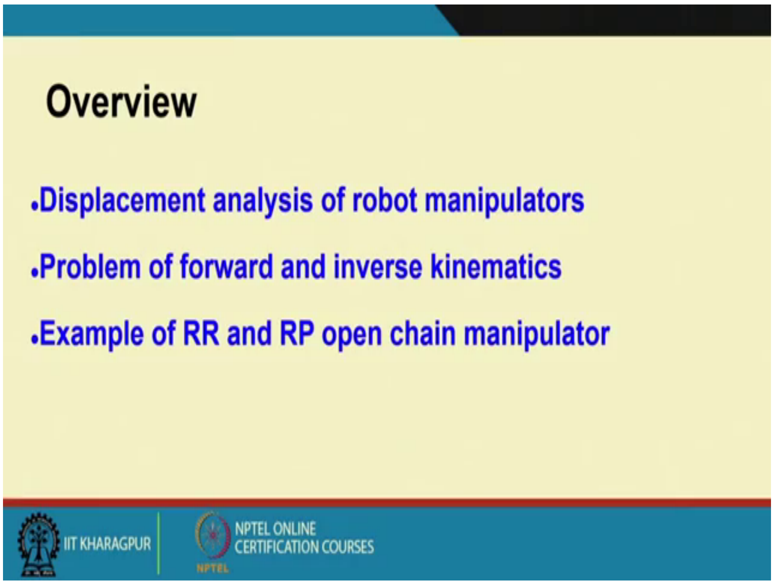
Kinematics of Mechanisms and Machines
Prof. Anirvan Dasgupta
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 15
Displacement Analysis of Robots-I

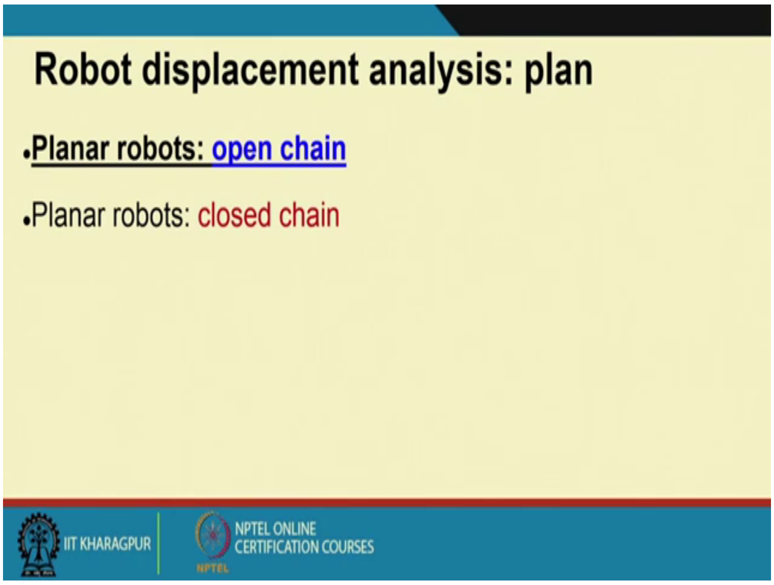
In this lecture I am going to discuss the Displacement Analysis problem for Robot Manipulators.

(Refer Slide Time: 00:27)



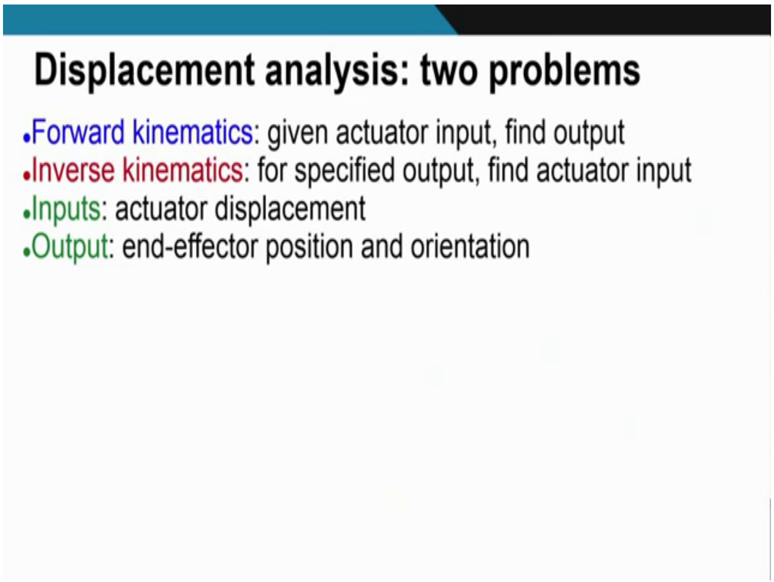
The overview of today's lecture is as follows: we look at the displacement analysis problem, the problem of forward and inverse kinematics with examples of RR and RP open chain manipulators. These are planar open chain manipulators, we are going to look at the forward and inverse kinematics problem for these manipulators.

(Refer Slide Time: 00:52)



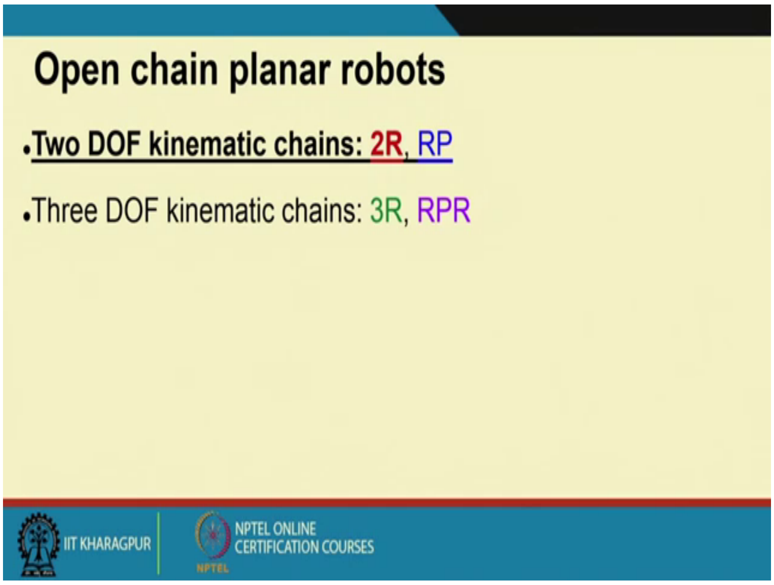
So, our plan is that we are going to look at the planar robots: open chain, then subsequently we are going to look at closed chain planar robots.

(Refer Slide Time: 01:06)



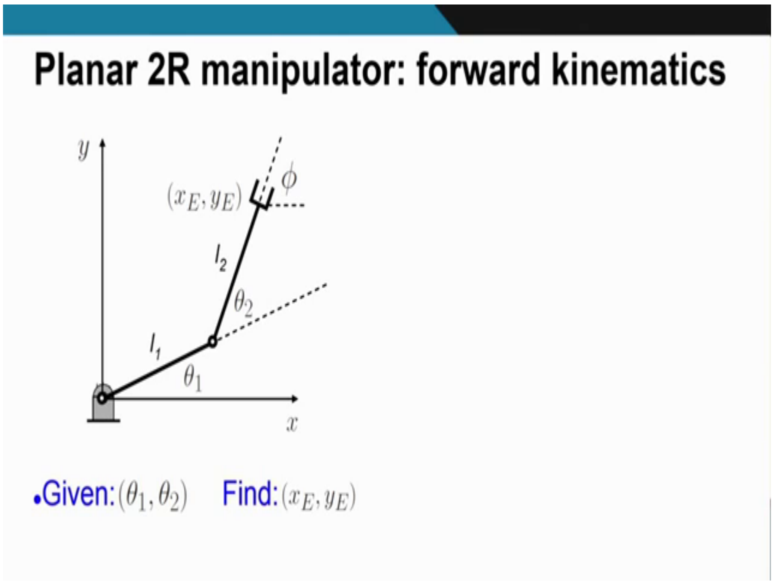
As already discussed before the forward kinematics and the inverse kinematics problem for kinematic chains is that the in the forward kinematics we are given the inputs or the joint angles may be joint displacements and we have to find out the output motion. The inverse kinematics is just the reverse, we are given the end effectors position and may be orientation and we are to find out what should be the actuator displacements.

(Refer Slide Time: 01:39)



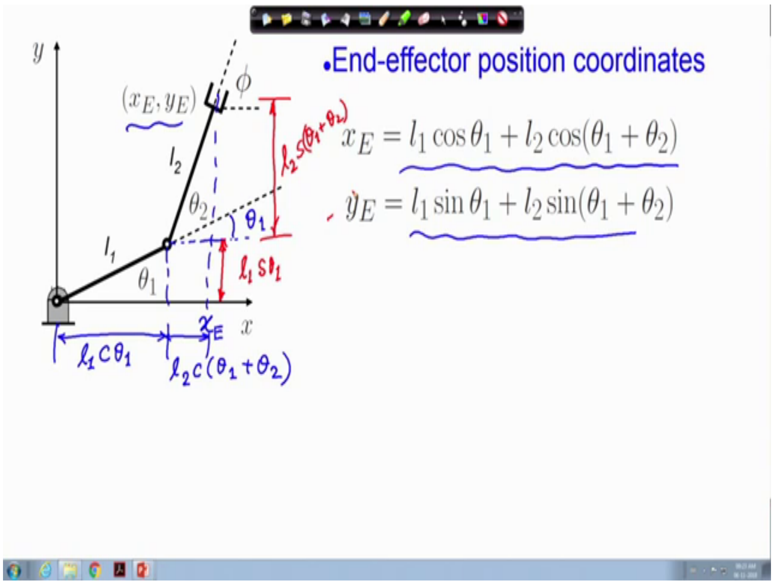
We are going to look at these two kinds of chains open chain manipulators one is 2R, the other is RP. Later on we are also going to look at 3 degrees of freedom kinetic chains.

(Refer Slide Time: 01:57)



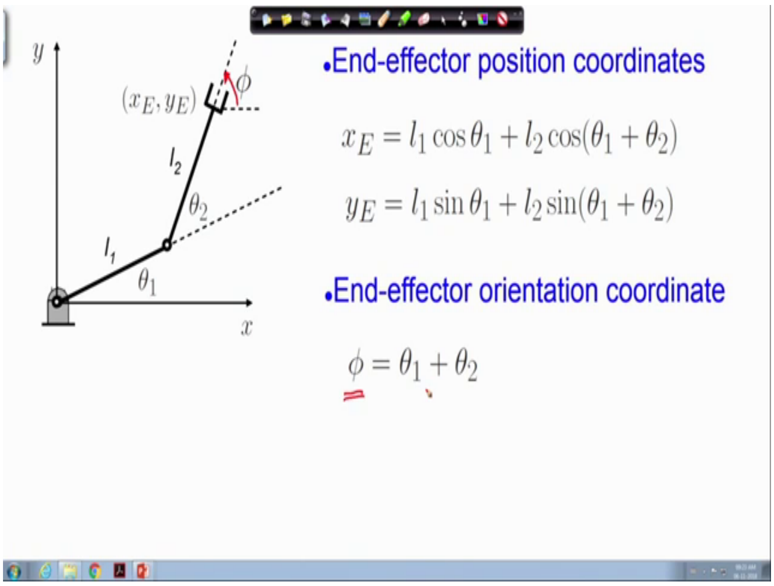
So, this is an RR open chain manipulator. So, the forward kinematics problem is as follows. We are given theta 1 and theta 2 which are these joint angles, note how theta 1 and theta 2 are defined; theta 1 is defined from the x axis to link l 1 and theta 2 is defined from link l 1 to l 2. Now, given theta 1 and theta 2 we have to find out the position of the end effector. So, that is the forward kinematics problem.

(Refer Slide Time: 03:14)



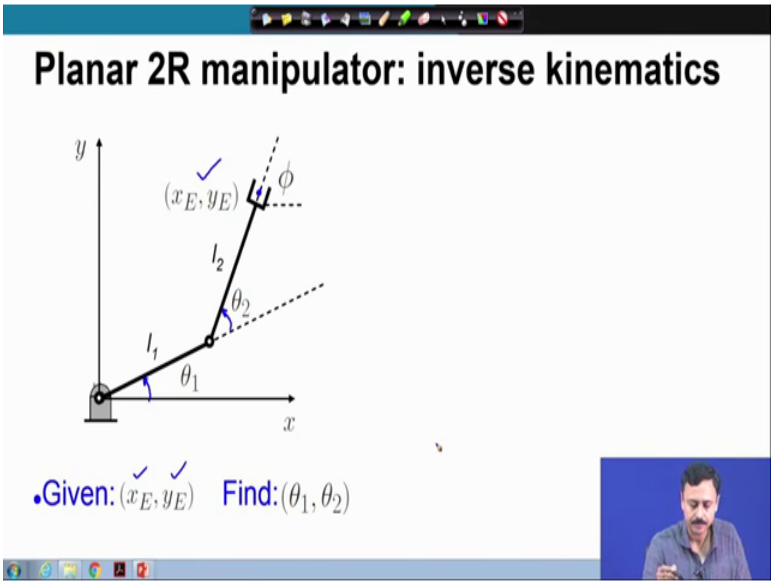
You can very easily write out the location or the position of the end effector which is denoted by x E and y E. For example, this x E so, this is x E this is nothing, but the sum of these two lengths. This length is l 1 cosine theta 1 and this length this angle is theta 1. So, this length becomes l 2 cosine of theta 1 plus theta 2 that gives us x E. Similarly, you can find out y E as l 1 sin theta 1 which is this length and add to that this length which is l 2 sin of theta 1 plus theta 2, that gives us y E.

(Refer Slide Time: 04:48)



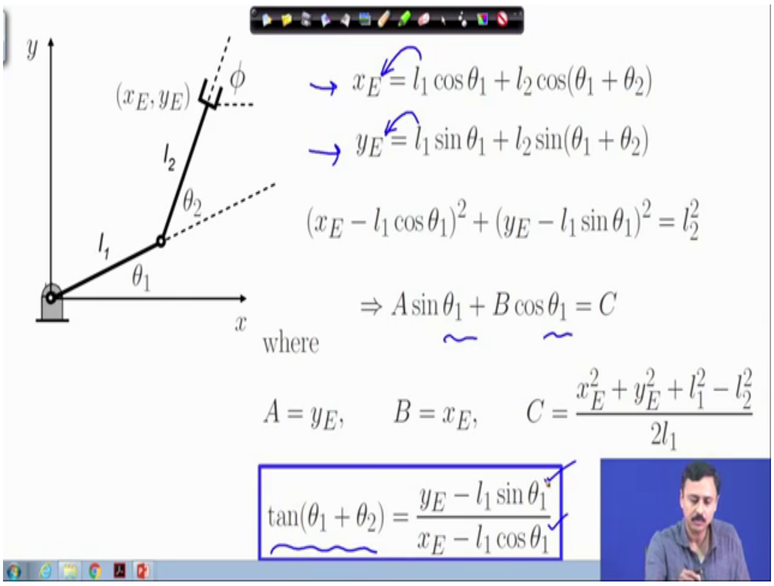
The end effector orientation coordinate given by this angle phi is theta 1 plus theta 2..

(Refer Slide Time: 05:01)



So, the forward kinematics problem is fairly straightforward, let us look at the inverse kinematics problem now. We are given x E and y E, this is the location of the end effector. We have to find out these angles theta 1 and theta 2. We start off with the forward kinematics relations x E in terms of theta 1 theta 2, similarly y E in terms of theta 1 theta 2. Then we are given x E and y E we have to find out theta 1 and theta 2.

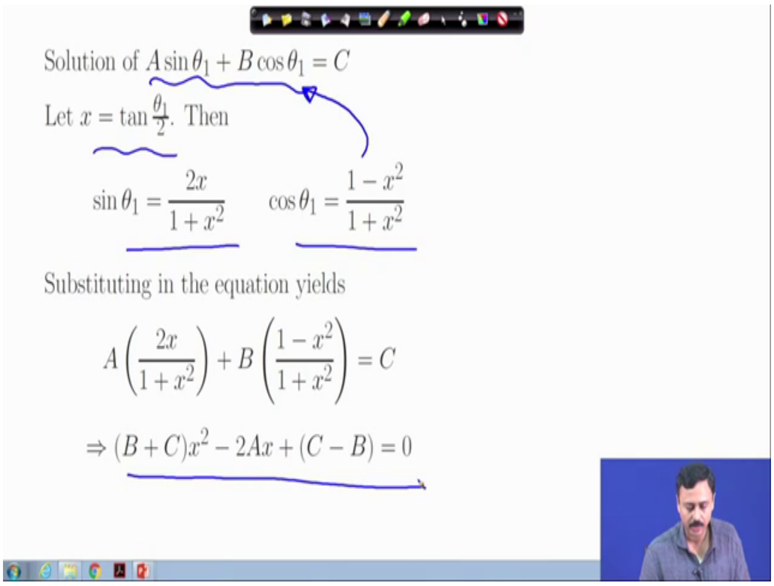
(Refer Slide Time: 05:32)



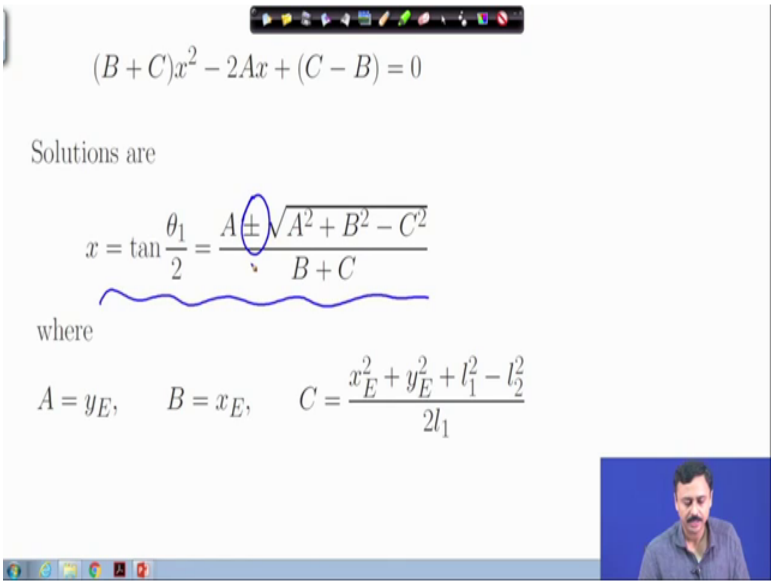
If somehow I can eliminate this sum of theta 1 and theta 2, then I can get a relation only in terms of theta 1 which is shown here. What I do is take it to the left hand side the first term and square and add that will give us l 2 that eliminates theta 1 plus theta 2. So, therefore, we have a relation between theta 1 and the other known quantities, now this can be expressed. So, this was our equation after eliminating theta 1 plus theta 2. This can be recast as A sin theta 1 plus B cosine theta 1 equal to C where, A is y E B is x E and C is this expression. This can be very easily derived by simplification.

Now, this kind of equation we have solved before. Once we have solved for theta 1 from this equation we go back to these two equations, we have taken this to the left hand side. Now, we will take the ratio to find out tangent of theta 1 plus theta 2 and that is given by y E minus l 1 sine theta 1 divided by x E minus l 1 cosine theta 1. Since theta 1 is now known, we can find out theta 1 plus theta 2 and once we can find out theta 1 plus theta 2 and since theta 1 is known we know theta 2.

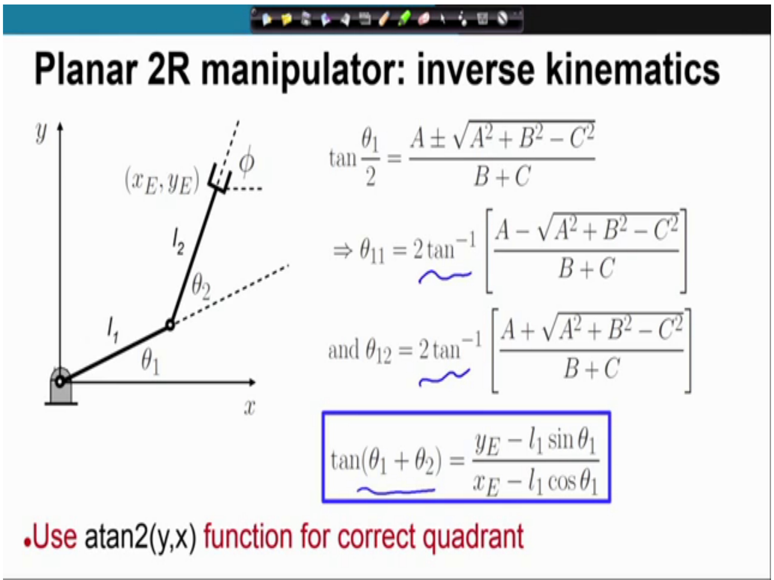
(Refer Slide Time: 07:47)



Now, this is a solution procedure that we have discussed already, for this kind of equation we substitute x as tan theta 1 by 2 right out sine theta 1 terms cosine theta 1 in terms of x substitute them back in our equation, obtain the quadratic equation. And finally, we have the two solutions given by these two signs.

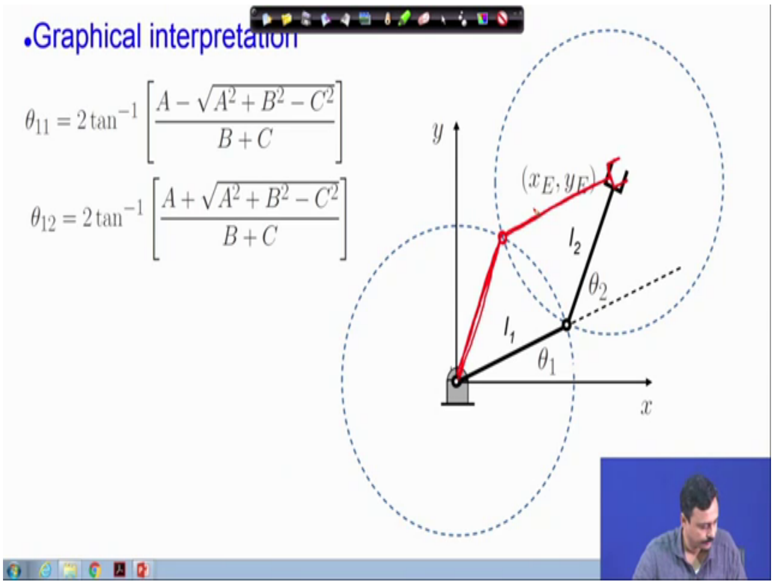


(Refer Slide Time: 08:21)



So, we have tangent theta 1 by 2 given by this expression in terms of A B and C which are known. Therefore, we have two solutions of theta 1 theta 1 1 and theta 1 2 and once we have theta 1 1 and theta 1 2, we can substitute in this expression for tangent of theta 1 plus theta 2 and determine theta 2. So, when we take these tangent inverses here also we have to take tangent inverse we must use the a tan 2 function to get the correct quadrant.

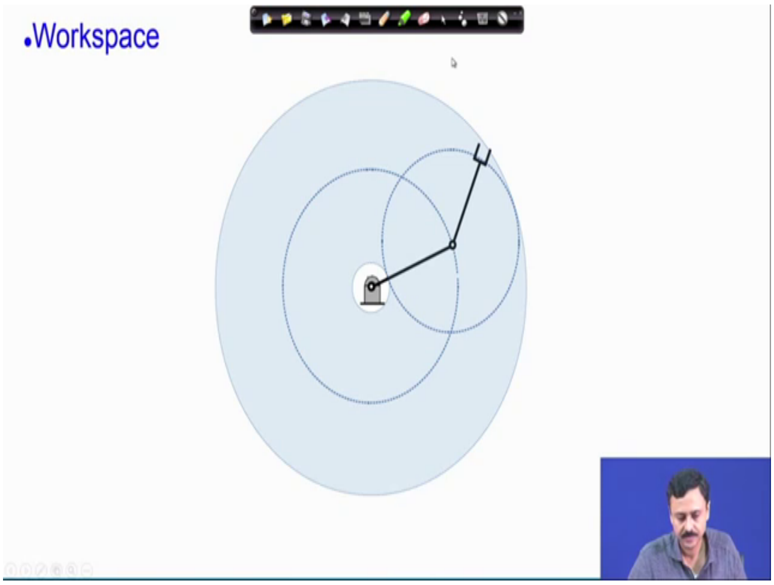
(Refer Slide Time: 09:19)



Let us try to understand the two solutions graphically these are the two solutions now what is fixed is this point x E y E that is fixed. Now, these two solutions can be thought of as the assembly of this manipulator at these two positions. So, this hinge as considered on l 2 will move on this circle and the same hinge as considered on l 1 will move on this circle.

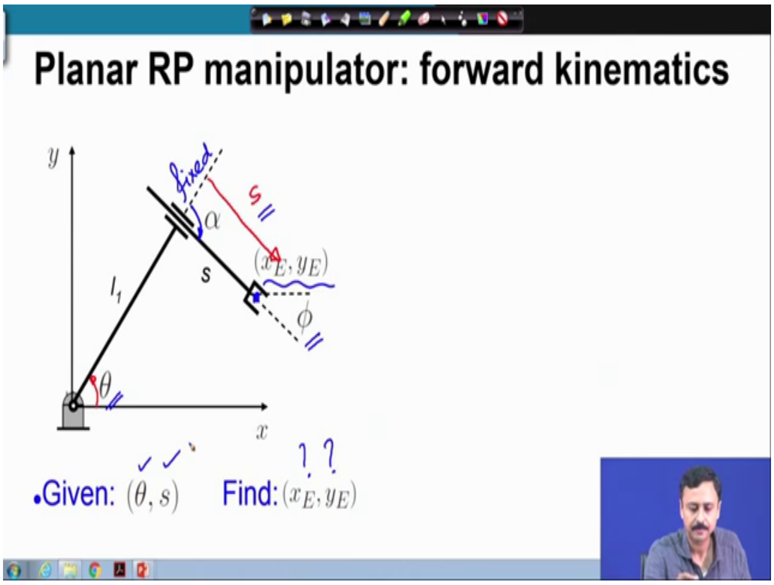
Therefore, the mechanism can be assembled at the intersection of these 2 circles there are 2 intersections 1 is shown here already in black the other configuration is this red configuration. So, these are the 2 configurations which will reach the same point x E y E.

(Refer Slide Time: 11:08)



Next let us look at the theoretical workspace of the manipulator where this manipulator can reach. As you can see this joint if it can rotate 360 degree and this hinge will rotate on this circle. In the same way when this joint rotates completely the end effector will rotate on the smaller circle. Therefore, the reach of the manipulator is now shown here it's within this blue region; it can reach anywhere within this shaded region, this is the workspace of the manipulator.

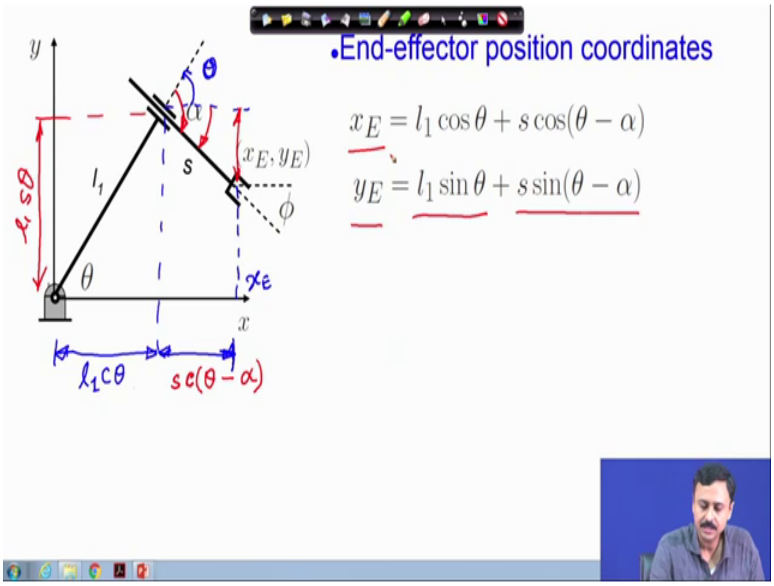
(Refer Slide Time: 12:14)



Next let us move on to another kind of planar open chain manipulator the RP this also has 2 degrees of freedom. The degrees of freedom here are this angle theta and this extension of this prismatic pair which is indicated by this s. The angle alpha is a fixed angle, this angle is fixed.

Therefore, we have 2 input variables 1 is theta the other is s which will position the end effector and the coordinates of the end effector are given by x E y E. The orientation of the end effector is given by this angle phi, this forward kinematics problem is finding x E and y E when we are given theta and s.

(Refer Slide Time: 13:36)

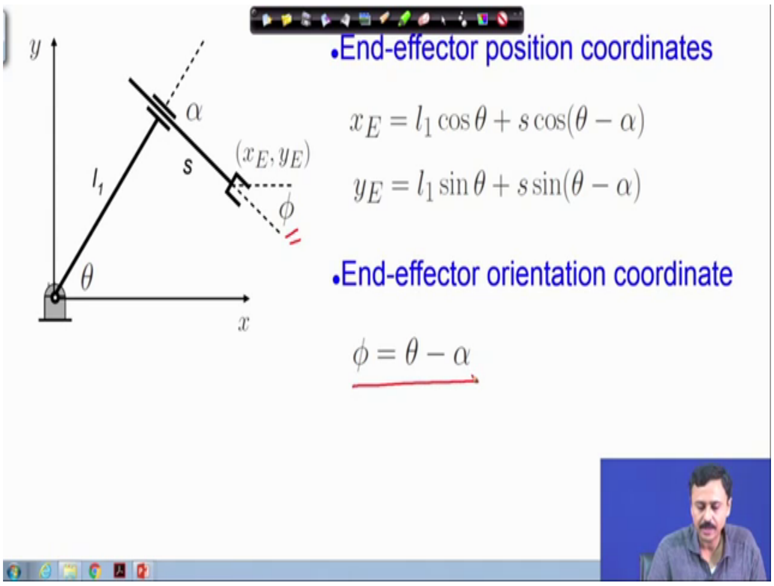


Let us look at these relations the expression of the coordinates which are straightforward x E. So, this is x E the summation of these two lengths this is l 1 cosine theta 1. Now this is s times this is theta 1, this is theta well this angle is theta this angle is theta and this angle is alpha which is fixed. Now since all angles we will be measuring counterclockwise as positive which is by our choice of coordinate system.

So, counter clockwise measurement of angles is positives clockwise measurement is negative therefore, this angle is theta minus alpha and therefore, this distance is s cos theta minus alpha. So, that is the total x E. Similarly, you can express y E as l 1 sin theta and add to that this distance which is s sin theta minus alpha.

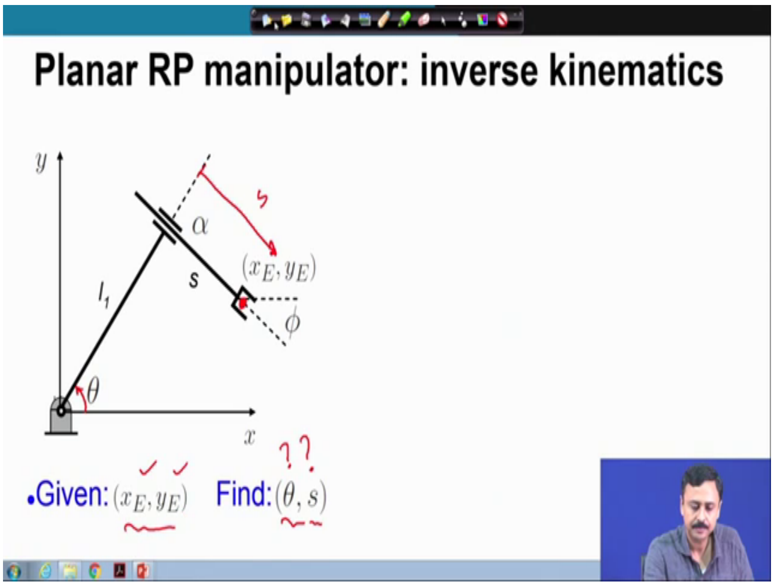
So, the angle will take care whether it is added or subtracted as you can see theta minus alpha will be something negative and sin of negative angle is negative. So, this ultimately gets subtracted from l 1 sin theta therefore, that gives us the y coordinate.

(Refer Slide Time: 16:30)



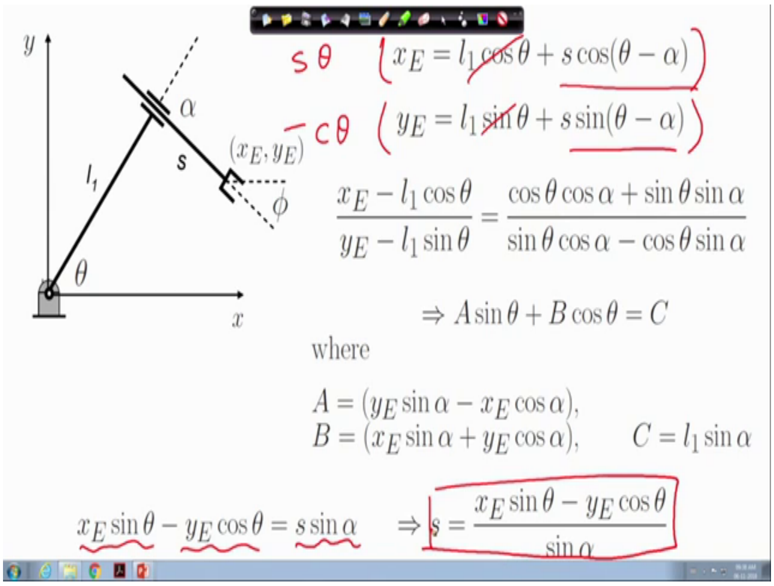
The end effector orientation coordinate which is phi is theta minus alpha.

(Refer Slide Time: 16:43)



So, once again the forward kinematics problem was straightforward let us look at the inverse kinematics problem now. In this we are given x E y E, which means we are given the location of the end effector we have to find out theta and s this angle and the throw of this prismatic actuator. Since we have 2 variables 2 degrees of freedom we can fix the location of the end effector.

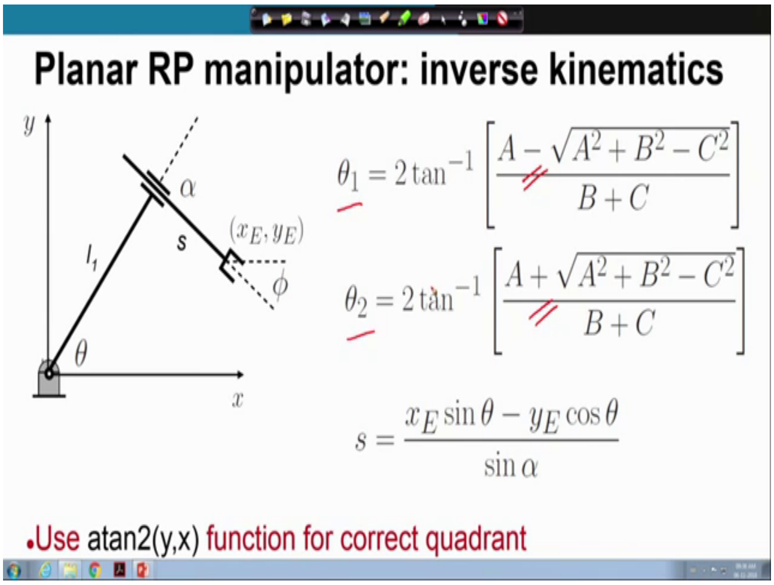
(Refer Slide Time: 17:31)



Let us again start with the forward kinematics relations which I have written out here x E in terms of theta and s. Here you must remember that this angle alpha is fixed what we have to determine a theta and s. What we do in the first step is eliminate s? So, to eliminate s we take this first term from the right hand side to the left hand side and divide. So, we come to this expression this can be simplified to obtain an equation of this form were A B and C are known since x E y E these are given to us therefore, B and C unknown we have to solve for theta.

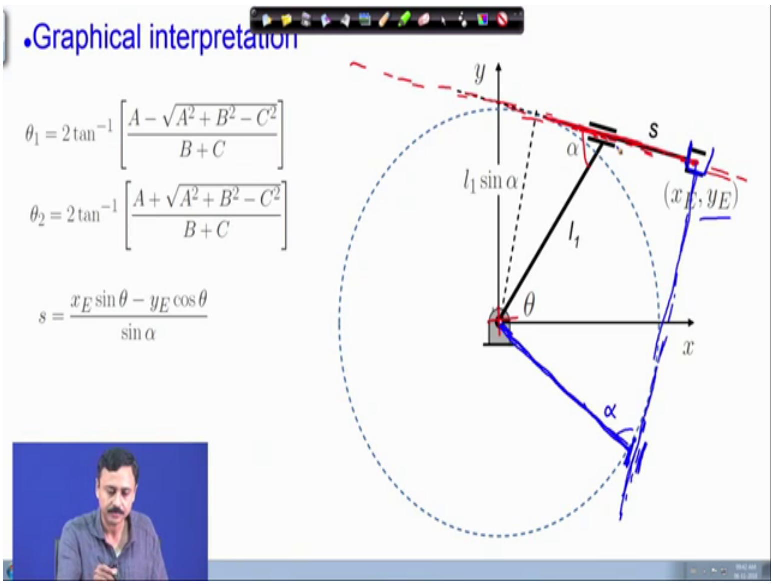
Once we solve for theta we need to find out s to do that we multiply the 1st equation with sin theta and the 2nd equation with cosine theta. And we subtract, if you subtract these two terms cancel off. And therefore, you are left with x E sin theta minus y E cosine theta. And this simplifies to s sin alpha; here everything is known except s therefore, we can solve s.

(Refer Slide Time: 19:57)



Let me directly show you the solutions. These are the 2 solutions that we obtained for theta 1 and theta 2, these are the 2 solutions of theta depending on the sign that we consider here. And, corresponding to the 2 values of theta we will get 2 values of s. Once again for the tangent inverse we need to use 8 and 2 for the correct quadrant.

(Refer Slide Time: 02:33)

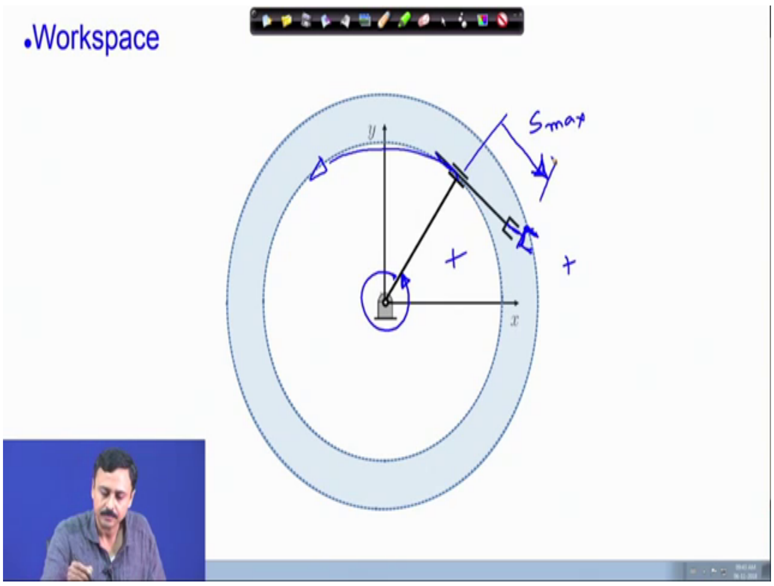


Let us try to understand the solutions the 2 solutions that we have obtained I have written out the solutions here in this inverse kinematics problem we were given this x E and y E this was known. We have to find out this angle theta and the throw of the actuator. Now, here I have drawn a circle which is tangent to the line of sliding of the prismatic actuator. This is the line of sliding of the prismatic actuator and this circle is the tangent circle centered at the origin. So, that base of the manipulator. Now, we draw this tangent and that must be the second link and the first link is obtained because we know this angle alpha.

So, these two values we know from the solution, the other solution that since we have 2 solutions as you can see the other solution is obtained by drawing a tangent to the circle from the same end effector point we have two tangents to the circle: one is shown in red, the other is shown in blue. In order to now locate the link l 1 we draw this line such that this angle is alpha. Now this second solution is unphysical as far as the physical construction of the manipulator is concerned because, we will note that the end effector has gone on the reverse side. The end effector has gone on the reverse side which is not possible to achieve in a physical manipulator. Though this is a mathematical solution the end effector reaches the desired point x E y E.

But it is physically not possible for a single manipulator to achieve these two solutions. So, this is a different manipulator altogether where the end effector is on the other side off of the prismatic pair. So, these are the two solutions that we obtained from the inverse kinematics problem.

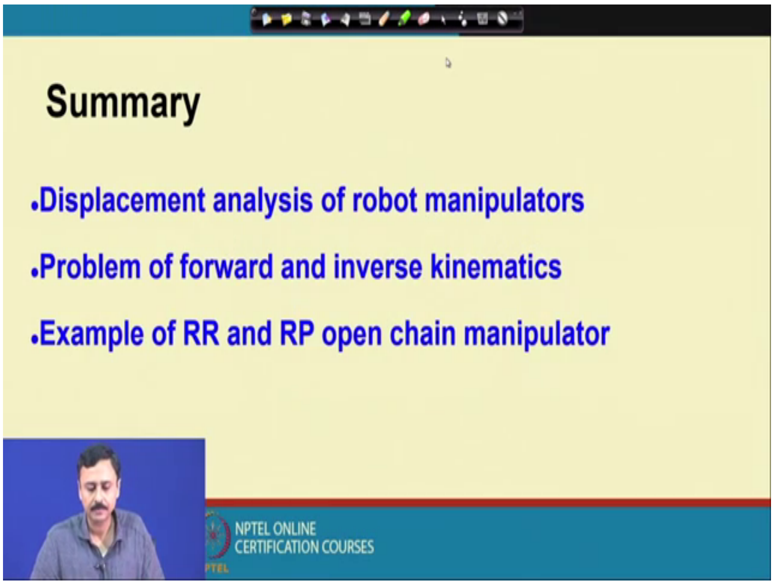
(Refer Slide Time: 24:00)



Now, the work space here as the joint one rotates 360 degrees is the theoretical workspace as it rotates 360 degree this point can move on this circle. And, the throw of this actuator will the maximum flow of this actuator will decide what is the reach of the manipulator, what is the outer boundary of the workspace.

So, this gives us the workspace of this manipulator. So, this blue shaded region is the workspace of the manipulator it cannot reach at any point here or any point here given that this is the s max.

(Refer Slide Time: 25:09)



So, that we summarize we have looked at the displacement analysis of cleaner robot manipulators we have looked at two examples the RR and RP manipulators discussed their forward and inverse kinematics. And, we have interpreted the solutions and looked at the workspace of the two manipulators with that, I will conclude this lecture.