Kinematics of Mechanisms and Machines
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Lecture – 12
Displacement Analysis Example - I

In today's lecture, I am going to discuss some problems related to Displacement Analysis. We have been discussing about displacement analysis of constraint mechanisms. I will look at I will discuss two problems or three problems related to displacement analysis over next two lectures.

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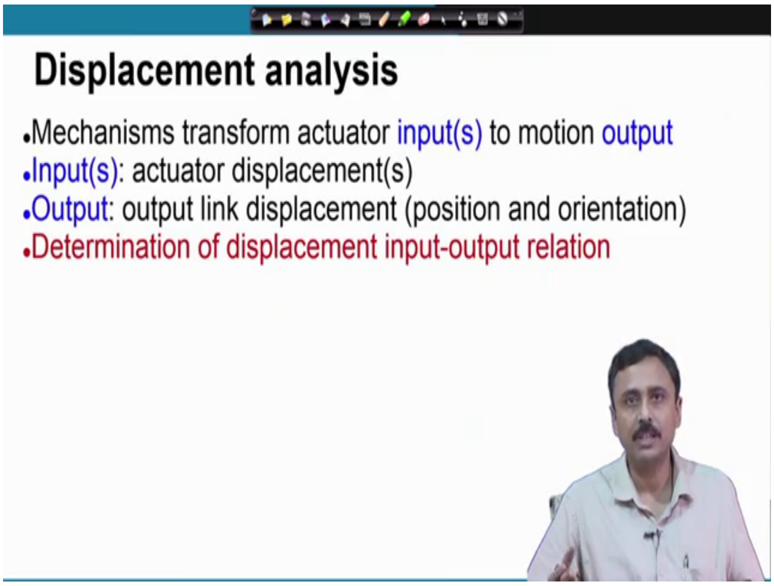

As you know displacement analysis problem is the determination of the displacement input-output relation.

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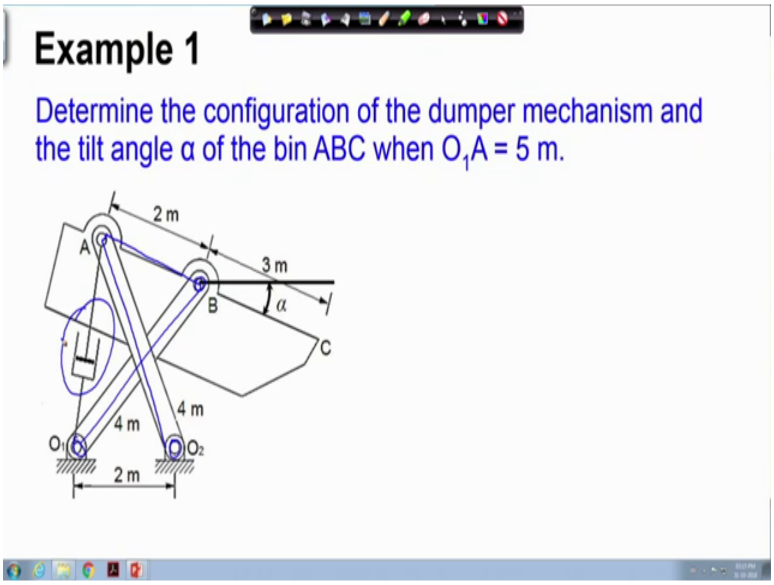

Let us look at this problem of a bin, which is driven by a mechanism a four r mechanism. So, this is a crossed four r mechanism as you can see which is actuated by this prismatic actuator. Now, the problem is to determine the configuration of the dumper mechanism and the tilt angle alpha of the bin ABC when O 1 A is 5 meter so, O 1 A is this length.

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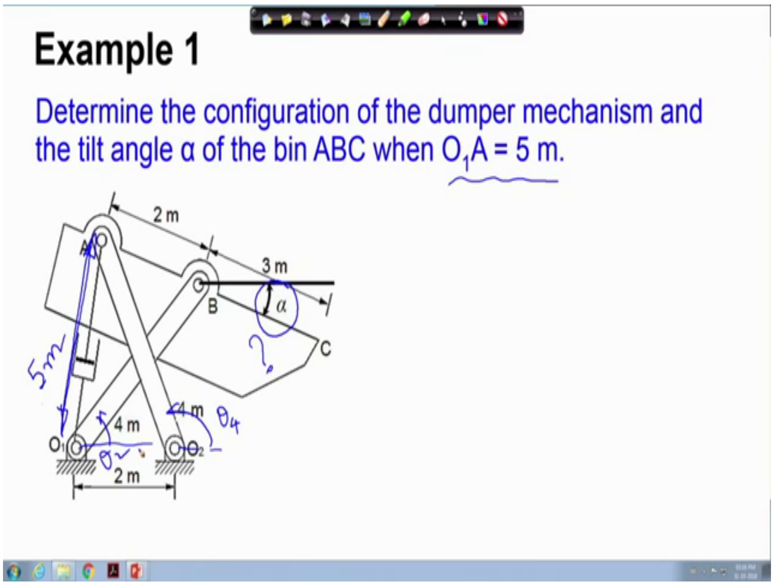

So, this length when it is 5 meter, we have to determine the configuration of the mechanism and this tilt angle alpha. Essentially therefore, the configuration the mechanism means finding out let us say this angle call it theta 4, this angle call it theta 2 and alpha.

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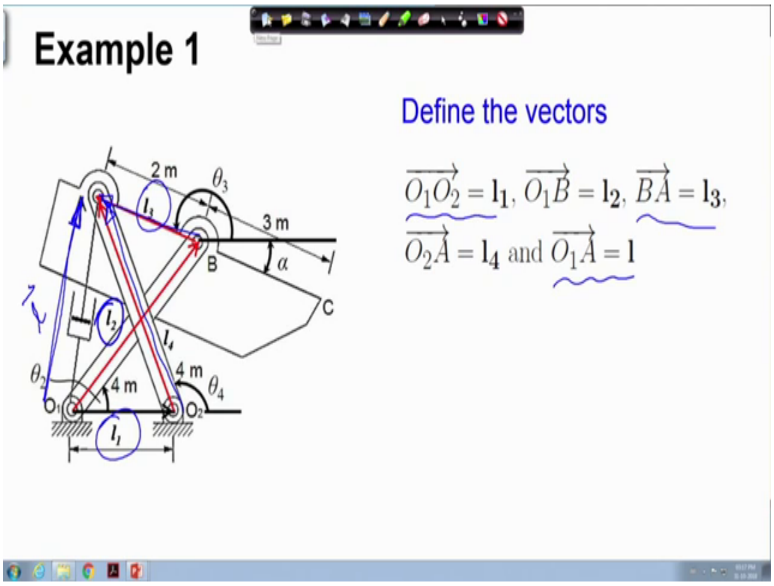

To start the problem, we first define these vectors. The first vector O 1 O 2 is this which is l 1 vector. O 1 B is this which is the l 2 vector, and B A, which is this vector l 3. You can note the directions the way the vectors have been considered. And O 2 A as this vector finally, O 1 A which is this vector. So, this is the vector O 1 A, which is denoted as the l vector. In order to represent these vectors, we will require a coordinate system. Let me draw that coordinate system.

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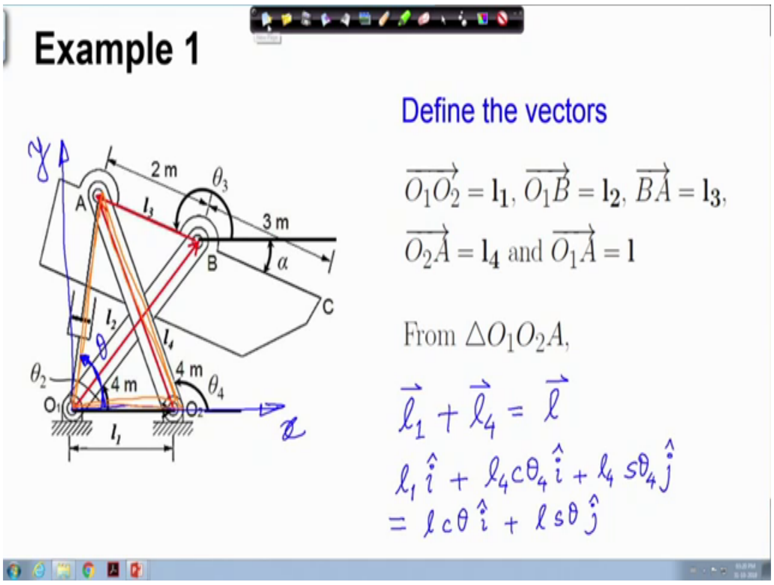

So, we will consider this as x and this as the y axis. Using these this coordinate system, we will represent all these vectors. If you look at this triangle O 1 O 2 A, which is this triangle O 1 O 2 A from here I can write vector l 1 plus vector l 4 is equal to vector l. Now, using the coordinate system, we can now represent the vectors l 1 is nothing but l 1 i cap it is along the x axis. l 4 is l 4 cosine theta 4 i cap plus l 4 sin theta 4 j cap and this must be equal to the vector l. This vector l, I will write as l cosine theta i cap plus l sin theta j cap, where theta is this angle, this angle I call theta.

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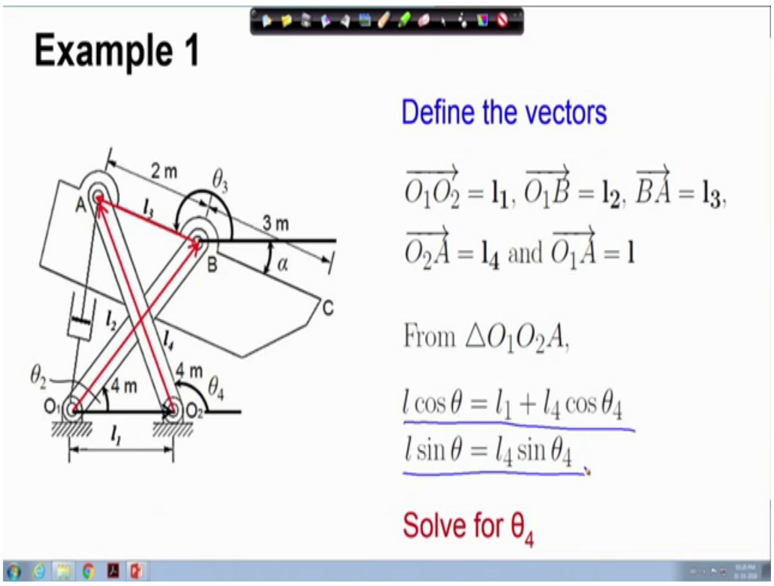

Now, if you compare the terms on both sides, then you can very easily see that l cosine theta must be equal to l 1 plus l 4 cosine theta 4 and l sin theta must be equal to l 4 sin theta 4. If I eliminate theta, then I can solve for theta 4, which I will show you here.

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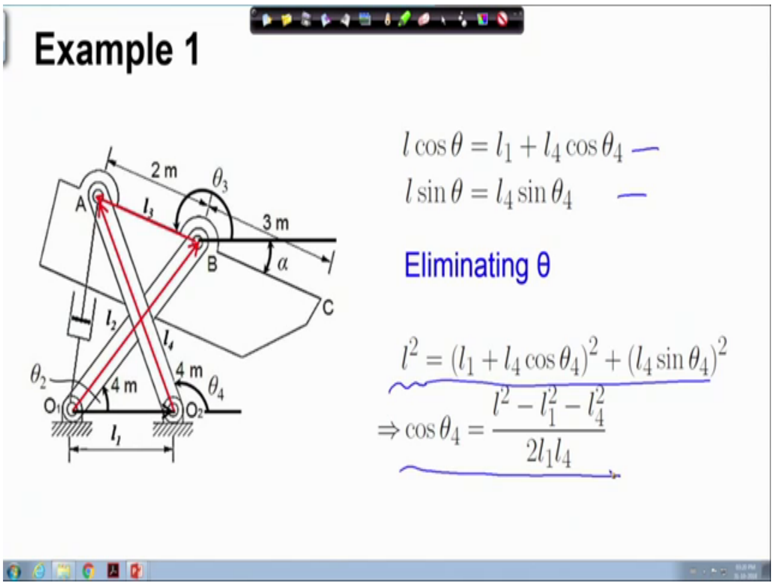

So, essentially we square and add these two relations and get l square. Now, this equation has only theta for as unknown. Therefore, I can solve theta for using this equation. Now, there will be two solutions as you know.

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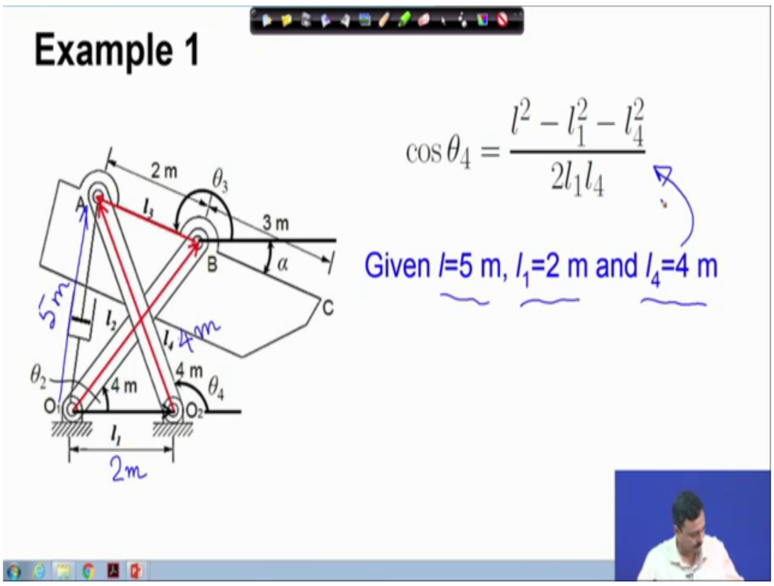

Given the parameters of the problem that l is equal to 5 meter, this is l remember, so this is 5 meter, l 1 is 2 meter and l 4 is 4 meter.

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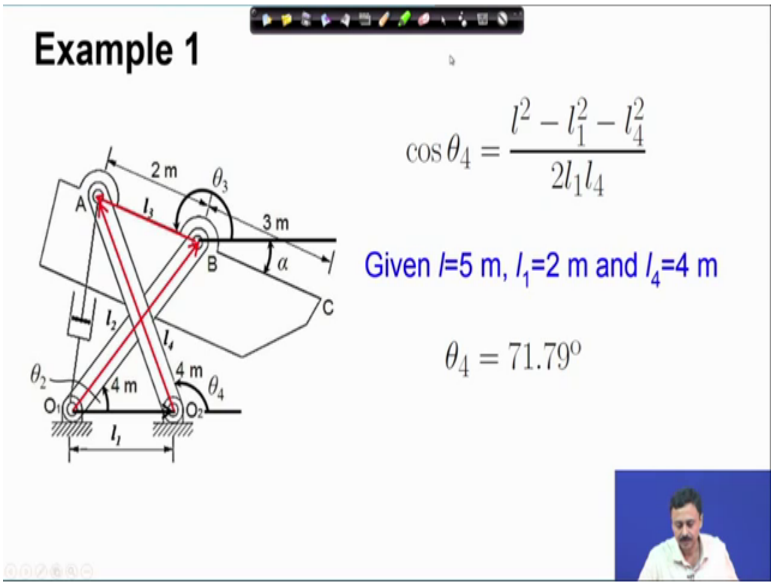

If you substitute in this you will get theta 4 as 71.79 degree. Now, this is the solution, which is relevant to the configuration.

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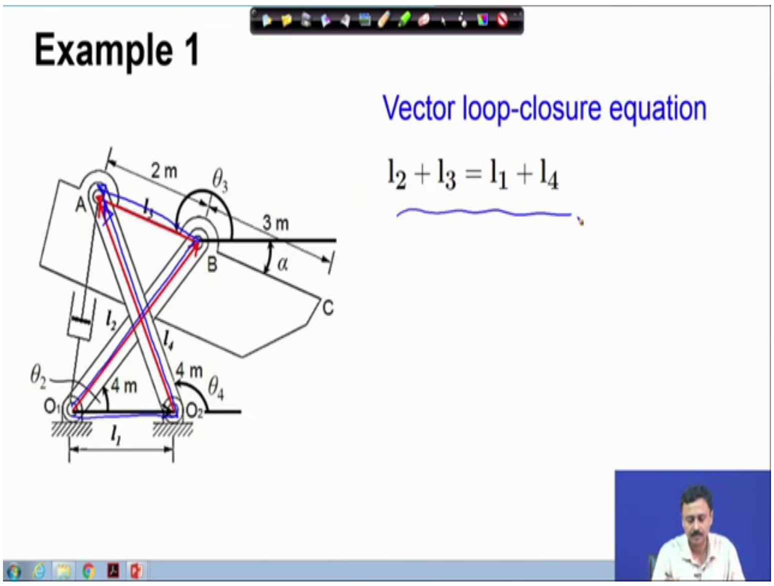

Next, we look at this vector loop equation l 2 which is this plus l 3 which is this must be equal to l 1 plus l 4. So, this vector loop equation.

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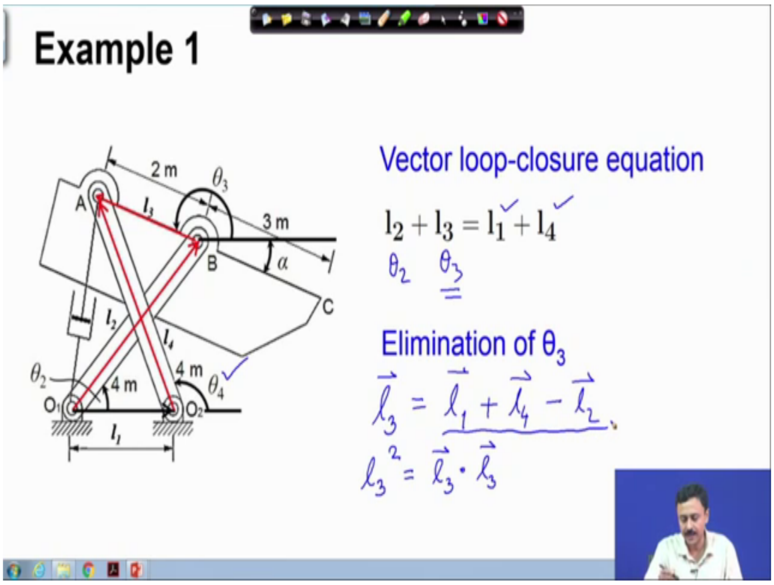

Here we have just now solved theta 4, we have solved theta 4. So, we know theta 4 here which means that we in completely know l 4, l 1 is completely known. Now, l 2 will involve theta 2 and l 3 will involve theta 3. If I eliminate theta 3, which I do as follows, I will write l 3 vector is equal to l 1 vector plus l 4 vector minus l 2 vector and take the dot product with itself, then theta 3 gets eliminated and what I get from this side is this equation.

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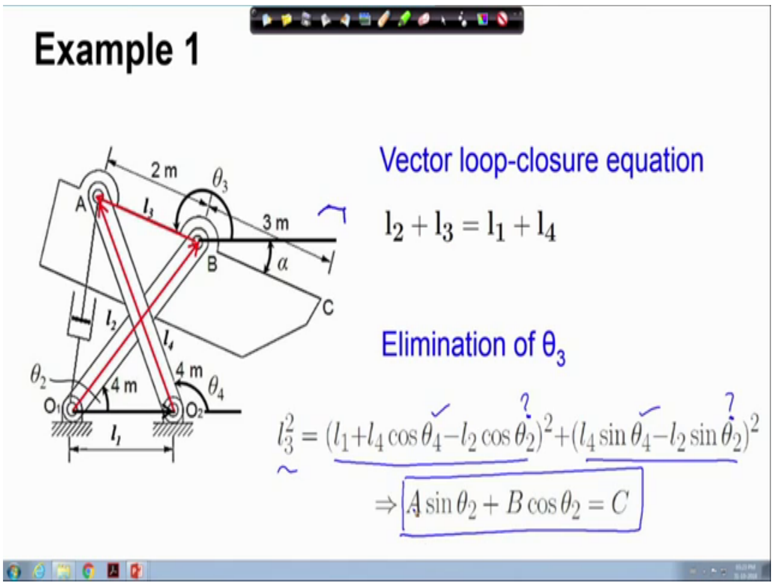

So, l 3 square is this quantity square plus this quantity square. Remember, we already know theta 4, we have to find out theta; theta 2. Therefore, we recast this equation in this form where A, B and C are defined here which involves theta 4, which is already known to us. And it also involves the other link lengths. Therefore we should be able to solve this equation, which we have solved before.

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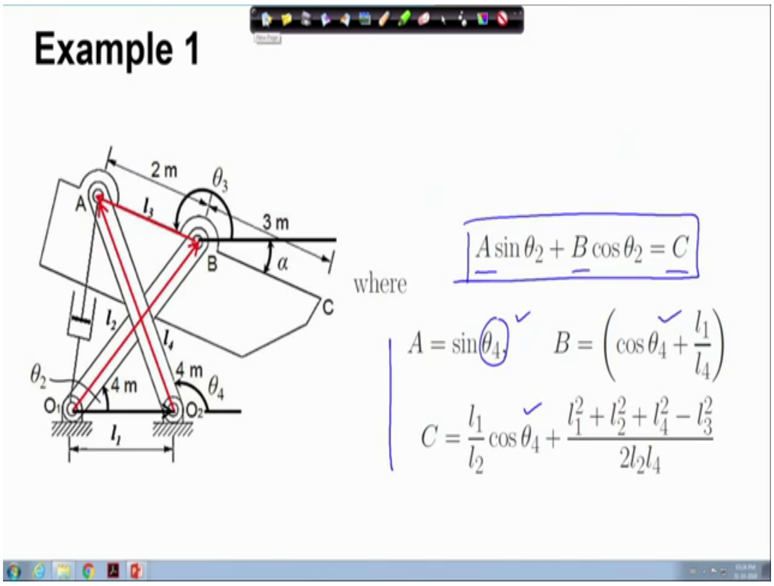

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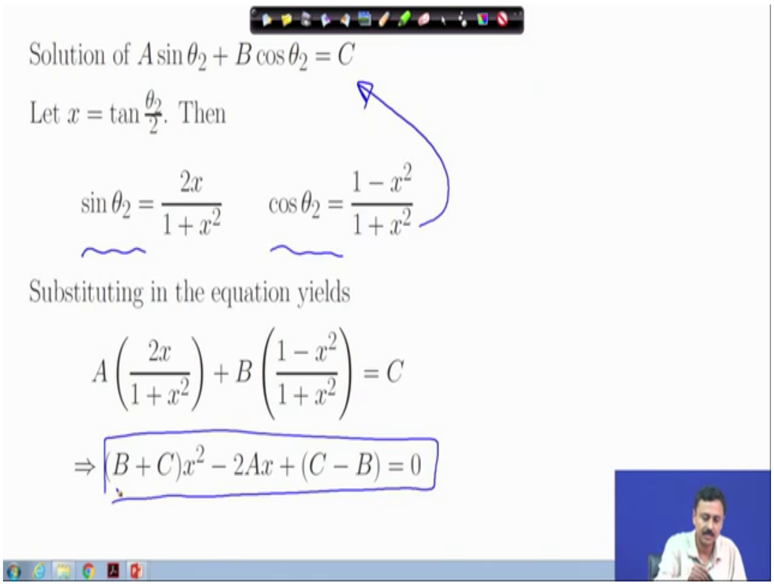

By making the substitution x equal to tan theta 2 by 2 representing sin theta 2 and cosine theta 2 in terms of x substituting in this master equation, finally getting a quadratic equation in x two solution is known to us.

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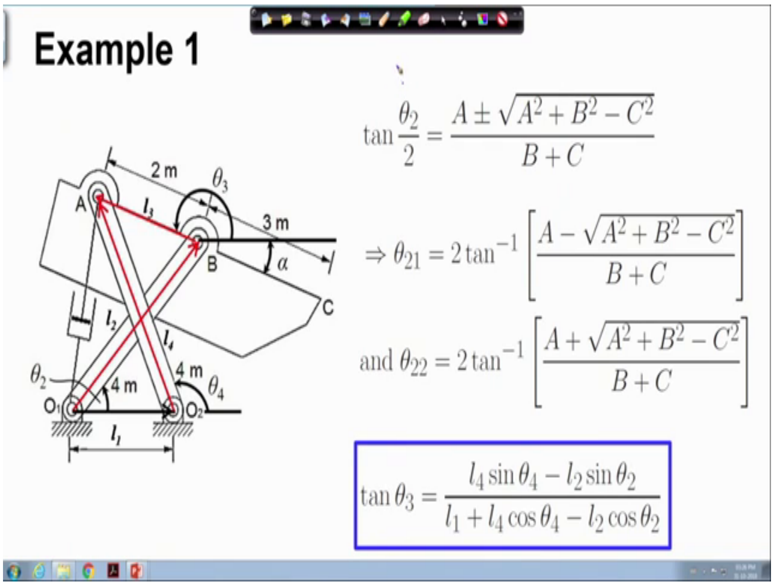

There are two solutions depending on the sign plus or minus. Let us get back to our mechanism now. So, we have these two solutions of theta 2, which are given by theta 21 and theta 22, from these expressions we can find them out. Therefore, given these link lengths and theta 4, we can now find out theta 2.

Once, we have theta 2 then we can find out theta 3 which we have discussed before. So, tangent of theta 3 is given by this expression, since we have solved for theta 4 and theta 2, we know the right hand side. And therefore, we can solve for theta 3. Always remember you have to use the a tan 2 function to determine the correct quadrant of theta 3.

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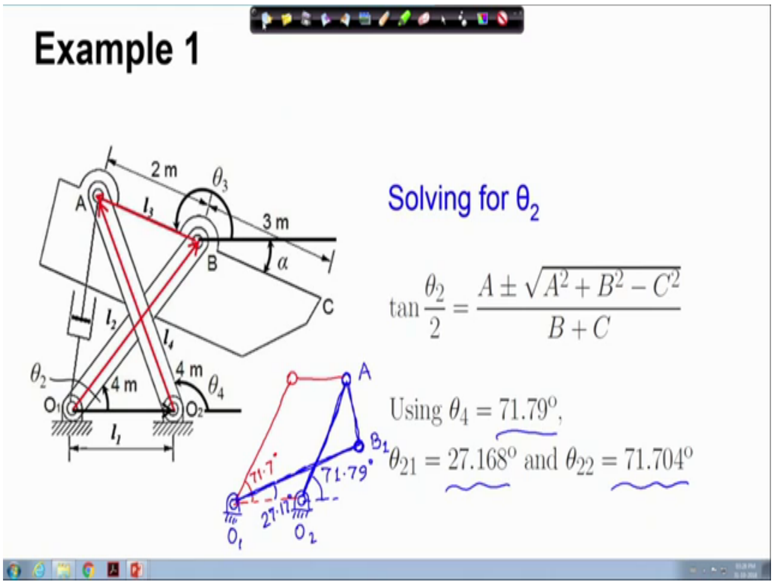

So, when we solve for theta 2 for the given data, we already have theta 4 as 71.79 degree, we get these two solutions. So, what we have let us look at the configuration. This is O 1 this is O 2, we have theta 4 as 71.79 degree, which is this angle. Therefore, this point this hinge is A. Now, we have two solutions for theta 2, one solution would look something like this, the other solution is essentially this.

So, this solution angle is 71.7 and this solution is 27.17 degree of we definitely know that we are started off with a cross configuration. Therefore, this is the configuration, the blue one is the configuration that we expect. The next thing that remains is to find out the configuration of bin or the inclination of the bin. Therefore, we choose the blue configuration for the present bin.

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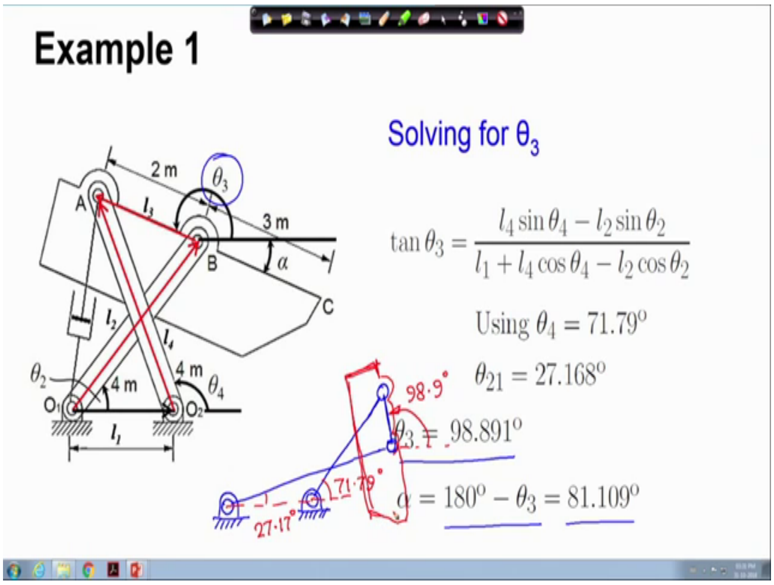

So, we use these two values of theta 4 and theta 2. And finally, we get theta 3 as 98.89 degree and alpha is nothing but pi minus theta 3. So, this is theta 3 which is which in the current configuration is 98.89 degree. And therefore, alpha which is pi minus theta 3 is 81.1 degree. So, let me once again draw out this configuration. This angle is 71.79 degree, this angle is 27.17 degree and this angle which is theta 3 is 98.9 degree. So, this is the configuration of the mechanism so, this is the bin.

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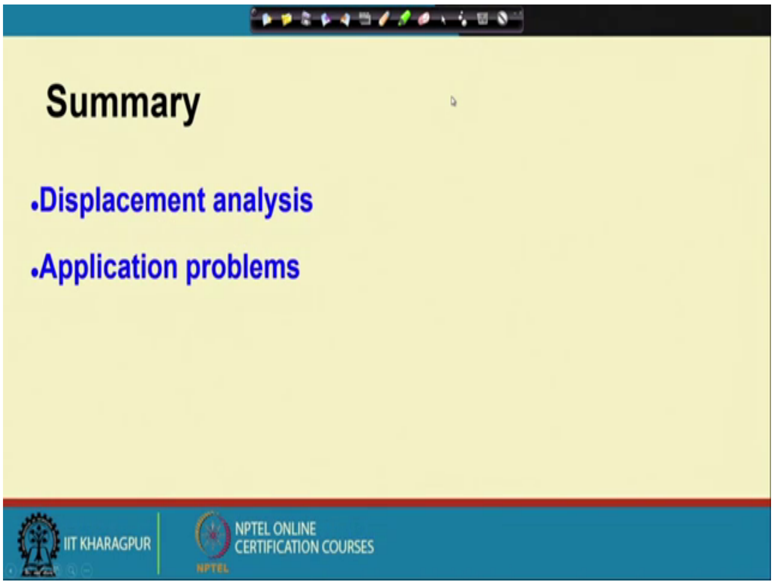

So, we have discussed an example of a four r mechanism of bin, which can be tilted using a prismatic actuator. So, with that I will conclude this example lecture.