Kinematics of Mechanisms and Machines
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Lecture – 16
Displacement Analysis of Robots – II

In this lecture we are going to look at the Displacement Analysis problem of closed loop parallel manipulators.

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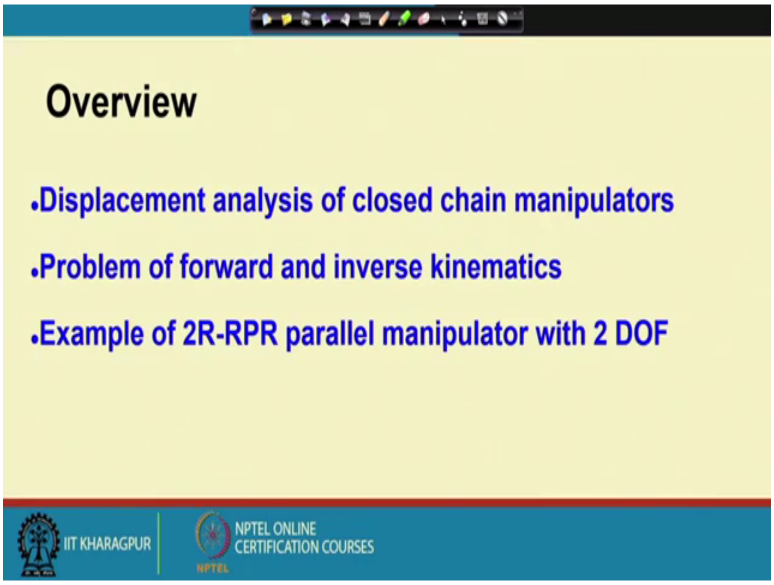

So, to give you the overview of what we are going to discuss in this lecture, we are going to look at the displacement analysis problem of closed chain manipulators, problem forward and inverse kinematics of a 2R-RPR parallel manipulator with 2 degree of freedom. So, here we have a nomenclature which we are which I am going to explain.

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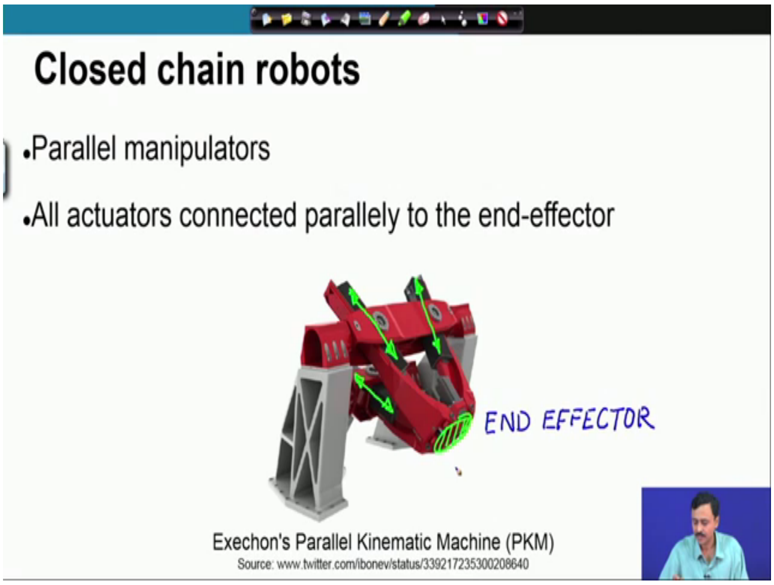

So, what are closed chain robots? They are also known as parallel manipulators. So, what are these closed chain robots? In normal robots as we have an idea of, we talk of serial chain manipulators. So, for example, my hand you can consider is a serial chain, why do we consider this as serial chain? Because the actuators and the joints they appear serially in the chain. So, this is the chain of my hand. So, here there is one actuator, the joint which is actuated, here is another joint which is actuated and they come serially.

In a serial manipulator therefore, the end effector which is my hand is connected with the links through these joints in a serial manner. As opposed to this in a parallel manipulator we have all the links which are actuated connected to the end effector directly, parallelly. So, that is why you also use this term parallel manipulator.

So, all actuators are connected parallely to the end effector. So, here I have this example of x accounts parallel kinematic machine, which is actually used for machining operations. So, let us understand why this is a parallel manipulator. Here you can see this is one actuator, this is the second actuator and underneath this is the third actuator and all these actuators are connected to the end effector. So, this is the end effector, this is the end effector where the machining tool or the gripper will be connected.

So, all these actuators parallely connect to the end effector and as you can very easily see that there are no singular links as expected in a closed chain, closed kinematic chain. So, there is no singular link, no link with only one kinematic pair. So, we have a closed chain robot in which all actuators connect parallely to the end effector.

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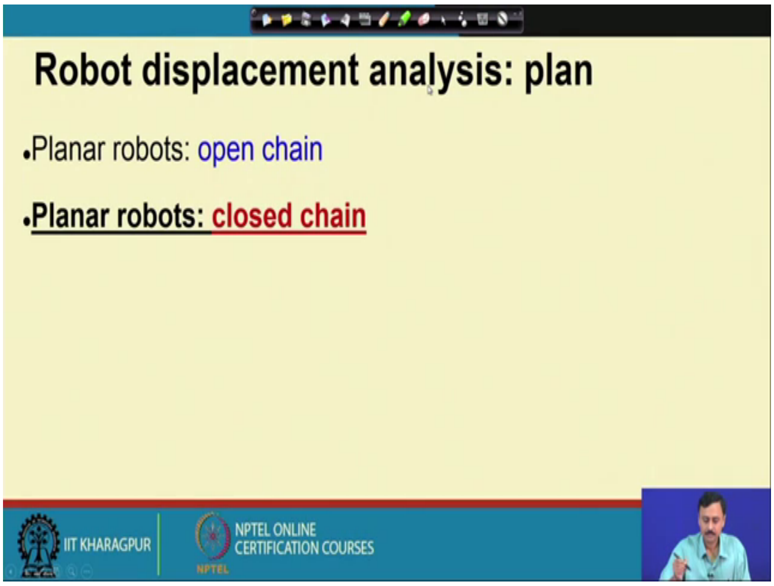

So, as per our plan we have discussed open chain planar robots previously. So, in this lecture we are going to start with closed chain planar robots.

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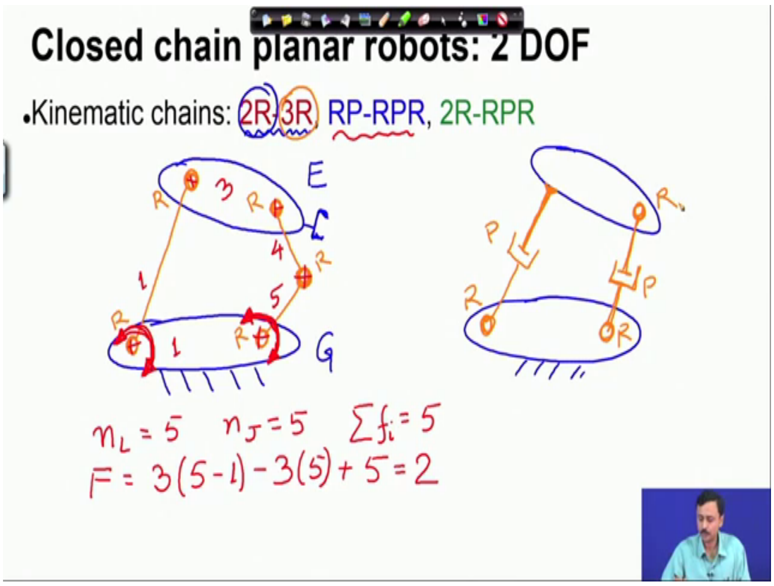

So, there can be various kinds of chains, let me explain this nomenclature and draw out. So, we have one link which is ground and the other link which is the end effector. Now in this nomenclature like 2R dash 3R this 2R stands for one of the legs of this parallel manipulator. So, therefore, this leg the 2R leg is like this. So, R R so you have R here and R here and the other leg is a 3R leg and this one is your end effector.

So, if you want to calculate the degree of freedom. So, this ground is 1, 2, 3, 4, 5. So, number of links is 5, number of joints 1, 2, 3, 4, 5 and summation of degree of freedom of each joint since they are all revolutes, there are 5 revolutes. So, summation of degree of freedom is 5. So, therefore, degree of freedom is 3 times number of links minus 1 minus 3 times number of joints plus summation of degree of freedom of each joint. So, this turns out to be 2.

So, this has 2 degrees of freedom. So, that is why this is a robot, there is no longer a constraint mechanism. So, it will require two joints to be actuated. So, possibly this joint and this joint so the two ground revolute pairs can be actuated. So, two joints will be required to be actuated, RP-RPR the next chain. So, once again we have a ground and an end effector link. So, we have R and P which is here it is welded.

So, R P and the other leg is RPR. So, this is the RPR leg here also you can calculate the degree of freedom, will turn out to be 2 the next chain is 2R-RPR which we are going to study.

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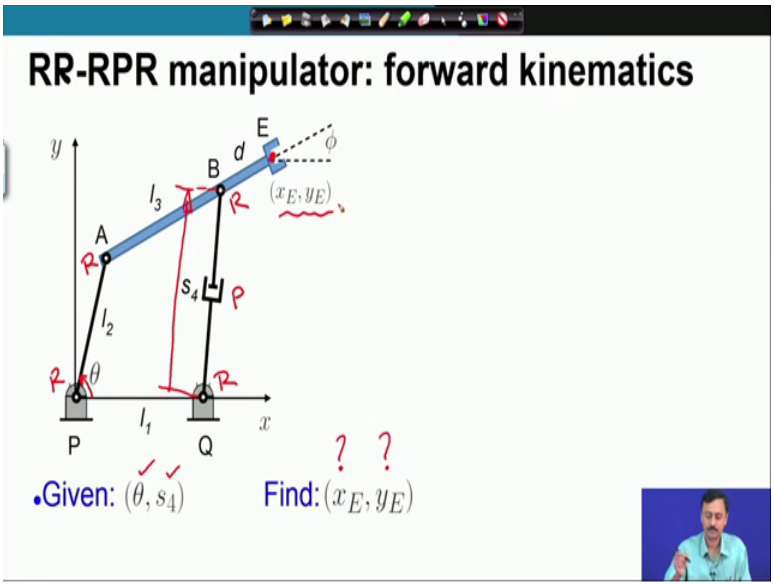

So, this is the 2R RPR which also has 2 degrees of freedom as you can easily check.

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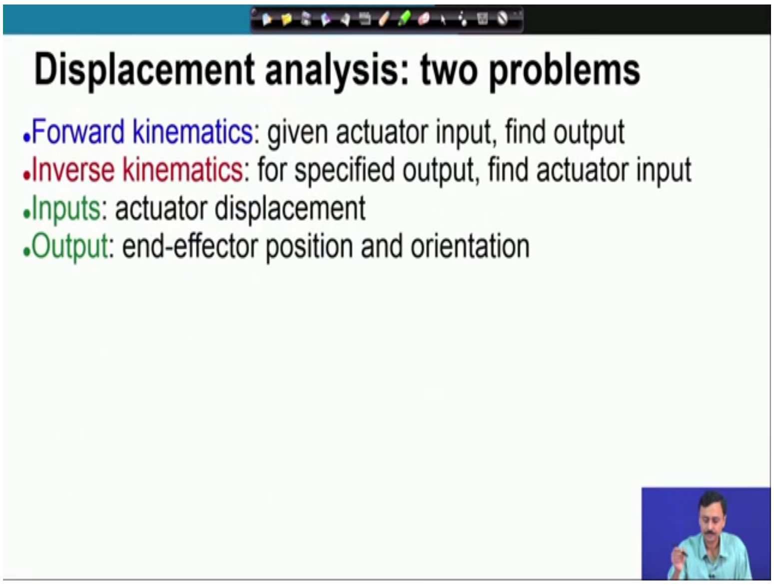

Now, we have two kinds of problems as you know the forward kinematics problem in which the actuator inputs are given, we have to find out the output, output is the end effector position or position and orientation depending on degree of freedom of the chain.

So, an inverse kinematics problem for a specified output; that means, the position and orientation of the end effector or just the position of the end effector, we have to find out the actuator input or inputs. So, in this RP RPR. So, here we have this RR-RPR. So, this actually is RR-RPR chain. So, we are going to discuss the forward kinematics problem of this RR-RPR chain. So, here you have RR-RPR R and the forward kinematics problem we are specified theta which is this angle and the throw of the prismatic actuator, which is this length, which is S 4.

So, we are given theta and S 4, these are to be actuated we have to find out x E and y E which are the coordinates of the end effector point, we have to find this in the forward kinematics problem.

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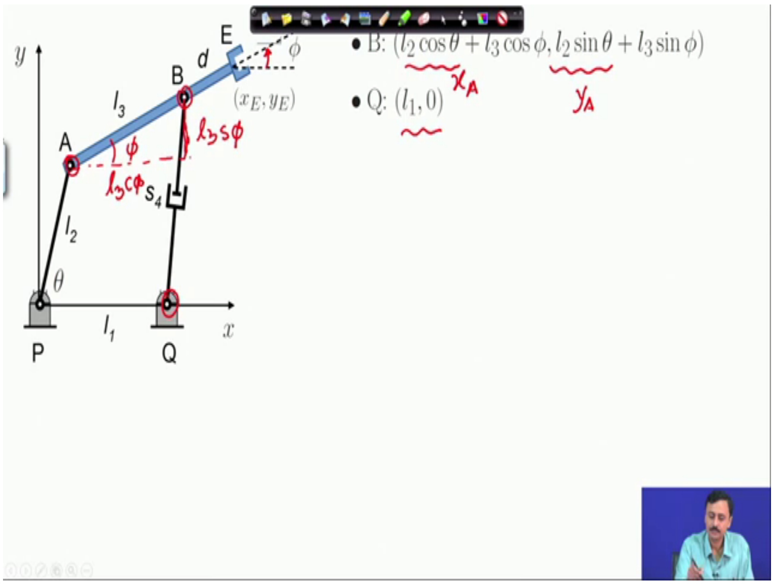

So, let us look at how we go about doing this. So, the point B here you see we have this point B whose coordinates you can now very easily find out is l 2 cos theta which is the coordinate of point A. So, this is the x coordinate of point A plus l 3 cos phi now this angle phi is an orientational coordinate. So, this gives the orientation of the end effector linked with the datum the x axis.

So, that is phi. So, I relate the coordinates of point b in terms of theta and this phi, I have brought in this additionally which I will show you how to calculate. So, the first term l 2 cos theta is the x coordinate of point A plus l 3 cosine phi is a this projection. So, that is the x coordinate of point B the y coordinate of point B, this is the y coordinate of point A and to that I add the y projection of A B, that is l 3 sine phi this is l 3 sine phi and this is l 3 cosine phi and we have this coordinates of point Q as l 1 comma 0.

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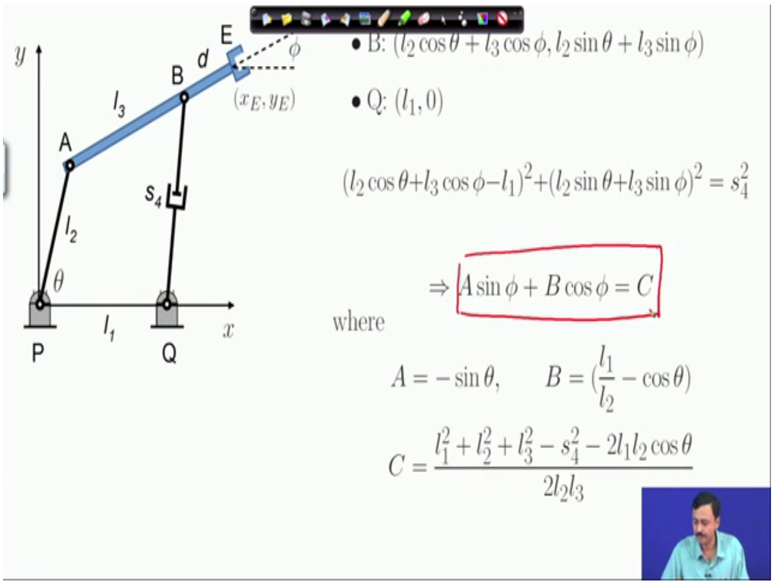

Therefore the length S 4 I can express. So, s 4 square is nothing, but x B minus x Q square plus y B minus y Q square.

Now, if you substitute these expressions the coordinates of point B and Q then you come to this expression. And when you open this up and arrange the terms then you can simplify this equation remembering that we are given theta and s 4 and the unknown here in this equation is phi, we are given theta theta and this s 4, the only thing that is unknown is phi. Therefore, I can assemble this equation I can simplify this equation and assemble it in the form some A sine phi plus B cosine phi equal to C which you can easily do where you will find that this A B and C are completely known because I know theta and I know s 4.

So, therefore, a b and c are completely known to me. So, what is unknown is phi which I need to solve from this equation. So, I need to solve this equation in order to find phi.

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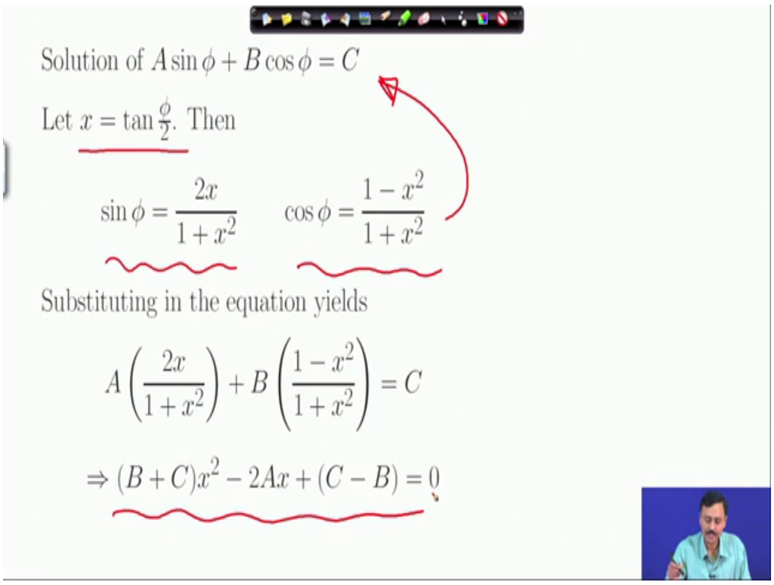

As discussed previously we will take this approach which can be very easily programmed on the computer and you can get the both, you can get all the solutions of this equation A sine phi plus B cos phi equal to c. So, in that we substitute, we make a make a definition x equal to tan phi by 2 and represent sine phi and cosine phi in terms of x which when substituted into our master equation finally, gives us this quadratic equation in x whose roots we can now easily find out. And hence we can find out tan phi by 2 and that is what we are going to do.

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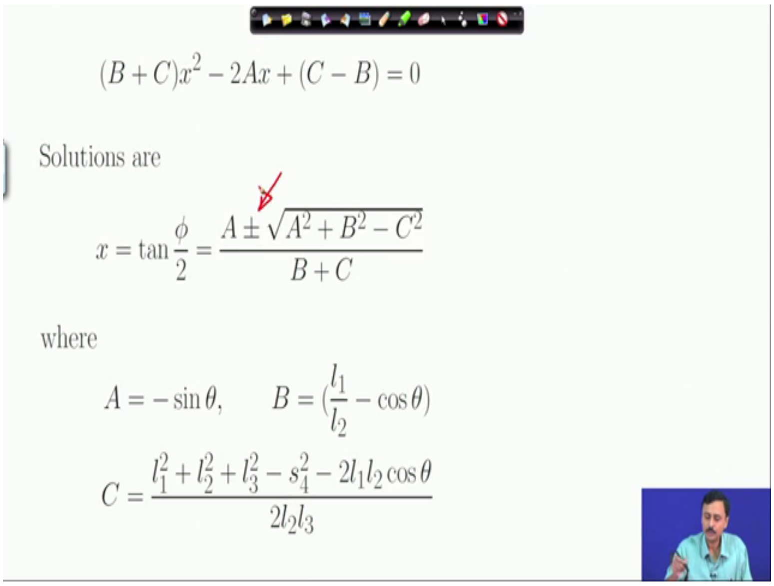

So, the solution solutions of this quadratic equation, we have these two solutions given by these two signs positive and negative. So, we get two solutions of x and hence two solutions of phi A B and C are completely known.

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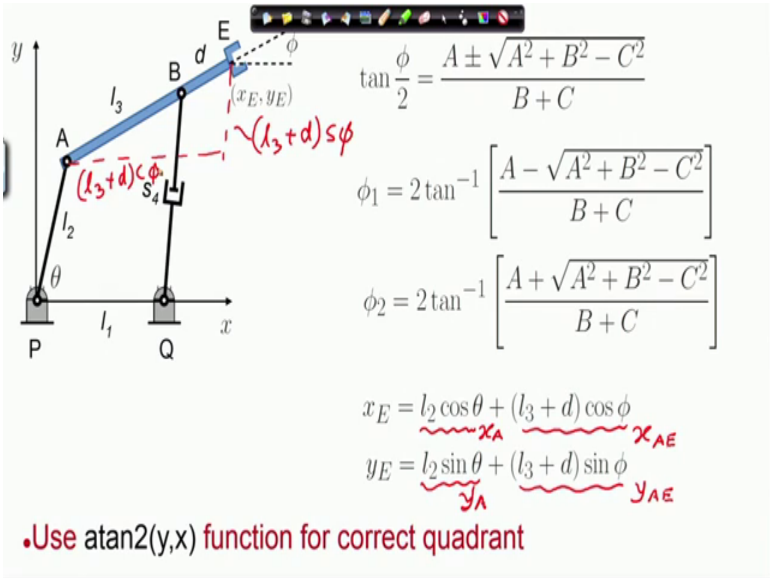

So, therefore, we have this tan phi by 2 expression in terms of A B C, these are the two solutions, once again you need to use this 8 and 2 functions so that you get the correct quadrant of phi 1 and phi 2 and finally, what we set out to calculate was the coordinates of this end effector. So, x E and y E.

So, x E x coordinate of the end effector is l cosine theta which is nothing, but x coordinate of point A and this part the second term in the expression of x E which is l 3 plus d cosine phi is nothing, but the vector A E, the x coordinate of the vector AE. So, I will I will write it like this, that this is the x coordinate of AE, similarly in the expression of y E you have l 2 sine theta, which is the y coordinate of a and the second term is nothing, but the y projection of this AE which is l 3 plus d sine phi. So, this is l 3 plus d sine phi and this is l 3 plus d cosine phi. So, that is x E and y E. So, we have obtained the coordinates of the end effector point E.

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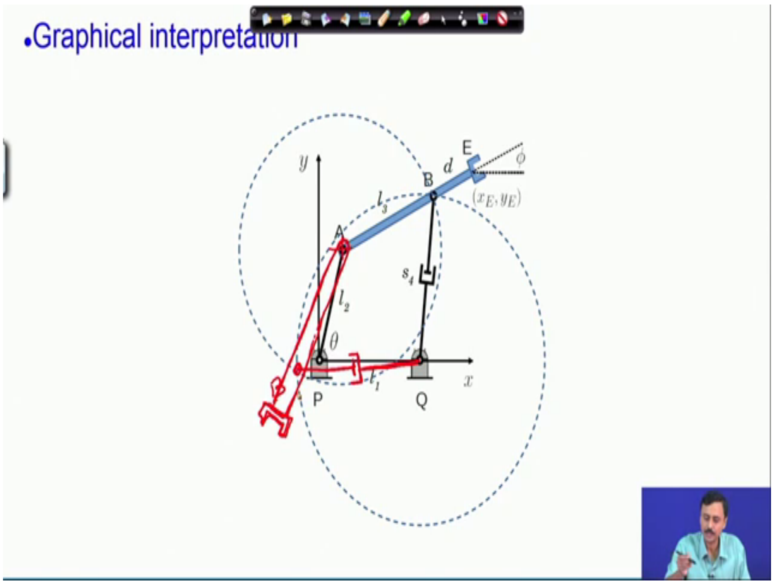

Let us understand the solution graphically, remember that we are given theta and s 4. So, therefore so theta and s 4 are given. Now if you see when theta is given then this point A gets fixed, what is not fixed is phi because this hinge B on the end effector link can rotate on this circle, while the hinge B on the actuator arm on this on this other leg can rotate on this circle.

So, the way to assemble the mechanism is where these two circles intersect for example, this is one intersection point. So, you have one configuration that is already shown, there is another solution which is given by this your hinge B can also lie here. So, therefore, the mechanism in this configuration will look like this. So, in the red configuration because point A is fixed remember, because theta is given since theta is specified a gets fixed and hence you have another assembly mode of this mechanism as shown by this red configuration.

So, these are the two solutions.

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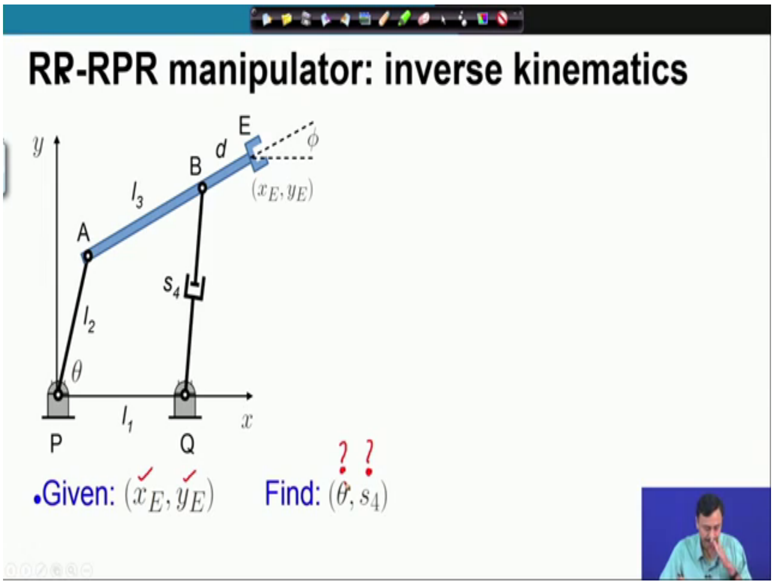

Now let us move on with the. So, this is the RR-RPR manipulator and we are going to study the inverse kinematics of this chain now. Here we are given the coordinates of the end effector and we are to find out the inputs the actuator inputs, which are given by theta and s 4.

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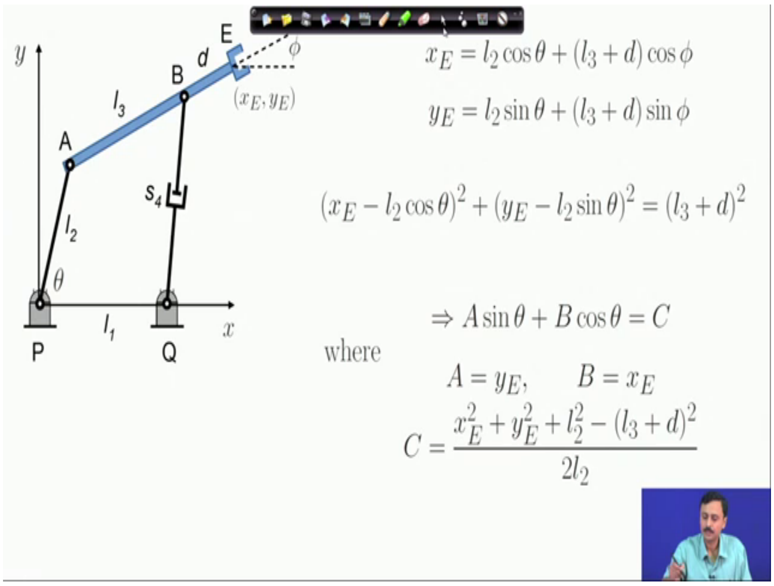

So, here I have written out the forward kinematics solution, you remember we had derived this, these expressions of x E and y E.

So, we start with the forward kinematics solution or relations, if I take this term so what I am given this x E and y E. So, these are known to be x E and y E are known to me, what I have to find out is theta let us say the first thing is theta. So, from these two, I can eliminate phi and this is what I have done in the next step. So, I have taken these terms to the left hand side and squared and added them to eliminate phi. So, phi is completely eliminated in this equation.

So, what I am left with we have in this equation x E and y E which are completely known and what is not known is theta. Now if you open up this expression on the left hand side and simplify then you can very easily arrive at this form. So, remember we have to find out theta and these terms A B and C, they are completely known because y E x E these are given to us. So, we need to solve this equation in order to solve for theta. So, this is a standard equation which we have been solving.

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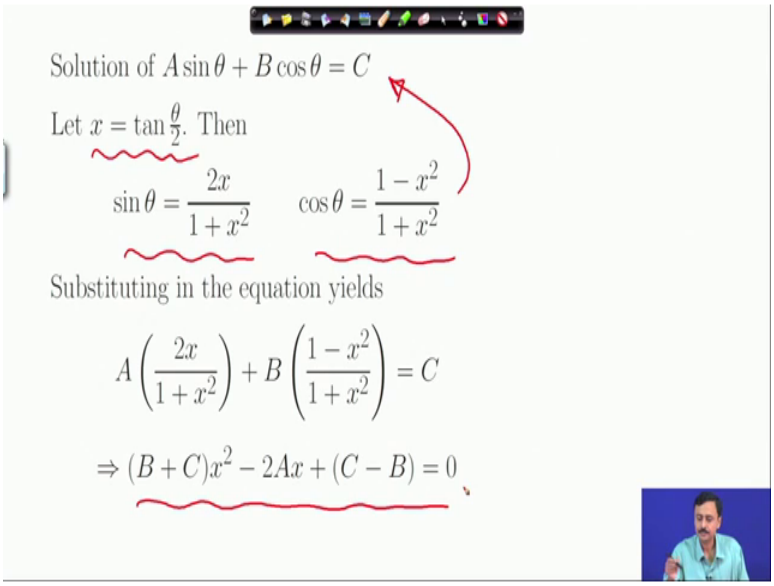

So, once again just to reiterate what we have done we have defined this x in terms of tan theta by 2, express sine theta and cosine theta in terms of x substituted into the equation that we want to solve and finally, obtain this quadratic equation.

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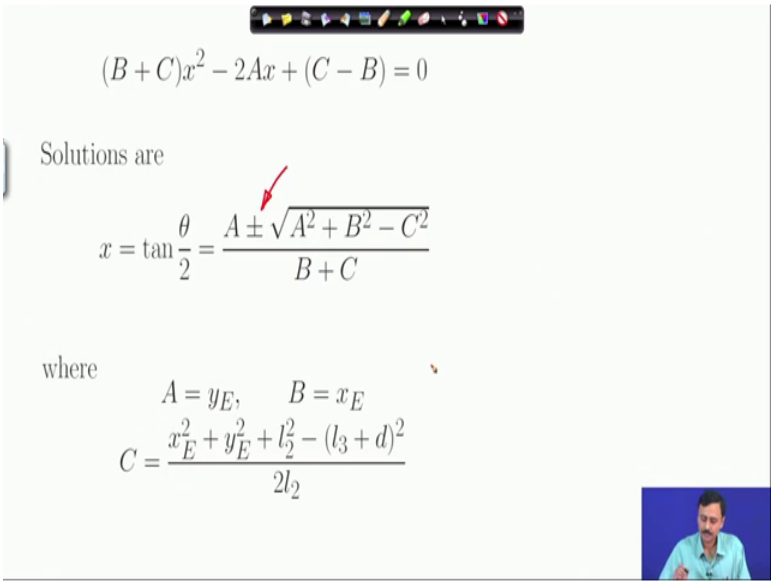

Which has solutions in terms of A B C which are completely known to us we have two solutions as you can see again here.

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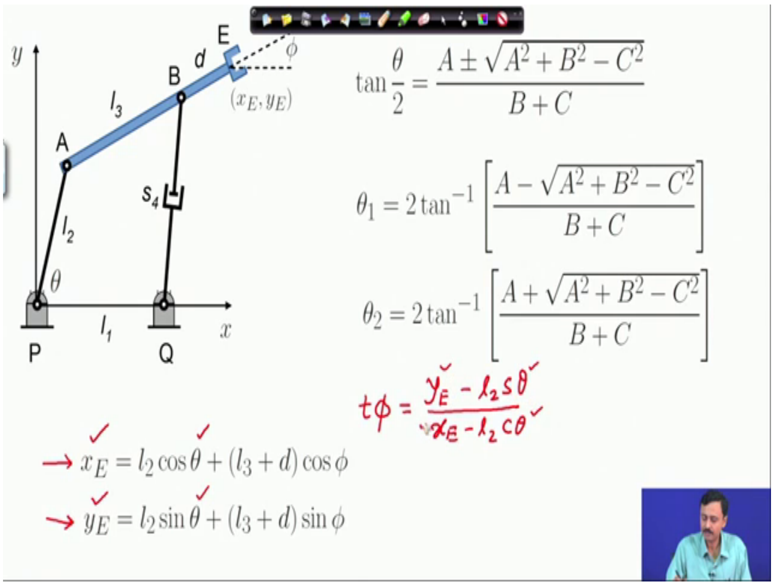

So, once we have these solutions we can obtain theta the two solutions of theta, theta 1 and theta 2 in terms of tangent inverse of this expression. So, for that again we need to use the a tan 2 function. Now once I have found theta I need to find out s 4. So, to find out s 4 we take recourse to these steps, first I will again look at these relations the forward kinematics relations which we have used. Now we know theta x E and y E of course given, we are now solved for theta; from these two equations we can now solve for phi.

So, we find out tangent phi. So, tan phi is nothing, but y E minus l 2 sine theta by x E minus l 2 cosine theta. Now, since I know theta and know x E and y E so, I can calculate phi.

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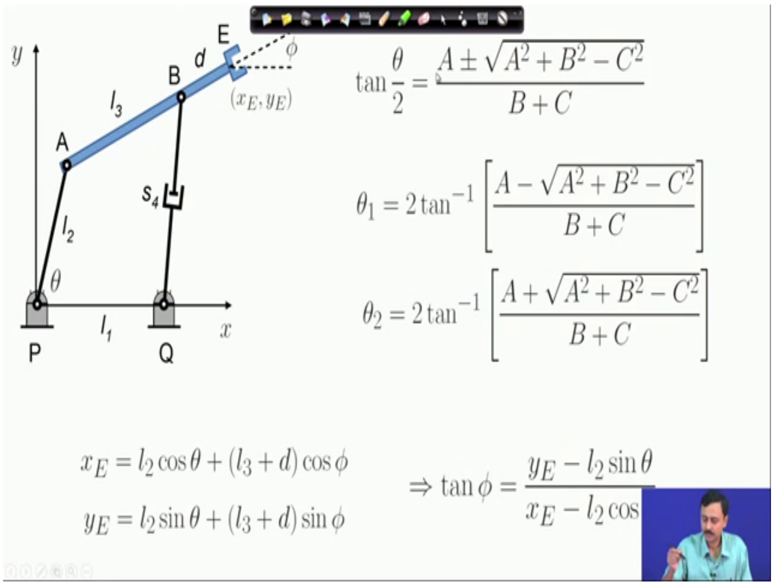

So, formally so this is the expression for tan phi. So, from here I can solve for phi, again using the a tan 2 function because I need to get the quadrant right.

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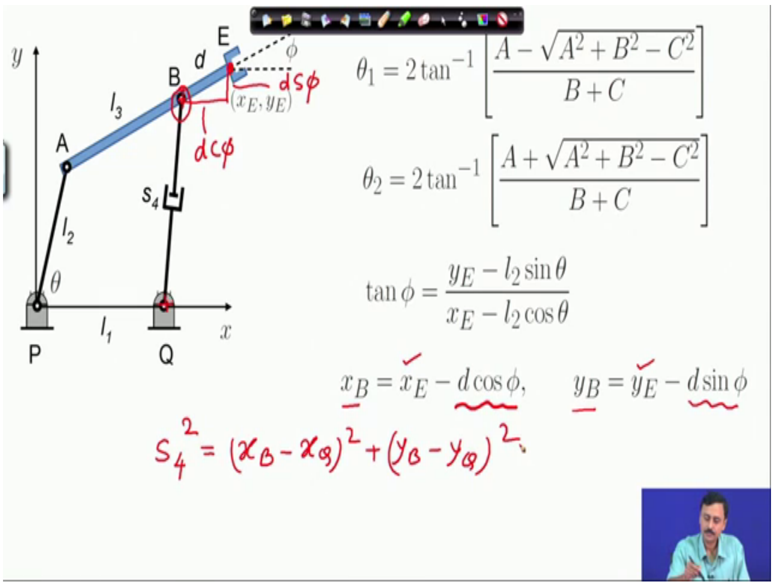

So, I have corrected these expressions now. So, we know theta 1, theta 2 in terms of x E y E then I calculate phi, once I have phi I can define the coordinates of point B. So, coordinates of point B is nothing, but coordinates of point E which is the end effector point which is given to me, which is known minus this d cosine phi which is the projection of B E this is a projection of B E along the x axis.

So, this is d cosine phi, similarly y B is equals to y E which is known to me minus d sine phi this is d sine phi, the vertical projection of B E. So, I know the coordinates of point B, once I know coordinates of point B I also know coordinates of point Q therefore, I can now find out this length s 4 because s 4 square is equal to x B minus x Q whole square plus y B minus y Q whole square that is s 4 square. So, from here I can find out the throw of this prismatic actuator.

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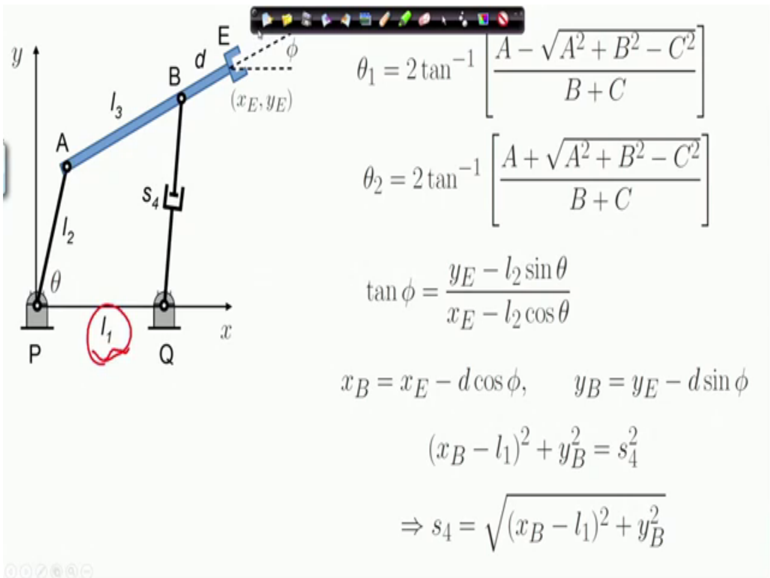

So, this is what I have written out. So, s 4 is square root of x B minus l 1. So, you have l one here the length P Q. So, x B minus l 1 whole square does y B square because y Q is 0 the y coordinate of point Q is 0.

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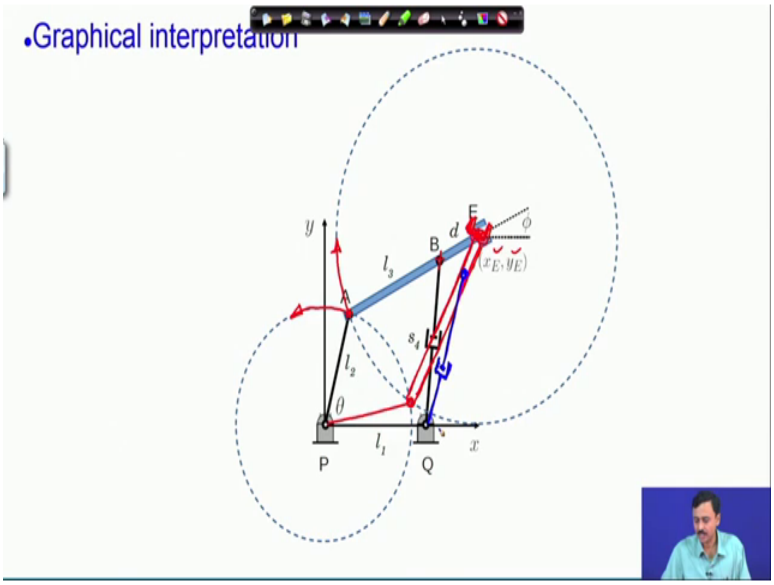

So, let us understand this solution graphically we have been given x E and y E. So, this point E is fixed what is not fixed is this hinge A on the end effector link A can move on this circle on the hinge on the link l 2 can move on this circle therefore, if I want to assemble the mechanism then it can happen only at these intersection points of the two circles. Now once A is fixed since E is also fixed therefore, B gets fixed and therefore, you can find out B Q as we have done there is another configuration which looks like this.

So, this is the end effector link and let me draw the prismatic actuator the other leg. So, here I have draught drawn it in blue. So, this red blue configuration that, I have drawn is the second configuration of the second solution for the inverse kinematics problem.

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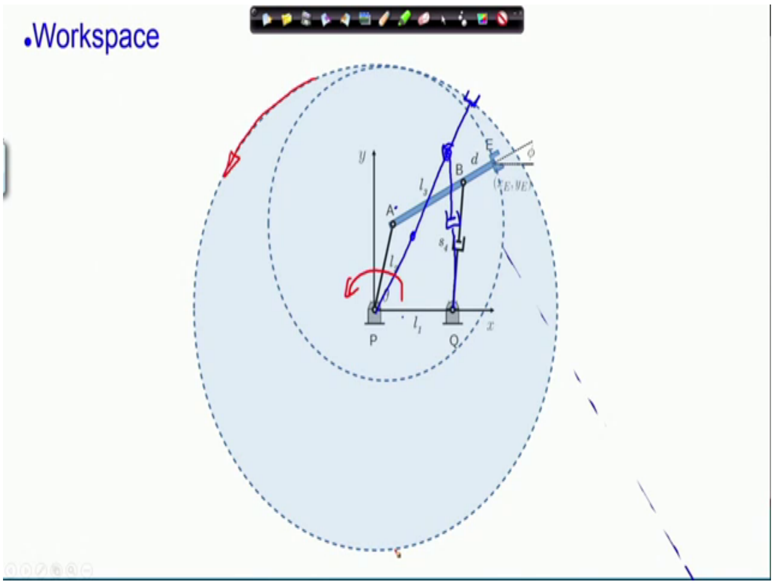

This is the workspace of the manipulator. So, if you completely extend, if you completely extend this link and then move it in the circle, you generate the outer circle which defines the workspace of this manipulator. Of course, with joint limits or actuator limits this workspace is going to get more complicated and will be reduced which you can find out based on geometry.

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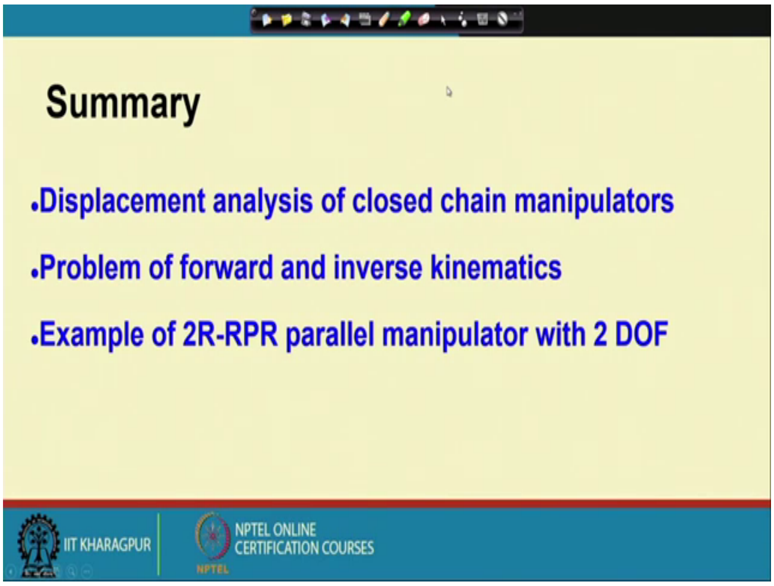

So, finally, let me summarize we have looked at the displacement analysis problem of closed chain manipulators with the example of a 2R-RPR kinematic chain. So, with that I will close this lecture.