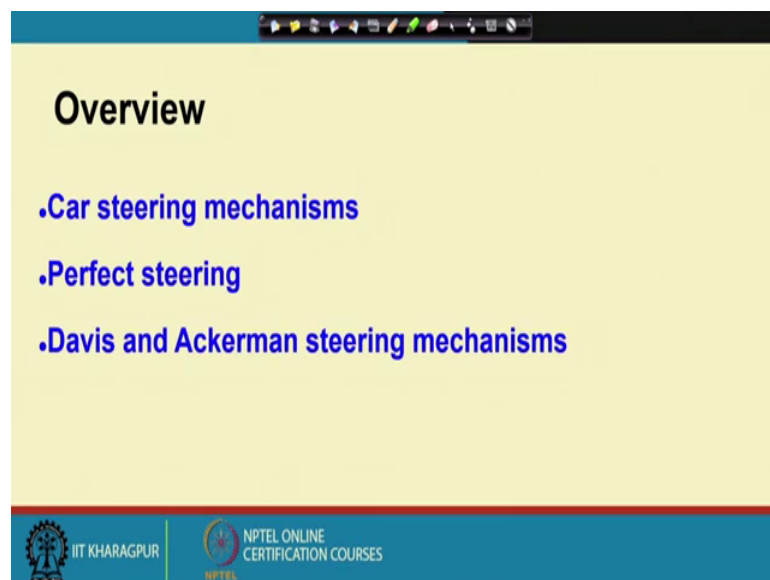


**Kinematics of Mechanisms and Machines**  
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**Lecture – 14**  
**Steering Mechanisms**

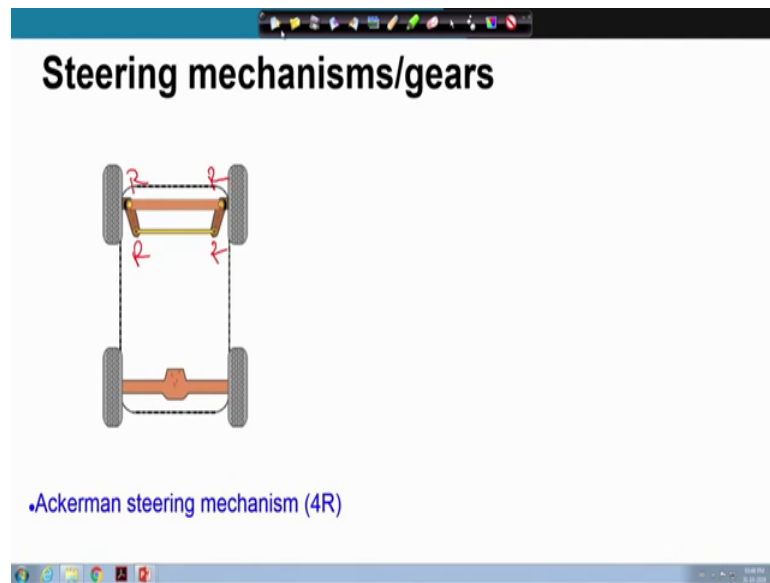
In this lecture, I am going to discuss about Steering Mechanisms. Steering mechanism is a very important mechanism for automobiles as you know. I am going to discuss two steering mechanisms and perform some analysis, and finally compare these mechanisms.

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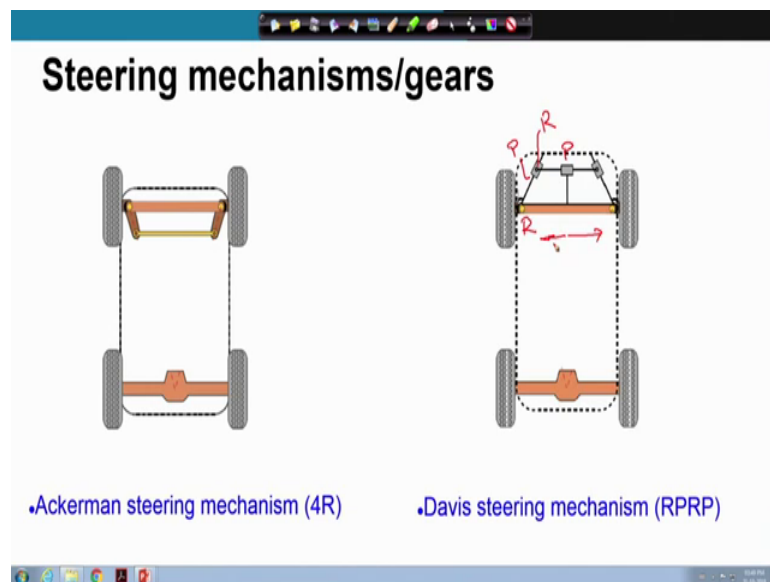
So, to give an overview of our discussions, we are going to first discuss what is a perfect steering in relation to the kind of mechanisms we are thinking of. We will essentially discuss two kinds of mechanisms, Davis and Ackerman steering mechanisms.

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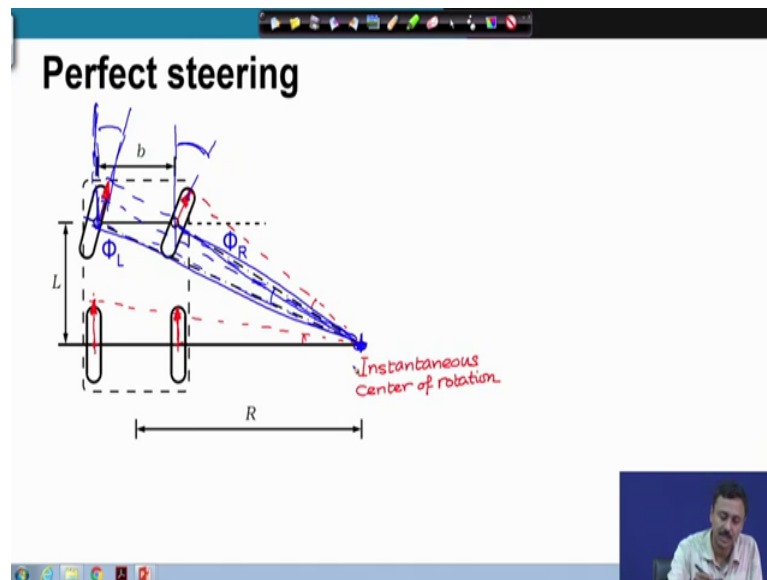
So, what is this Ackerman steering mechanism? You have seen it before, it is a 4R chain; so R, R, R, R, this is quite simple in construction as you can see.

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And we have the Davis steering mechanism, which uses two RPRP configurations. Two RPRP chains, here is R, then we have a P and then R, and here we have another P. And this is repeated symmetrically on the other side. So, both sides are symmetric. So, this is the Davis steering mechanism.

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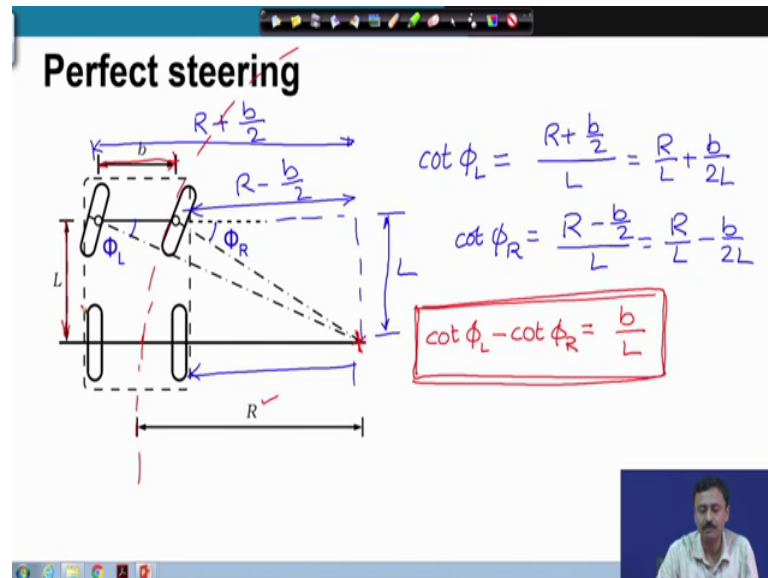
Let me first discuss, what is perfect steering? You know whenever rigid body is in motion in a plane, you can think of the motion as an instantaneous rotation about a point. If that be so, this is the plan or plan view of 4 wheeled vehicle, these are the rear wheels and the front wheels are steered. The velocity of this wheel suppose is this, in that case the velocity of this wheel gets fixed if you consider this as the instantaneous center of rotation.

We have more to discuss on instantaneous center of rotation in the future course. So, this is the velocity, then you know that velocity of the left rear wheel is this. And this wheel is this, this angle being the same. And therefore, the velocity of this wheel you have to maintain this angle. So, there must be a common point through which the centerline of all the four-wheels pass, which means that the angles these two wheels make with the forward direction, they will be different so that these centre lines pass through a single point that defines the instantaneous center of rotation. If it does not pass through a single point, then there is a chance of slippage. One slippage starts, then you lose stability of the vehicle and control of the vehicle.

Therefore, it is the purpose of the steering mechanism to see that the steerable wheels, the centre lines of the steerable wheels pass through a single point. We have to design the steering mechanism, so that so as to achieve this condition. So, this is known as the

perfect steering condition. Let us look at, what this perfect steering condition looks like mathematically.

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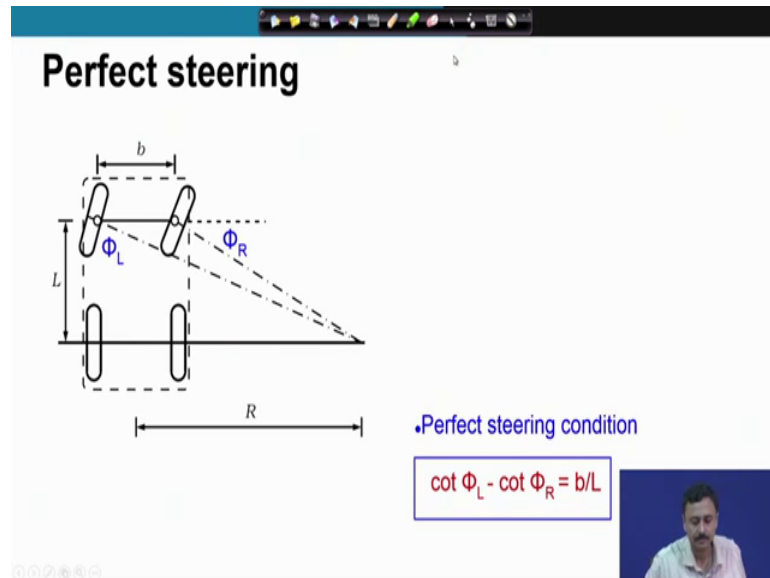
These two angles I have defined as phi R and phi L. I can write, cotangent of phi L is equal to this distance by this distance, this distance is L; now this distance is nothing but R plus b by 2. Therefore, cotangent of phi L will be R plus b by 2 over L. Similarly, cotangent of phi R is equal to R minus b by 2, this distance over L which is R over L minus b by 2 L.

Here of course, R is known as the radius of curvature. So, essentially it is as if that the car is rotating about this point, once I have this R is something, which has been introduced; I can eliminate R by subtracting the second equation from the first to get cotangent of phi L minus cotangent of phi R as b over L this relates the angle phi L and phi R. So, what are these angles phi and phi R, they are the steering angles of the wheels; the rotation of the wheels from their normal straight position.

So, the right wheel rotates by phi R and left wheel rotates by phi L. And the relation between phi L and phi R, so that we have an instantaneous center of rotation is this, cotangent of phi L minus cotangent of phi R is b over L. Now, b is the width of the axle of the two wheel attachment points. And L is the wheelbase, the length the distance between the front and the rear wheel axles. So, we have a relation between the functional

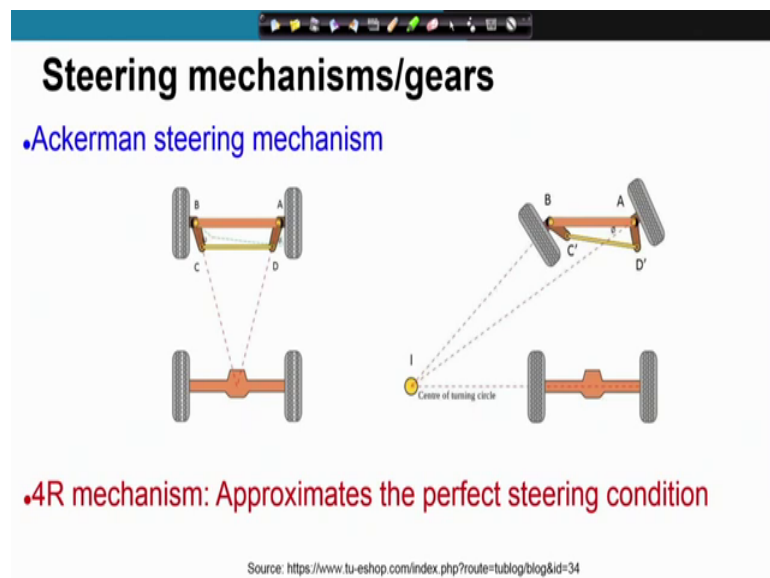
relation between  $\phi_L$  and  $\phi_R$  for perfect steering. So, this is called the perfect steering condition.

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So, this is the perfect steering condition.

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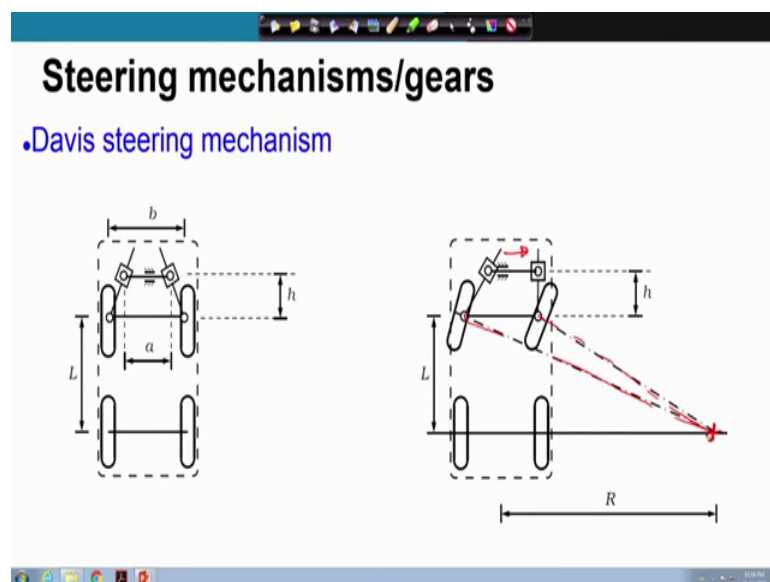


Now, let us look at the Ackerman steering mechanism. When the turn is made these two central lines, they must pass through a common point which lies on the axis of the rear wheels, these angles of rotation  $\phi_L$  and  $\phi_R$  they must satisfy that relation. Now, we have looked at this relation between  $\phi_L$  and  $\phi_R$  for a 4R mechanism.

And definitely it is very different from this condition, this is the perfect steering condition, but a 4R mechanism can never achieve this perfect steering condition for all configurations of steering; that means, all steering angles it cannot achieve this, it can only approximate at certain points it will exactly satisfy this relation, but not at all the points in the range of steering.

So, therefore this Ackerman steering mechanism will only approximate this perfect steering condition. Therefore, there will be small amount of there might be small amount of slip. So, this 4R Ackerman steering mechanism it only approximates the perfect steering condition, it can never achieve it exactly.

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Let us now discuss the Davis steering mechanism which uses this R P R P chain, when it is steering this is the configuration figure shown on the right. So, this bar has shifted to the right, and we expect that the center lines the axis of the steering steerable wheels, they meet at the center of rotation.

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### Steering mechanisms/gears

•Davis steering mechanism

•Straight drive condition

$$\cot \alpha_0 = \frac{1}{h} \left( \frac{b}{2} - \frac{a}{2} \right)$$

$$= \frac{b-a}{2h}$$

Now, let us analyze this Davis steering mechanism. In the straight drive configuration as I have shown here on the in the figure on the left, we have these lengths the length of this bar is  $a$ , and the distance between the two wheels is  $b$ . Therefore, in the straight drive condition we have this angle  $\alpha_0$  on both the sides. I can write cotangent of  $\alpha_0$  is equal to this distance divided by this distance; this distance as shown is  $h$ , now this distance is nothing but  $b$  by  $2$  minus  $a$  by  $2$ . Therefore, cotangent of  $\alpha_0$  is  $1$  over  $h$   $b$  by  $2$  minus  $a$  by  $2$ ; so that is the angle  $\alpha_0$ , which is the which is in the straight drive condition.

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### Steering mechanisms/gears

•Davis steering mechanism

•Straight drive condition

$$\cot \alpha_0 = (b-a)/(2h) = A$$

•Right turn

$$\cot \alpha_L = \frac{\frac{b}{2} - \left( \frac{a}{2} - x \right)}{h}$$

$$= \frac{b-a}{2h} + \frac{x}{h}$$

$$= A + \frac{x}{h}$$

$$\cot \alpha_R = A - \frac{x}{h}$$

$\phi_L = \alpha_0 - \alpha_L$ ,  $\phi_R = \alpha_R - \alpha_0$

Let us name this as  $A$ , because these are all constant  $b$ , is the constant  $a$ , is the constant  $h$  of the constant we call that as  $A$ . Now, let us condition look at this right turn situation, this bar has been pushed to the right. Now, we look at this angle, now these are different  $\alpha_L$  and  $\alpha_R$ ; initially they were  $\alpha_{naught}$  both were  $\alpha_{naught}$ , but now we have  $\alpha_L$  and  $\alpha_R$ . A cotangent of  $\alpha_L$  is this distance divided by this distance remains the same which is  $h$ .

Now, this distance is now this was  $b$  by  $2$  minus  $a$  by  $2$  minus  $x$ . So, this becomes  $b$  by  $2$  minus  $a$  by  $2$  minus, so this is I am considering that this shift is  $x$ , shift of this bar is  $x$  to the right. So, this distance was  $b$  by  $2$ , now this distance which is being subtracted is less than  $a$  by  $2$ . So, it is  $a$  by  $2$  minus  $x$  divided by  $h$ . So, this becomes  $b$  minus  $a$  by  $2$   $h$  plus  $x$  by  $h$ ; which is  $A$  plus  $x$  over  $h$ .

In a similar manner, if you look at the right side, then cotangent of  $\alpha_R$  is  $A$  minus  $x$  over  $h$ , this can be easily derived following the similar procedure. So, we have quote cotangent of  $\alpha_L$  and cotangent of  $\alpha_R$ . Now, what is the steering angle. So, this is the next thing that we are going to determine. This is  $\phi_L$  and this is  $\phi_R$ , as per our definition we have to determine  $\phi_L$  and  $\phi_R$ ; a relation between  $\phi_L$  and  $\phi_R$ .

Now, what is this  $\phi_L$ ? Remember that initially in the state drive condition this arm was oriented like this. Now, it has so it was  $\alpha_{naught}$ ; initially it was  $\alpha_{naught}$ , now it is  $\alpha_L$ . So, what is  $\phi_L$ ?  $\phi_L$  is nothing but  $\alpha_{naught}$  minus  $\alpha_L$ . So, this  $\phi_L$  is nothing but  $\alpha_{naught}$  minus  $\alpha_L$ .

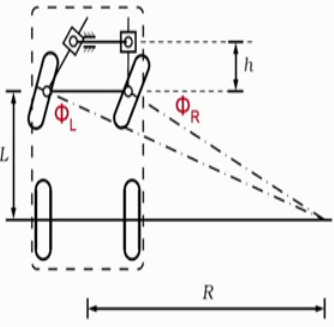
Similarly,  $\phi_R$  remember that initially it was oriented like this, now it has become  $\alpha_R$ , so  $\phi_R$  must be  $\alpha_R$  minus  $\alpha_{naught}$ ,  $\alpha_R$  minus  $\alpha_{naught}$ . So, this angle is  $\phi_R$ , and this angle is  $\phi_L$ . We have to determine cotangent of  $\phi_L$  and cotangent of  $\phi_R$ , which is cotangent of  $\alpha_{naught}$  minus  $\alpha_L$ , and cotangent of  $\alpha_R$  minus  $\alpha_{naught}$  and by expanding these we can very easily write the cotangent in terms of these relations that we have already derived.



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### Steering mechanisms/gears

•Davis steering mechanism



•Straight drive condition

$$\cot \alpha_0 = (b - a)/(2h) = A$$

•Right turn

$$\cot \alpha_L = A + (x/h)$$

$$\cot \alpha_R = A - (x/h)$$

•Steering angles

$$\cot \phi_L = \cot(\alpha_0 - \alpha_L)$$

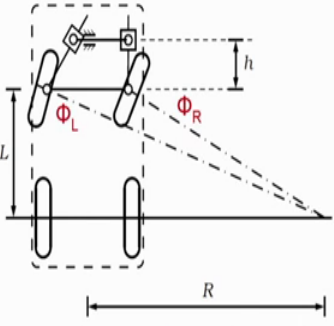
$$\cot \phi_R = \cot(\alpha_R - \alpha_0)$$

To see that, so these are the cotangent alpha L value and alpha R R value, you have to expand the cotangents. So, cotangent phi L will be cotangent of alpha naught minus alpha L. And similarly cotangent of phi R, will be cotangent of alpha R minus alpha naught. Now, using the trigonometric properties of cotangent alpha naught minus alpha L, which is 1 plus it will become 1 plus cotangent alpha naught, cotangent alpha L divided by cotangent alpha L minus cotangent alpha naught.

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### Steering mechanisms/gears

•Davis steering mechanism



•Straight drive condition

$$\cot \alpha_0 = (b - a)/(2h) = A$$

•Right turn

$$\cot \alpha_L = A + (x/h)$$

$$\cot \alpha_R = A - (x/h)$$

•Steering angles

$$\cot \phi_L = \cot(\alpha_0 - \alpha_L) = \frac{(h + A^2h + Ax)/x}{(h + A^2h - Ax)/x}$$

$$\cot \phi_R = \cot(\alpha_R - \alpha_0) = \frac{(h + A^2h - Ax)/x}{(h + A^2h + Ax)/x}$$

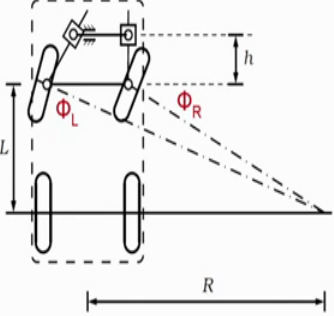
$$\cot \phi_L - \cot \phi_R = 2A = \frac{b - a}{h}$$

So, if you use these properties and finally use these expressions, then cotangent of phi L is obtained here and cotangent of phi R is here. Now, let us look at this expression of cotangent of phi L minus cotangent of phi R. If you look at the right hand side now, you will get equal to 2 A, other terms cancel off. And remember what is A, is obtained here, therefore, this must be equal to b minus a by h.

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**Steering mechanisms/gears**

•Davis steering mechanism



$$\cot \Phi_L - \cot \Phi_R = (b - a)/h$$

•Perfect steering condition

$$\cot \Phi_L - \cot \Phi_R = b/L$$

$$\frac{b - a}{h} = \frac{b}{L}$$

$$\frac{b}{h} - \frac{b}{L} = \frac{a}{h}$$

$$\Rightarrow a = b - \frac{bh}{L} = b \left(1 - \frac{h}{L}\right)$$

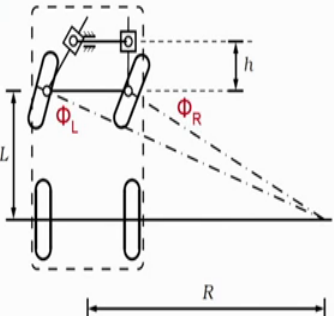
Now, recall that for the perfect steering condition. We must have cotangent theta phi L minus cotangent phi R is equal to b over L, which means that if I want to achieve perfect steering in the case of Davis steering wheel, I must have b minus a by h is equal to b over L. So, given the dimensions of the vehicle if I want to find out a, then I have b over h minus b over L must be equal to a over h, which means a must be equal to b minus b over L h.

This goes in the numerator, this becomes b 1 minus h over L, so that should be the length of the bar. Now, once this is done, once we set A as per this prescription, then we have perfect steering. Therefore, Davis steering mechanism can achieve perfect steering. You always have center of rotation through which the wheel centerline, the axis the wheel will pass.

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### Steering mechanisms/gears

- Davis steering mechanism



$\cot \Phi_L - \cot \Phi_R = (b - a)/h$

- Perfect steering condition

$\cot \Phi_L - \cot \Phi_R = b/L$

- Comparing

$a = b(1 - h/L)$


- Davis steering mechanism achieves perfect steering

So, this is our perfect steering condition for the Davis steering mechanism.

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### Comparison of steering mechanisms

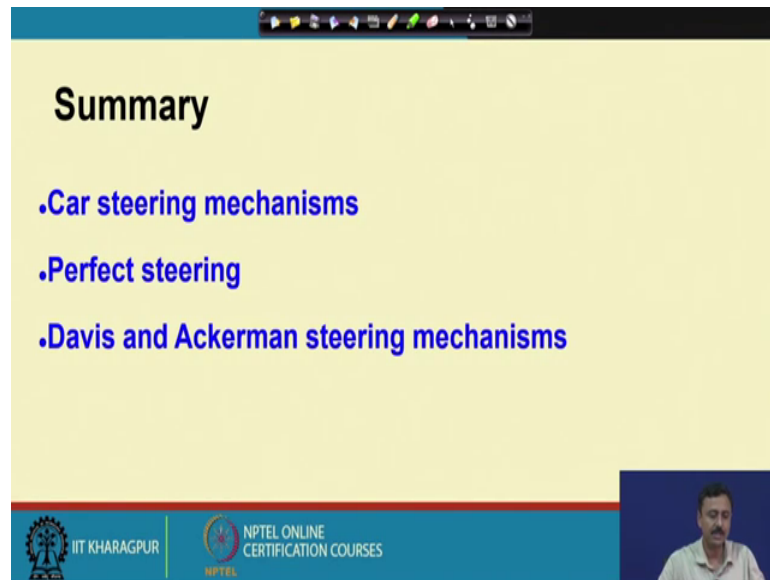
- Ackerman steering mechanism
- Simple construction and robust (uses R pairs)
- Imperfect steering
- Davis steering mechanism
- Perfect steering
- Uses P pairs (prone to wear)



Let us now compare these two steering mechanisms. So, in the Ackerman steering mechanism, the advantages it has got a simple construction. And it is very robust, you have only revolute pairs, which are frictionless. You can make them almost frictionless, it is very robust, and because it uses our pair the construction is very simple, but it has this disadvantage which we have seen before that it has imperfect steering, it does not achieve the perfect steering condition. So, the steering is not perfect.

On the other hand, the Davis steering mechanism the advantages, it has perfect steering, it achieves perfect steering, but it uses P pairs. Now, P pairs are prone to wear. And once they wear, it introduces free play and inaccuracies in steering.

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The image shows a presentation slide with a yellow background. At the top, there is a blue header bar with a navigation menu. The main title 'Summary' is in bold black font. Below it, there is a list of topics in blue text: '.Car steering mechanisms', '.Perfect steering', and '.Davis and Ackerman steering mechanisms'. At the bottom of the slide, there is a blue footer bar containing the IIT Kharagpur logo and the text 'NPTEL ONLINE CERTIFICATION COURSES'. In the bottom right corner, there is a small video inset showing a man with a beard and mustache, wearing a light blue shirt, speaking.

So, let me summarize. We have looked at two kinds of steering mechanisms for automobiles. We have looked we have started with we started with the perfect steering condition, and we have analyzed the Ackerman and the Davis steering mechanisms. We have found that the Ackerman mechanism while it is very simple in construction and robust; it does not achieve the perfect steering condition. Therefore, it has this little inaccuracy in the in the steering condition.

On the other hand, the Davis steering mechanism, it achieves perfect steering, but then it has this disadvantage of the P pairs, which is prone to wear which introduces inaccuracies and play. So, these are the two steering mechanisms, which are very you very much used in automobiles. Mostly it is Ackerman mechanism because of its simple construction. So, with that I will close this lecture.