times number of links minus 1, minus 3 times number of joints plus summation of degree of freedom of each joint. So, this gives us 3.

So, this closed chain planar manipulator has 3 degrees of freedom.; Next is the 3 times R PR which we are going to look at for, that also the degree of freedom calculation remains the same because some of the revolute pairs have been replaced by prismatic pairs. So, three revolute have been replaced by prismatic three prismatic pairs.

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So, let us move forward and look at our 3 RPR chain as shown here, we have this end effector link in blue and there are three legs which are which carry a prismatic and actuated prismatic pair. So, you can see that the coordinates of the end effector point x E and y E are to BE and the coordinates of the end effector point and the orientation. Since this has 3 degrees of freedom so we can control three things. So, position of the end effector and the orientation of the end effector link and we have actuations at these three prismatic actuators, in terms of s 2, s 4 and s 5.

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Displacement analysis: two problems

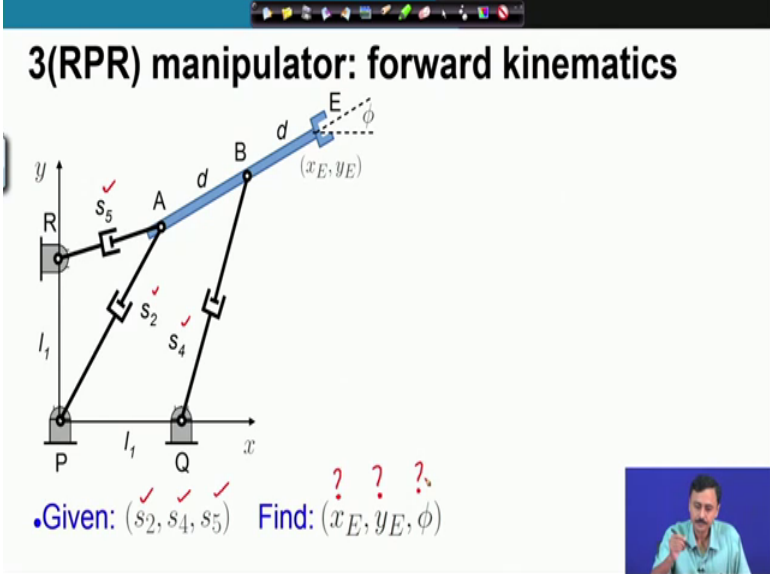
- **Forward kinematics:** given actuator input, find output
- **Inverse kinematics:** for specified output, find actuator input
- **Inputs:** actuator displacement
- **Output:** end-effector position and orientation




We know that the displacement analysis problem has two parts, one is the forward kinematics problem in which the actuator inputs are given and we are to find out the output. Output means, the end effector pose position and orientation. In the inverse kinematics problem for a given output which means the end effector position and orientation is specified we have to find out the actuator inputs.

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3(RPR) manipulator: forward kinematics



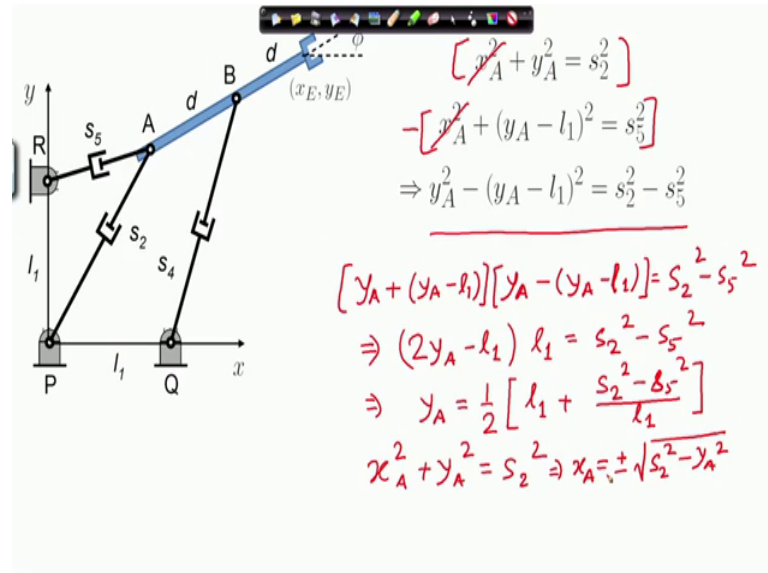
• **Given:** (s_2, s_4, s_5) **Find:** (x_E, y_E, ϕ)



So, let us look at the forward kinematics problem for the three RPR parallel manipulator, here we are given the actuator throws s_2 , s_4 , s_5 . So, we are given the lengths of these

actuators or these legs, we have to find out the coordinates of the end effector x_E and y_E and the orientation of the end effector link. So, this is the forward kinematics problem.

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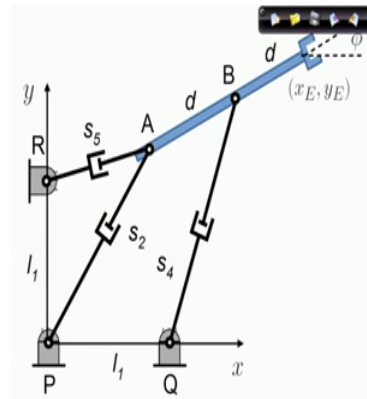


So, let us begin with writing out the coordinates of the points on the end effector link. So, here I have written out the core, the relations between the coordinates of point A and the actuator throws s_5 and s_2 . So, you can very easily derive these expressions.

So, this is the origin of our coordinate system so x a square. So, x coordinate of A square plus y coordinate of A square is the length of actuator 2 s_2 square. So, its length square the length R A square which is s_5 square is given by x_A square plus y_A minus l_1 whole square. So, essentially these are equations of circles. So, the 1st equation is is a circle about the origin with radius s_2 , the 2nd equation is the equation of a circle with radius s_5 and centre $0 \ l_1$ so, R essentially the point R. So, therefore, if I subtract the second equation from the first, if I subtract the 2nd equation from the first then x_A cancels off, what I arrive at is this which can be written as y_A plus y_A minus l_1 into y_A minus y_A minus l_1 is s_2 square minus s_5 square.

So, that gives us 2 times y_A minus l_1 times l_1 is equal to s_2 square minus s_5 square. So, from here I can solve for y_A . So, one half of so this is the solution for y_A . So, I have found the y coordinate of point A, then finding out x coordinate of point A is trivial because, I have this equation this is the 1st equation. So, I can solve for x_A . So, we have two solutions with plus or minus signs. So, let us look at this formally.

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$$x_A^2 + y_A^2 = s_2^2$$

$$x_A^2 + (y_A - l_1)^2 = s_5^2$$

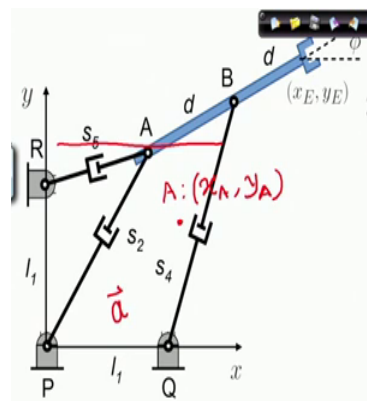
$$\Rightarrow y_A^2 - (y_A - l_1)^2 = s_2^2 - s_5^2$$

$$\Rightarrow y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1}$$

$$x_A = \pm \sqrt{s_2^2 - y_A^2}$$

So, this was the solution of y A and x A this has two solutions. So, I have written out these two solutions here, these are the coordinates of point A.

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$$y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1}$$

$$x_A = \pm \sqrt{s_2^2 - y_A^2}$$

$$\vec{a} = x_A \hat{i} + y_A \hat{j}$$

$$\begin{Bmatrix} x_A \\ y_A \end{Bmatrix}$$

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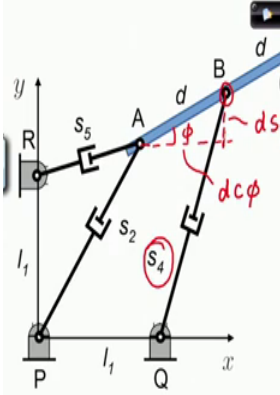


Diagram illustrating a mechanism with a slider block and a rotating link. The slider block has a vertical guide and a horizontal guide. The rotating link has a pivot at point P and a slider at point B. The distance between P and Q is l_1 . The distance between Q and B is s_4 . The distance between A and B is d . The angle between the horizontal and the line AB is ϕ . The diagram shows the geometry and the equations for the mechanism.

$$y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1}$$

$$x_A = \pm \sqrt{s_2^2 - y_A^2}$$

$$x_B = x_A + d \cos \phi, \quad y_B = y_A + d \sin \phi$$

$$(x_B - l_1)^2 + y_B^2 = s_4^2$$

$$\Rightarrow (x_A + d \cos \phi - l_1)^2 + (y_A + d \sin \phi)^2 = s_4^2$$

$$s_4^2 = (x_B - x_Q)^2 + (y_B - y_Q)^2$$

Handwritten notes in red ink include: $ds\phi$, $dc\phi$, and ϕ .

So, the two solutions are specified in terms given in terms of these two signs. Now I can express x_B in terms of coordinates of x_A and I will now introduce this ϕ . So, x_B so, I am expressing the coordinates of point B in terms of coordinates of point A which is now known to me, plus d times cosine ϕ . So, this distance AB is d .

As you can see this is $d \cos \phi$ this angle being ϕ and this is $d \sin \phi$. So, therefore, x_B x coordinate of point B is x coordinate of point A plus $d \cos \phi$ and y_B is equal to y_A which is not also known to us plus $d \sin \phi$. So, once I have the coordinates of point B of course, in terms of this unknown ϕ ; ϕ is unknown as yet then I can express the length of s_4 I can express the length s_4 square as x_B minus x_Q whole square plus y_B minus y_Q whole square.

So, that is s_4 square that is what I have written here by substituting the expressions of x_B and y_B and also x_Q y_Q is of course, 0. In this equation you will find what is not known is ϕ , ϕ is as yet unknown x_A is known y_A is known and s_4 is given. So, ϕ is the only quantity that is not known.

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$$y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1} \quad x_A = \pm \sqrt{s_2^2 - y_A^2}$$

$$x_B = x_A + d \cos \phi, \quad y_B = y_A + d \sin \phi$$

$$(x_B - l_1)^2 + y_B^2 = s_4^2$$

$$\Rightarrow (x_A + d \cos \phi - l_1)^2 + (y_A + d \sin \phi)^2 = s_4^2$$

$$\Rightarrow A \sin \phi + B \cos \phi = C$$

where

$$A = -y_A, \quad B = (l_1 - x_A)$$

$$C = \frac{x_A^2 + y_A^2 + d^2 + l_1^2 - s_4^2}{2d}$$

So, when I open up and simplify this equation I can express it in this form which you can very easily do. So, since phi is unknown to us I express it in terms of sin phi and cosine phi where A B and C they are completely known. Because, y A we have calculated x A we have calculated and s 4 is given to us.

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Solution of $A \sin \phi + B \cos \phi = C$

Let $x = \tan \frac{\phi}{2}$. Then

$$\sin \phi = \frac{2x}{1+x^2} \quad \cos \phi = \frac{1-x^2}{1+x^2}$$

Substituting in the equation yields

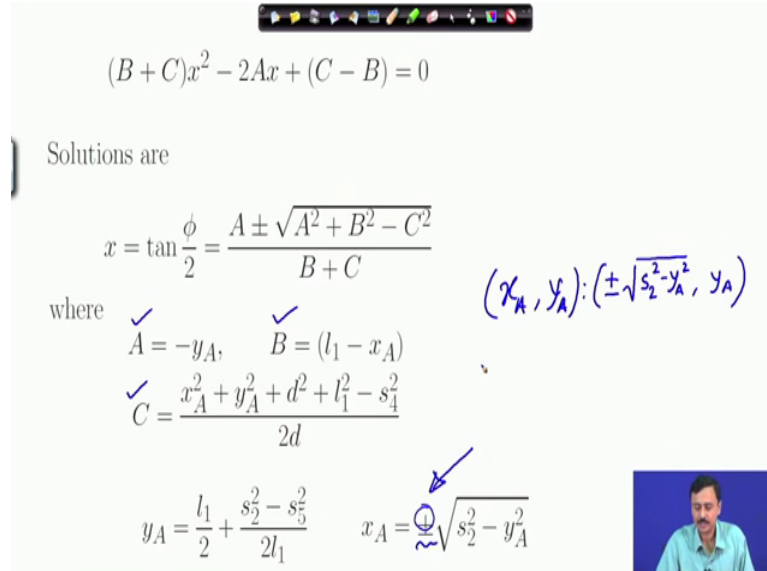
$$A \left(\frac{2x}{1+x^2} \right) + B \left(\frac{1-x^2}{1+x^2} \right) = C$$

$$\Rightarrow (B+C)x^2 - 2Ax + (C-B) = 0$$

So, the only thing that is not known is phi and we know how to solve this equation, we make this substitution we define x as tan phi by to express sin phi and cosine phi in terms

of x , substitute back in our equation get this quadratic in x whose solution is known to us.

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$$(B+C)x^2 - 2Ax + (C-B) = 0$$

Solutions are

$$x = \tan \frac{\phi}{2} = \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B+C}$$

where

$$A = -y_A, \quad B = (l_1 - x_A)$$

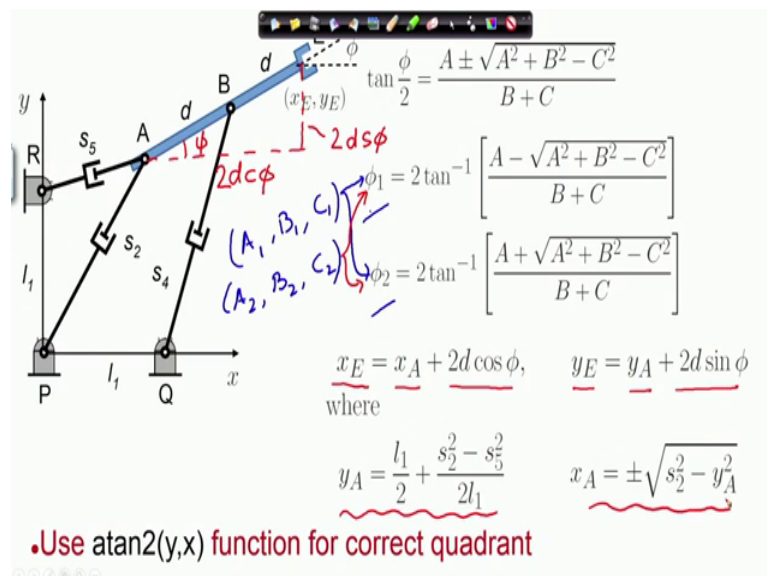
$$C = \frac{x_A^2 + y_A^2 + d^2 + l_1^2 - s_4^2}{2d}$$

$$y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1}, \quad x_A = \pm \sqrt{s_2^2 - y_A^2}$$

$(x_A, y_A): (\pm \sqrt{s_2^2 - y_A^2}, y_A)$

So, we know $\tan \phi$ by 2 in terms of A B C which are known to us, once we have ϕ we have this expression of $\tan \phi$ by 2.

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$$\tan \frac{\phi}{2} = \frac{A \pm \sqrt{A^2 + B^2 - C^2}}{B+C}$$

$$\phi_1 = 2 \tan^{-1} \left[\frac{A - \sqrt{A^2 + B^2 - C^2}}{B+C} \right]$$

$$\phi_2 = 2 \tan^{-1} \left[\frac{A + \sqrt{A^2 + B^2 - C^2}}{B+C} \right]$$

$$x_E = x_A + 2d \cos \phi, \quad y_E = y_A + 2d \sin \phi$$

where

$$y_A = \frac{l_1}{2} + \frac{s_2^2 - s_5^2}{2l_1}, \quad x_A = \pm \sqrt{s_2^2 - y_A^2}$$

• Use $\text{atan2}(y,x)$ function for correct quadrant

So, I have written out the expression of $\tan \phi$ by 2 in terms of A B C and there are two solutions of ϕ from here, ϕ_1 and ϕ_2 which are because of these the sign positive

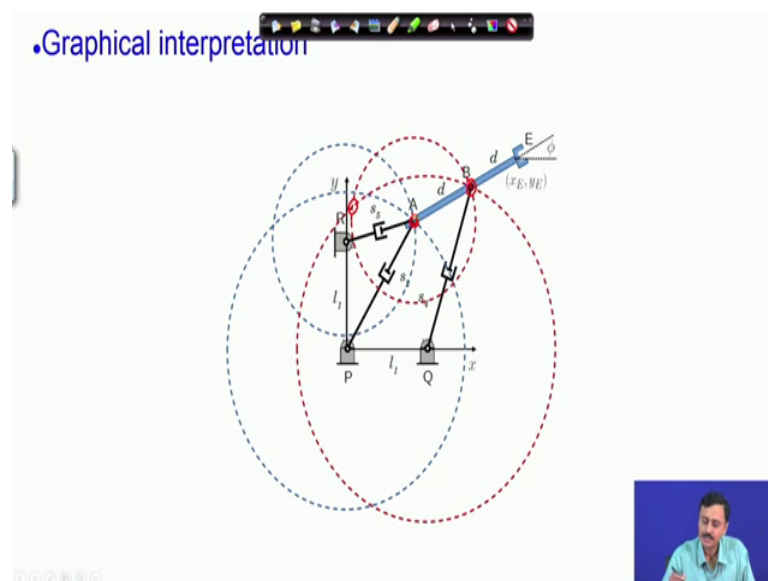
choice of the sign positive negative here. So, this is with a negative sign and this is with the positive sign.

So, these are the two solutions of ϕ , here we need to go back once to look at this expression and recall that there were two solutions of $x_A y_A$. So, $x_A y_A$ had two solutions, one with positive sign of x_A , the other with a negative sign of x_A . So, there were two solutions of x_A and y_A corresponding to these two solutions of x_A and y_A you will have two values of $A B C$, you may call them A_1, B_1, C_1 and A_2, B_2, C_2 . So, therefore, for each value of $A B C$ you will get two solutions of ϕ this is what we have looked at. So, for each value of $A B C$ you have two solutions of ϕ , ϕ_1 and ϕ_2 .

Now, since there are two sets of $A B C$ call them A_1, B_1, C_1 and A_2, B_2, C_2 we will now have 4 solutions of ϕ . So, two solutions for this and two solutions for this set so, there will be 4 solutions.

Now, once we have the solutions of ϕ we can now express the end effector coordinates x_E in terms of x_A plus 2 times $d \cos \phi$, now this is what is d so, $2 d \cos \phi$ this being ϕ and this is $2 d \sin \phi$. So, x_E will be x_A plus $2 d \cos \phi$ and y_E , similarly will be y_A plus $2 d \sin \phi$ and remember that we have these solutions of x_A and y_A .

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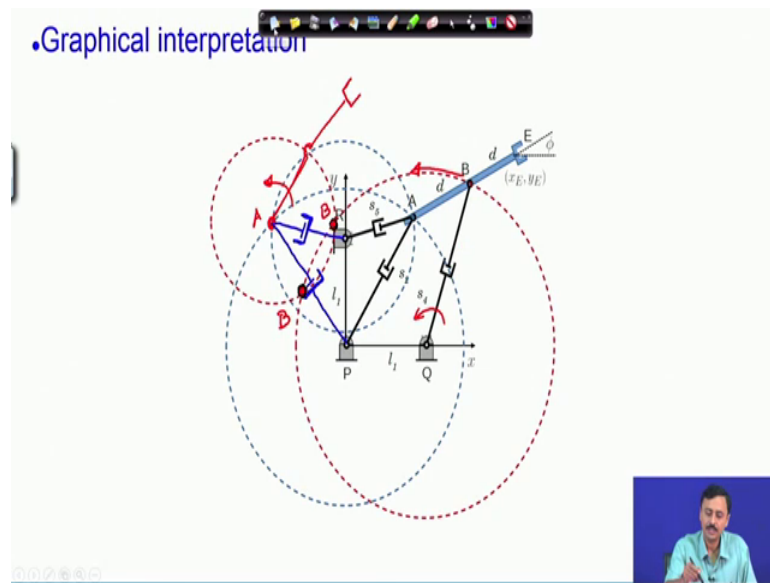


So finally, we have these 4 solutions, which we try to understand graphically. So, we were given s_2, s_5, s_4 . So, these were given, we were to find out x_E, y_E and ϕ , now we think of how we will assemble this mechanism. So, this point a belonging to this link, this leg can move on this circle. So, as we rotate this leg a will move on this circle, since s_5 is also specified if we rotate this link then A will move on another circle given by this. So, this is the circle with radius R_A or s_5 , same as s_5 .

So, wherever you have intersections of these two circles a can lie; so, a can lie on the intersections of these two circles. So, there are two possibilities; one is this possibility that has been shown the other is this possibility. So, these are the two solutions of x_A , you can see you can check that y_A remains the same, the y coordinate of A remains the same. So, the x coordinate of A has two solutions. So, as an ordered pair A can have two coordinates, one is here the other is here. Now corresponding to this solution let us say the first solution as shown.

Now, since s_4 is also given. So, the revolute pair B can lie on the intersection of the circle obtained by rotating this leg. So, when you rotate this leg. So, B moves on this circle, now because A is fixed here you can rotate the end effector link. So, therefore, B can move on another circle which is this. So on the end effector link B can rotate on this circle. So, therefore, the intersections of these two red circles which are here and here are the possible solutions, where B can be assembled and that will fix up the end effector link. So, therefore, x_E, y_E and ϕ get fixed. Now A has this solution there is a second solution of A .

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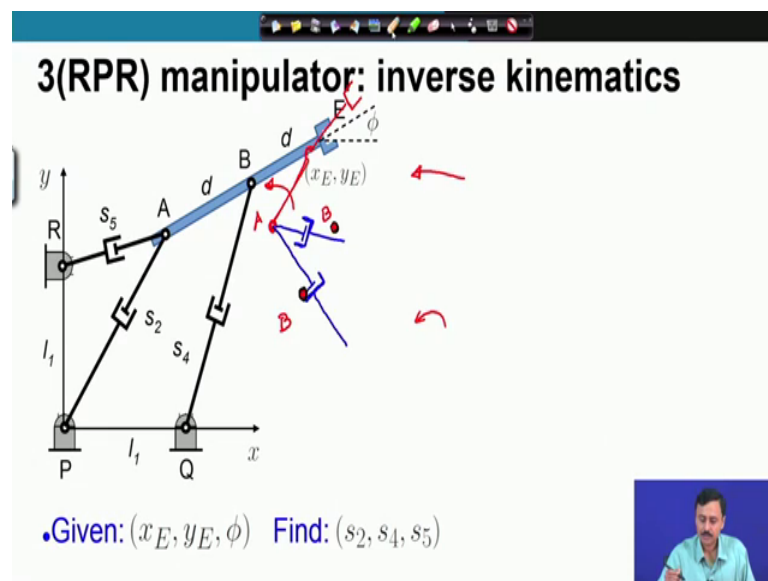
So, therefore, when you choose this solution of A then this end effector link when you rotate. So, let me draw out these two legs. So, this is the second solution for A and the end effector link can now be made to rotate about this point A and the s_4 link still rotates. So, B on the s_4 link still rotates on this circle. So, there are two intersections of this as you can see here and here. So, this is one intersection, this is the other intersection. So, this is where B can lie.

So, there are two solutions of B, for this second location of A, for the first location of A there were two solutions for B, for the second location of A there are 2 locations of B. So, there are four solutions. So, these are the four solutions that we have. So, let us look at this once again. So, we started off with this circle of radius s_2 , we want to first locate A. So, we first make a circle with radius s_2 about the origin which is P, then this second circle with center at R and radius s_5 .

So, there are two intersections of these two circles, these two intersections correspond to the two solutions of A. Now once A is fixed then we have to locate B. So, the end effector link A E can rotate about A and hence B will describes a circle and similarly the leg Q B of length s_4 can rotate about Q as a center with radius s_4 . So, I have drawn this circle with center at Q and radius s_4 and with centre A the end effector link can rotate with radius.

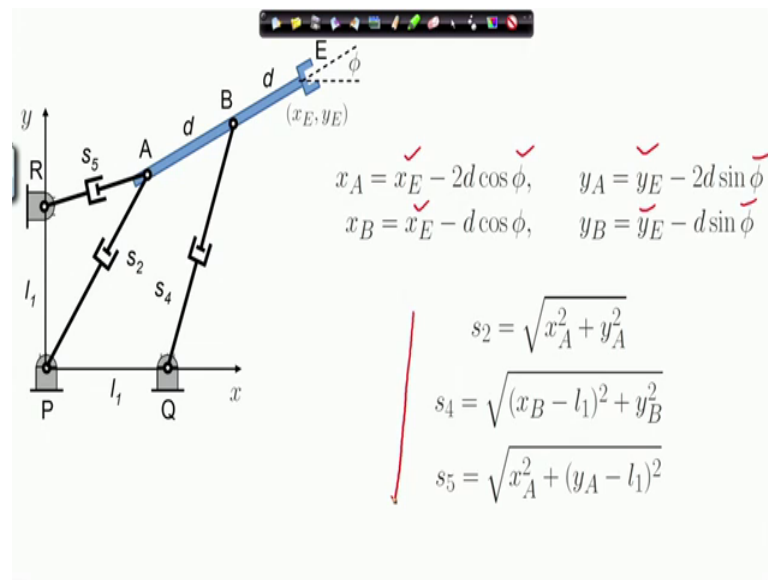
So, then rotate about A the radius is d . So, that is the second red circle. So, the intersections correspond to the point B, the intersections correspond to point B. So, there are two intersections so two solutions for B this is for the first choice of A, for the second choice of A the center at A shifts to the second solution. So, B can lie on the smaller red circle on the end effector link. So, the intersection of the two red circles again correspond to the two solutions of B corresponding to the second solution of A. So, these are the four solutions.

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Let us quickly look at the inverse kinematics problem. So, here we are specified x_E, y_E and ϕ which are the position and orientation of the end effector and we have to find out s_2, s_4, s_5 which are the actuator throws, the prismatic actuators.

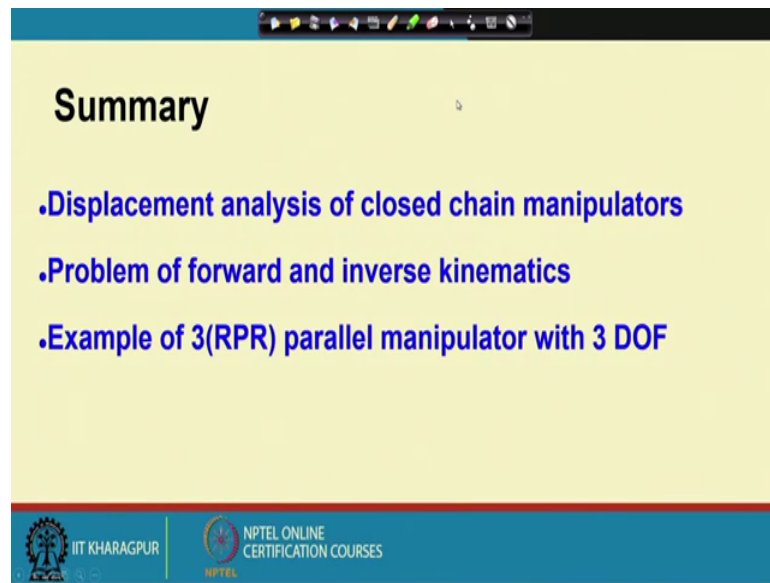
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So, here I have written out the coordinates of point A in terms of the known quantities you see x_E is known ϕ is known y_E is known. So, all these are known x_E y_E and ϕ these are known. So, therefore, I can calculate x_A y_A x_B y_B , the coordinates of these two points. Now, once I know the coordinates of these two points it is a matter of simple geometry to express s_2 .

Now, see s_2 is this, s_2 is nothing, but square root of x_A square plus y_A square then s_4 is nothing, but square root of x_B minus l_1 whole square plus y_B square from simple geometry and similarly s_5 is square root of x_A square plus y_A minus l_1 whole square. So, given x_E y_E ϕ so, these are given to us, we have been able to calculate the lengths of the actuators s_2 , s_4 and s_5 . So, this is absolutely straight forward.

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Summary

- Displacement analysis of closed chain manipulators
- Problem of forward and inverse kinematics
- Example of 3(RPR) parallel manipulator with 3 DOF

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So, let me summarize, we have looked at the displacement analysis of a closed kinematic chain manipulator and we have discussed the example of the 3 RPR kinematic chain, we looked at the forward and the inverse kinematics problem for this chain. So, with that I will close this lecture.