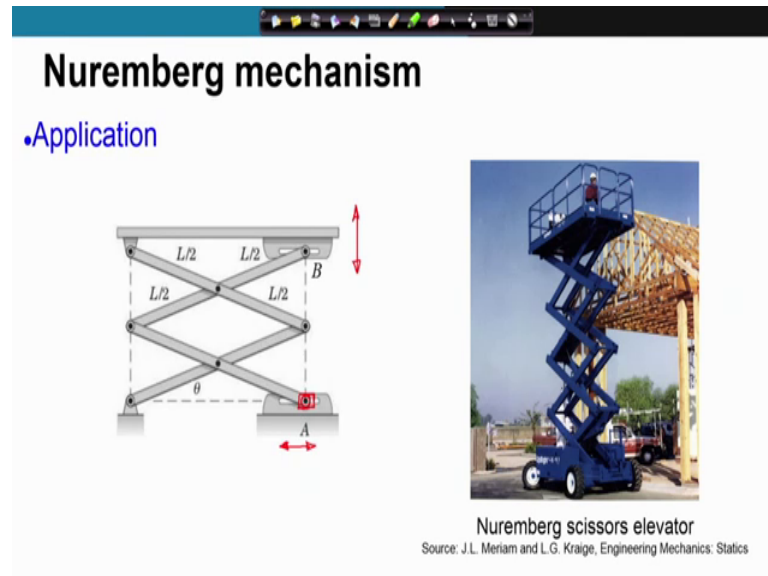


**Kinematics of Mechanisms and Machines**  
**Prof. Anirvan Dasgupta**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

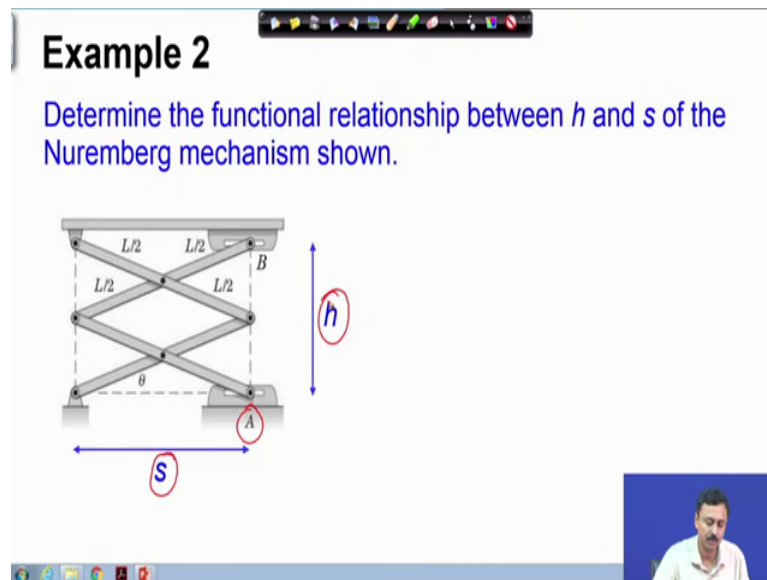
**Lecture – 13**  
**Displacement Analysis Example - II**

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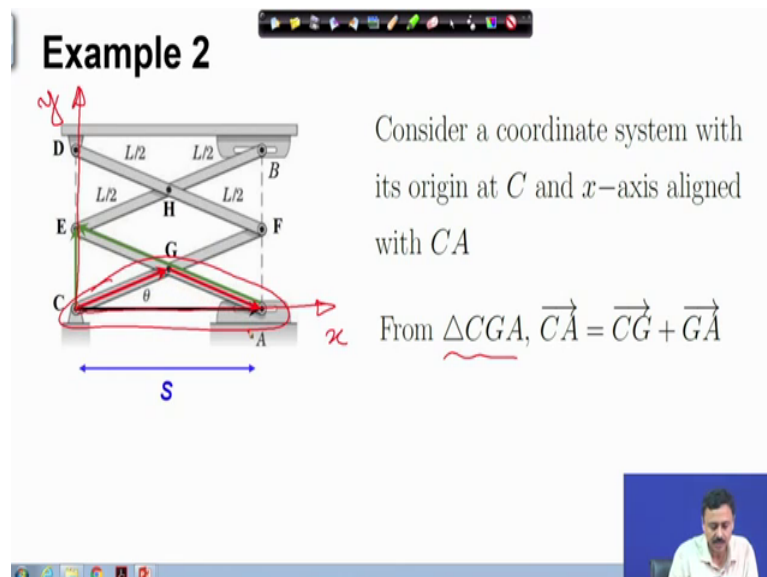
I am going to now discuss another example related to Displacement Analysis of Constraint Mechanisms. This is the Nuremberg mechanism; you must have seen it in many places. So, we have here a slider, which moves in this slot. And as it moves this platform moves vertically up or down. This mechanism finds a number of applications. One application is shown on the right, this called the Nuremberg scissors elevator. As you can see that this is a very compact mechanism, which can expand and can reach to different heights and this is adjustable.

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So, this example asks us to determine the functional relationship between  $h$ , which is the height of the platform, and  $s$  which captures the displacement of the slider pin be A. Essentially the location of A is captured by  $s$  and correspondingly  $h$  is the height of the platform; so, to determine a functional relationship between  $h$  and  $s$ .

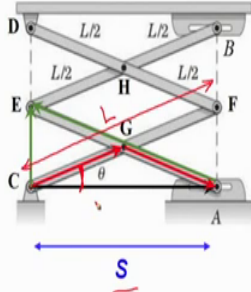
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We consider a coordinate system; this is  $x$   $y$  coordinate system. Now, from this triangle  $CGA$ , so this triangle; now,  $CA$  you can easily read out,  $CA$  is equal to the vector  $CG$  plus vector  $GA$ .

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### Example 2



Consider a coordinate system with its origin at  $C$  and  $x$ -axis aligned with  $CA$

From  $\triangle CGA$ ,  $\vec{CA} = \vec{CG} + \vec{GA}$

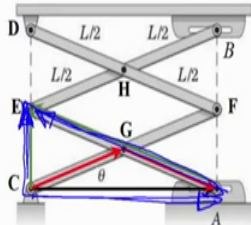
Using configuration variable  $\theta$

$$s = L \cos \theta$$

We use this variable theta this angle, therefore this  $s$  this displacement  $s$  is equal to  $L \cos \theta$ , where  $L$  is this length of this link. So,  $s$  equal to  $L \cos \theta$ .

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### Example 2



From  $\triangle CAE$ ,  $\vec{CE} = \vec{CA} + \vec{AE}$

$\vec{CA} = (s, 0)^T$

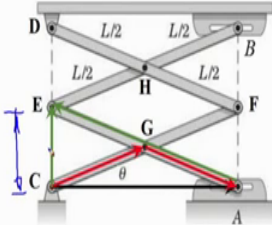
$\vec{AE} = \{L \cos(\pi - \theta), L \sin \theta\}^T$

$\vec{CA} = s \hat{i}$

From triangle  $CAE$ , this is this triangle  $CE$  this vector must be equal to the vector  $CA$  plus the vector  $AE$ . Now,  $CA$  the vector  $CA$  is nothing but  $s \hat{i}$  in the coordinate system, we have chosen. And  $AE$  this vector can be represented in this form, therefore substituting these two here.

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### Example 2



From  $\triangle CAE$ ,  $\vec{CE} = \vec{CA} + \vec{AE}$

$$\vec{CA} = (s, 0)^T,$$

$$\vec{AE} = \{L \cos(\pi - \theta), L \sin \theta\}^T$$

Therefore,  $\vec{CE} = (s - L \cos \theta, L \sin \theta)^T$

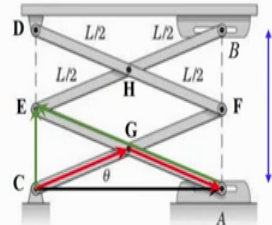
Or,  $\vec{CE} = (0, L \sin \theta)^T$

(Using  $s = L \cos \theta$ )

We get the CE vector, but because s is already L cosine theta, which we have already seen. Therefore, CE is nothing but so this is L sin theta definitely.

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### Example 2



$\vec{CD} = (0, 2L \sin \theta)^T$

Therefore, height  $h = 2L \sin \theta$

Also,

$$s = L \cos \theta$$

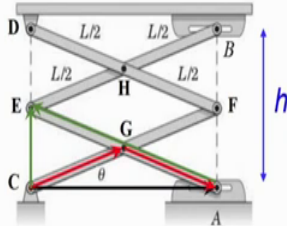
$$\left(\frac{h}{2L}\right)^2 + \left(\frac{s}{L}\right)^2 = 1$$

Now, we have two stages in this mechanism this and this, both are L sin theta. Therefore, the total height is 2L sin theta, and we have s equal to L cosine theta. We have two relations h as 2L sin theta, and s as L cosine theta. If we eliminate theta between the two, which is done this way h by 2L is sin theta, so sin the square theta plus s over L is cosine

theta, so that square is equal to 1. Therefore, the relation between h and s is given by this equation, which is an equation of an ellipse.

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### Example 2



Therefore, height  $h = 2L \sin \theta$

Also,

$$s = L \cos \theta$$

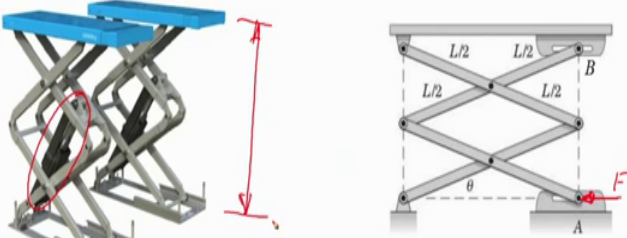
Eliminating  $\theta$

$$\frac{h^2}{4L^2} + \frac{s^2}{L^2} = 1$$

So, that solves our problem.

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### Alternate actuation

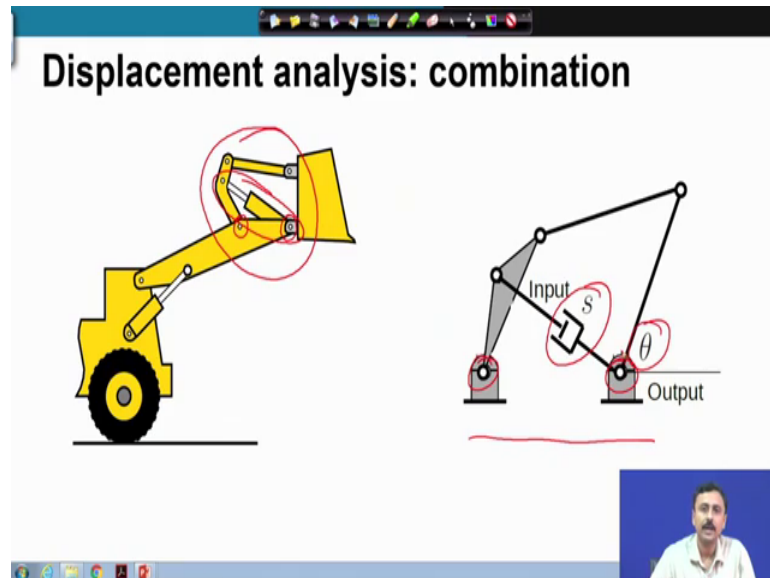


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There can be alternate actuation mechanism as shown here, rather than applying force here. This is one possibility, you apply a force here and shift pin A. There are other possibilities, you can have prismatic actuators arranged like this here also from

geometry, you can determine the relation between the actuator throw and the height of the platform.

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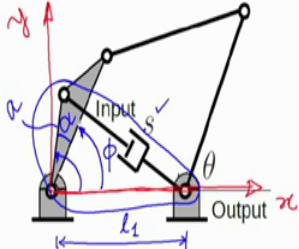
Let us look at another example. This I will discuss given idea of how to solve the displacement analysis problem for more complicated mechanisms. Here I have considered this part of the mechanism, which is represented here. I have made these two hinges as the ground hinge of this mechanism, there are two ground hinges.

s this throw of this actuator, which is represented by this prismatic actuator gives us the input, and the bin angle represented by theta is the output. Suppose, I want to find out theta given s, which is the forward kinematics problem all I can ask the reverse question given theta what should be the throw of the actuator.

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### Displacement analysis: combination

•Overview


$$s^2 = l_1^2 + a^2 - 2l_1 a \cos(\alpha + \phi)$$

The diagram shows a slider-crank mechanism. A crank of length  $l_1$  is pivoted at the origin of a coordinate system  $(x, y)$ . The crank makes an angle  $\alpha$  with the positive  $x$ -axis. A connecting rod of length  $a$  is attached to the end of the crank and makes an angle  $\phi$  with the crank. The other end of the connecting rod is attached to a slider of length  $s$  that moves horizontally along the  $x$ -axis. The output angle  $\theta$  is indicated. The input  $s$  is the horizontal displacement of the slider.

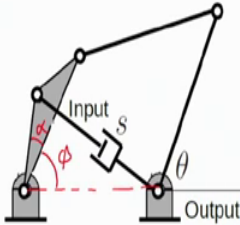
Let us look at these problems. Let me put this coordinate system. If you look at this 3 r 1 p chain, there are two chains as you can very easily recognize. There is a 3 r 1 p chain and a 4 r chain, which are coupled. If you look at this 3 r 1 p chain, if you are given  $s$ , then you can find out this angle.

Let me call this angle, which is a fixed angle as  $\alpha$ . And this angle let us say, I call it  $\phi$ . This length let me call it  $a$ , then given  $a$ ,  $\alpha$  and  $s$ . And of course, this length  $l_1$ , I will be able to find out the angle  $\alpha + \phi$  using the cosine rule  $s^2$  is equal to  $l_1^2 + a^2 - 2l_1 a \cos(\alpha + \phi)$ . Now, in this equation I know  $s$ ,  $l_1$ ,  $a$ , and  $\alpha$ , what is unknown is  $\phi$ ?

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### Displacement analysis: combination

•Overview



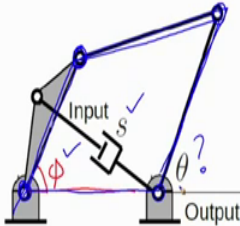
$\phi + \alpha = \theta_2$

Therefore, we can solve this to find out phi. Once, I know phi, then I identify the angle phi plus alpha as let us say theta 2, it is only phi.

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### Displacement analysis: combination

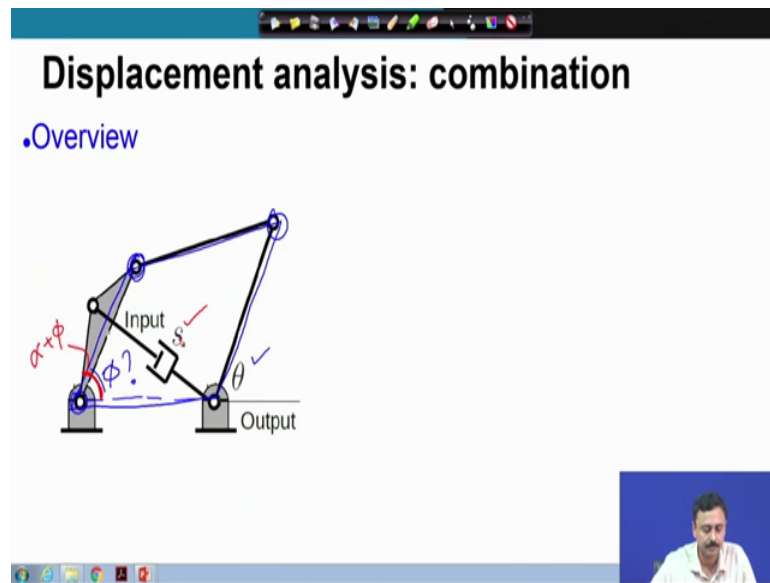
•Overview



If this is phi, then we can look at this chain this 4 r chain. Look at this 4 r chain, where phi is known. We have to find out theta, which is the problem which we have already solved. Therefore, using two steps you can find out theta given s. So, first we find out phi, and from there we find out theta.

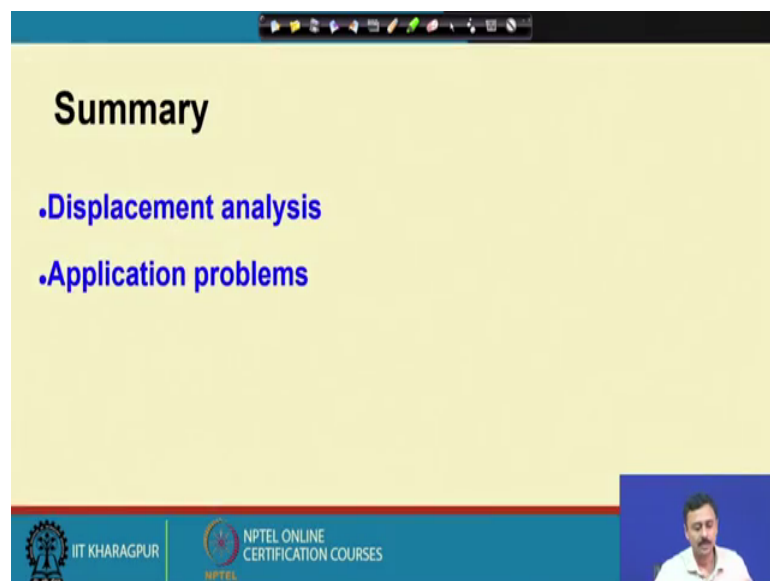


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If you ask the reverse problem, which means if we are given  $\theta$ , then we just move in the reverse direction, we find out  $\phi$ , which we have already discussed given the  $\theta$ . Once we know  $\theta$ , we can find out  $\phi$  from the 4-bar chain. Once we know  $\phi$ , since  $\alpha$  is known to me, so this total angle is known. This angle is  $\alpha + \phi$  and using the cosine rule I can find out  $s$ . So, given  $\theta$  we can find out  $s$  just by the reverse procedure. So, this is just an overview, the detailed calculations can be very easily be performed.

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With that I will close this lecture.