Kinematics of Mechanisms and Machines
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Lecture - 11
Displacement Analysis – II

In this lecture, I am going to continue our discussion on Displacement Analysis.

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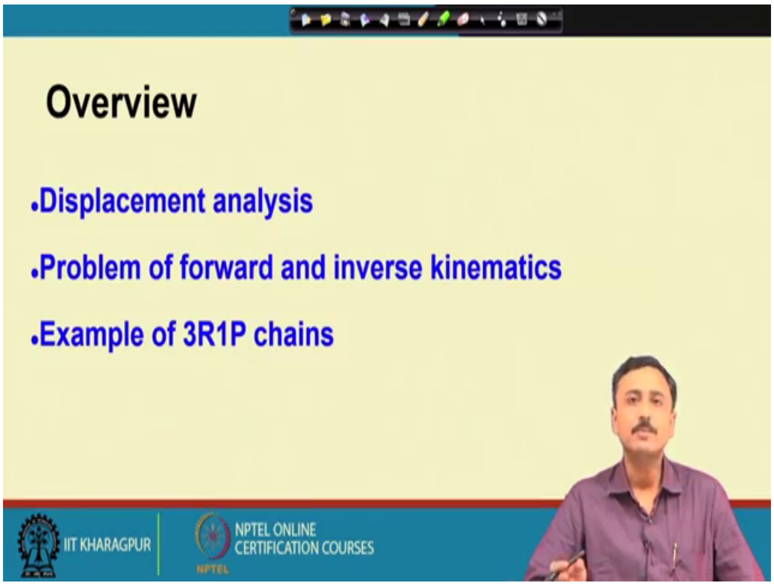

So, the overview is the Displacement Analysis problem, we will talk about the Forward and Inverse kinematics of 3R1P chains. We will consider 2 types of 3R1P chains which I will show you in a moment.

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As per our plan we are discussing the Constrained Mechanisms. We will subsequently discuss about open chain robot manipulators, and then we will go to closed chain robots.

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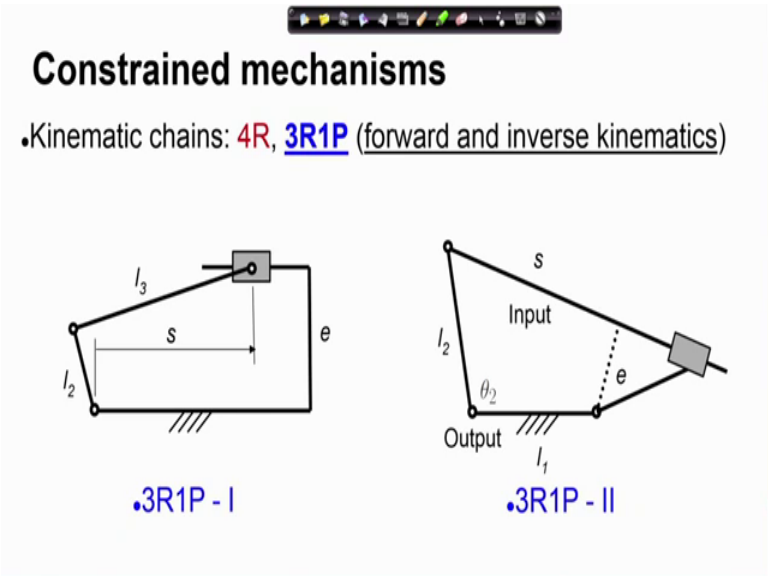

So, in this Constrained Mechanisms, today we are going to look at 3R1P chains. So, there are two 3R1P chains as shown on the left I have 3R1P chain 1 and on the right, I have 3R1P chain 2; they are essentially inversions. But there are Displacement Analysis, they differ in certain ways.

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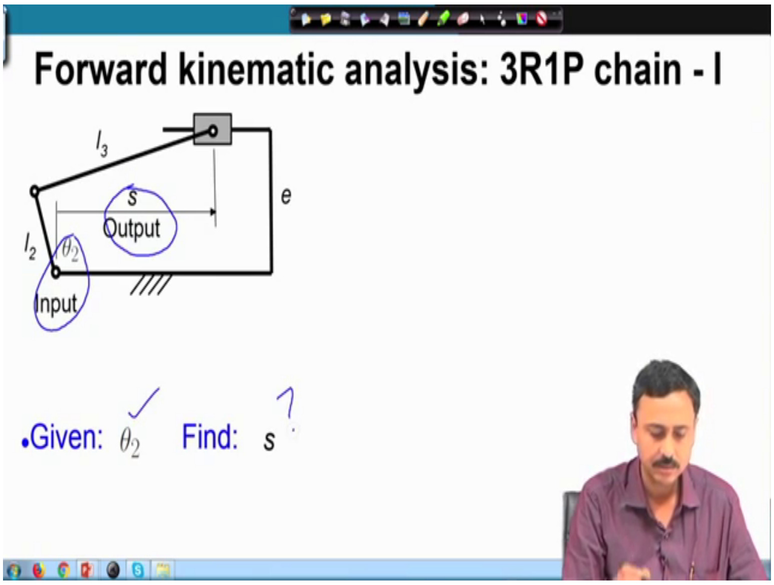

Let us start with the Forward kinematics analysis of this 3R1P chain type 1. So, in forward kinematics, as I have mentioned before that we are given the input we have to find out the position of the output link? Here, the input as shown is theta 2 and the output is s input is theta 2 and output is s: s is the displacement of the slider from a certain coordinate in a certain expressed a certain coordinate system.

So, in this problem we are given this theta 2 and we have to find out s.

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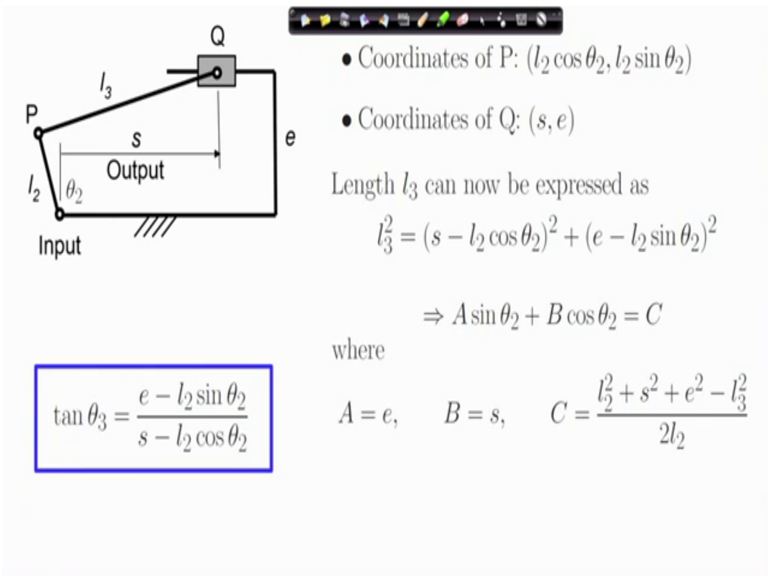

So, as we have discussed before, we consider coordinate system theta 2 is the angle made by the link l 2 with the x axis. Therefore, you can easily read the coordinates of point P which is this one as l 2 cosine theta 2 and l 2 sin theta 2. So, P x is l 2 cosine theta 2 and P y is l 2 sine theta 2. The coordinates of Q which is the slider; the x coordinate is e, the x coordinate is s and the y coordinate is e.

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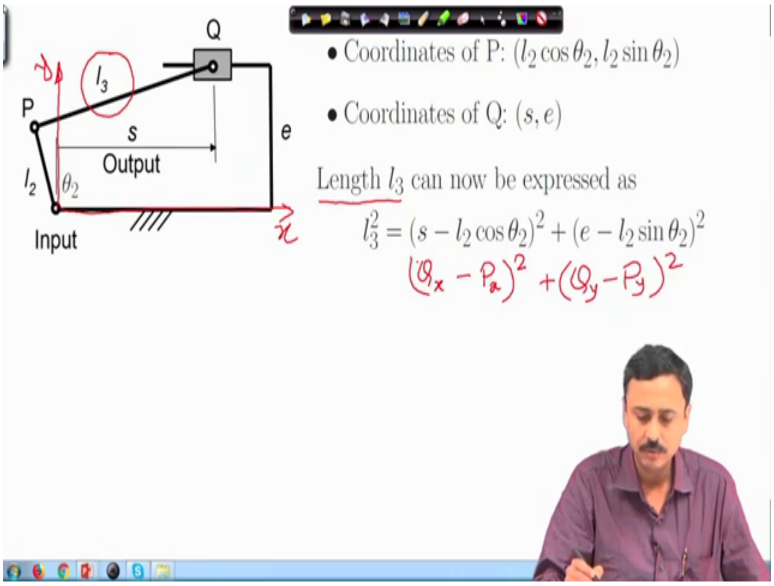

Given the coordinates of P and Q, the length l 3 which is this link essentially the length of PQ: so l 3 square can be written as this is Q x minus P x whole square plus Q y minus P y whole square and that is l 3 square. Now, if you open this up considering that we are given theta 2 and we have to find out s; I can obtain this quadratic equation in s.

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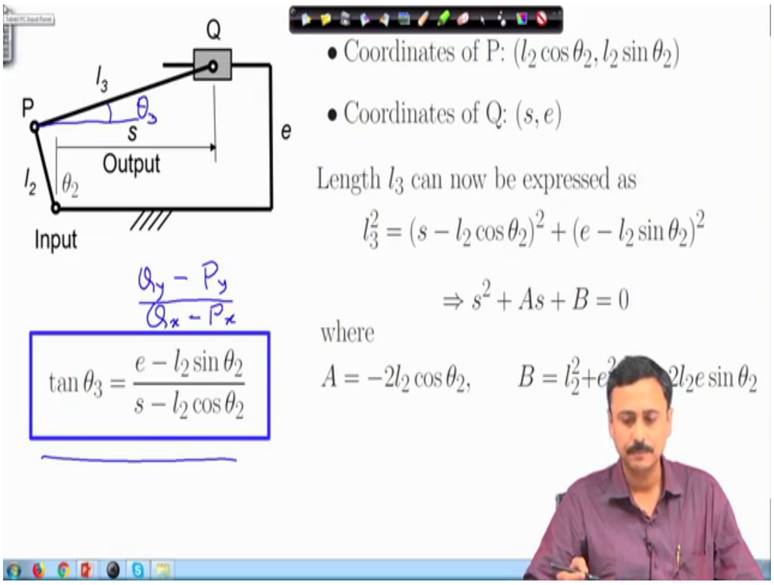

I can obtain this quadratic equation in s which can be solved because I know these coefficients A and B in terms of theta 2 and the other link lines because A and B are known to me I can solve for s.

Once I know s and since I already know theta 2, I can find out this angle theta 3. So, this angle is theta 3, I can write tan theta 3 as shown here. So, tan theta 3 will be Q y minus P y by Q x minus P x that will be tan theta 3 and you will get this expression.

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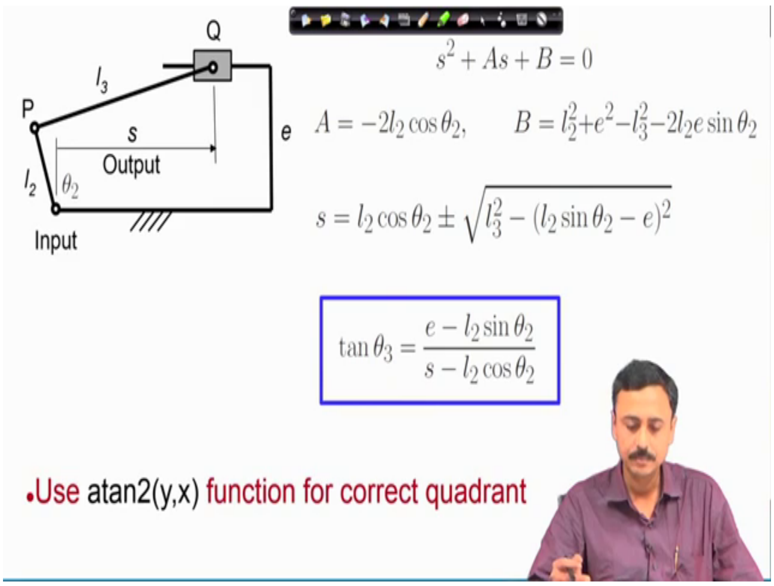

Finally, I have put together all these equations which can be solved[noise] now easily to find out s, and finally theta 3. Now when you solve for theta 3, remember to use the atan2 function which gives the correct quadrant of theta 3.

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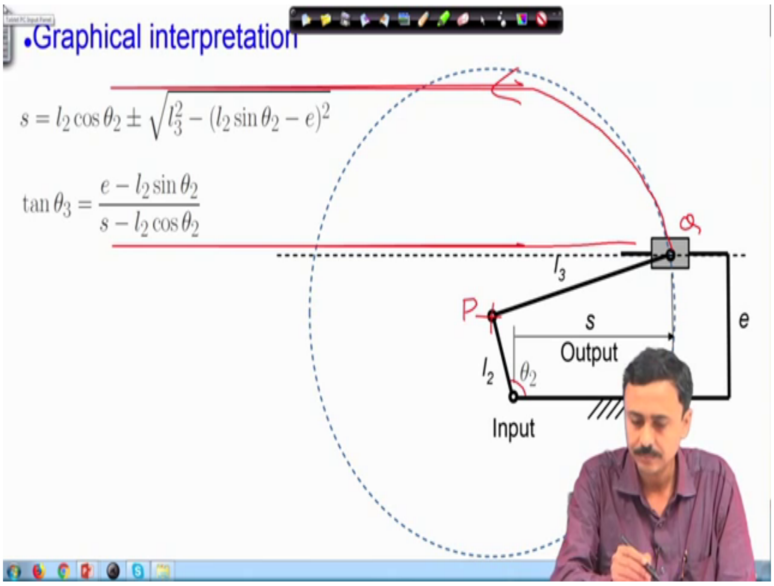

How do I interpret the solution? There are there can be two solutions as you can very well figure out. Because s is obtained from a quadratic equation and here we have this plus or minus, there are two solutions for s, and hence two solutions for theta 3.

Now, how do we interpret these two solutions? Here, I have written out the solutions as you can see the angle theta 2 is given to me, it is known. Therefore, this point is fixed. This point is fixed since theta 2 is known to me this point is fixed. Therefore, to assemble a mechanism; that means, connecting the link l 3 to the slider this point was called P and this point was called Q. Now Q can move on this circle. Q as a part of this link l 3 can move on the large circle, where as Q as a point on the slider can move on this straight line. Therefore, the intersection points of this circle and the straight line one is here the other is here these intersection points are points where I can assemble a mechanism, I can connect the slider block to the link.

Therefore, we have 2 possibilities of assembling the mechanism; one is already shown in this black, the other possibility is this where the slider comes here. Therefore, the black configuration is one configuration, the red configuration is the other configuration and these are the two solutions the two assembly modes of this kinematic chain. If this is a grush off chain then these 2 are distinct if this is a non grash of chain, then you can go from one configuration to the other.

Next, we go over to the inverse kinematics of this 3R1P chain of type 1. Now, what is given is s and what we have to find out is theta 2? So, this displacement of the slider is given, we have to find out this angle. So, that is the inverse kinematics problem. So, output is given the location of the slider is given, we have to find out the input angle. Once again, you can write the coordinates of P and Q. The coordinate system used once again is this. Once again you can write out l 3 square as we have done before.

But now, we are given s, we have to find theta 2. Therefore, we simplify the expression and cast it as this equation in which we have to solve for theta 2, A B C are now known because s is known and the other link length parameters are known. Therefore, in this equation theta 2 is unknown which we have to solve. We have solved such an equation before and of course, theta 3 will then be found from this tangent condition.

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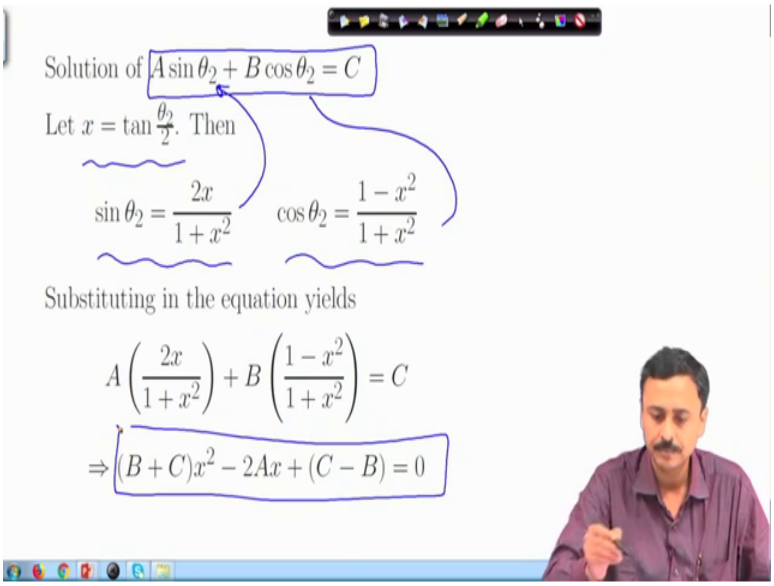

So, we now have to solve this equation for theta 2. We make this substitution x is equal to tan theta 2 by 2 and write sin theta 2 and cosine theta 2 in terms of x and substitute these expressions in our equation to be solved. And finally, obtain this quadratic equation now in terms of x.

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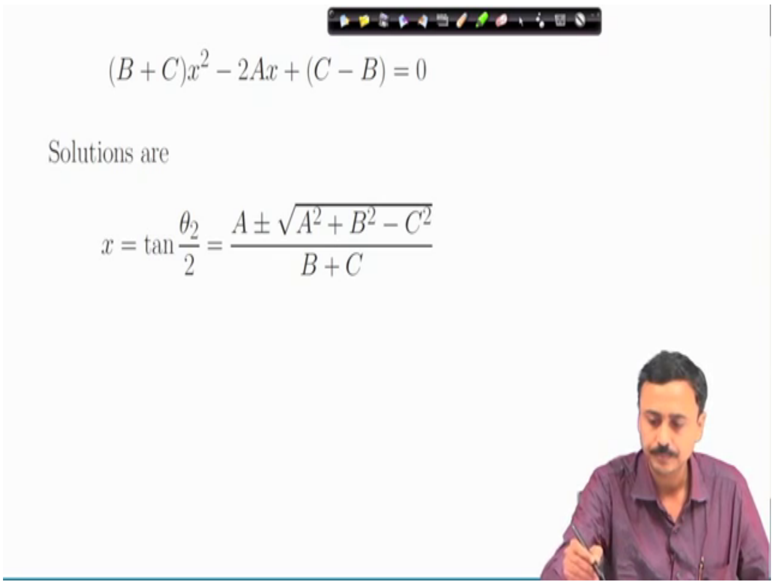

We know the solution of a quadratic equation. The solutions there are 2 solutions like this and the solution of x; x is essentially a tan theta 2 by 2. So, we have 2 solutions of theta 2.

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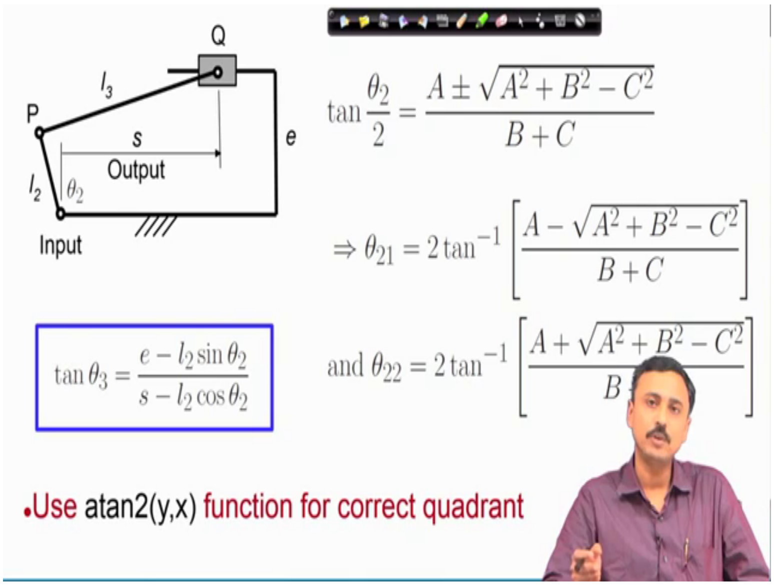

Here, are the 2 solutions theta 2 1 and theta 2 2 that depends on the sign that we take and once I have theta 2, I can substitute that in the expression of tan theta 3 and obtain theta 3. Once again, let me remind you that you have to use this atan2 function which will give you the correct quadrant.

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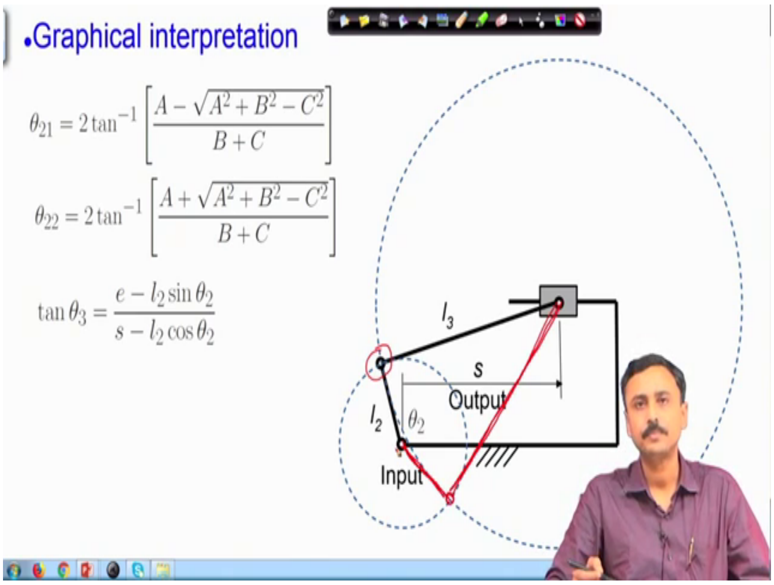

Now, we look at the interpretation of the solutions there are two solutions as we have mentioned. In this case, I am given s. Therefore, this point is fixed, the slider position is fixed. We can now think of assembling the mechanism by looking at the location of this hinge P. This P is a point on link l 3 will move on this circle larger circle; whereas, P as a point on the link l 2 will move on this smaller circle.

Therefore, the mechanism can be assembled whenever these 2 points coincide which means they are the intersection of the 2 circles. There are 2 intersections corresponding to these two solutions; one intersection is already shown and the black configuration of the mechanism is for this intersection, for the other intersection the mechanism will look like this. So, this is l 3 and this is l 2. So, red configuration is one assembly mode, the black configuration is the other assembly mode. So, these are the two solutions.

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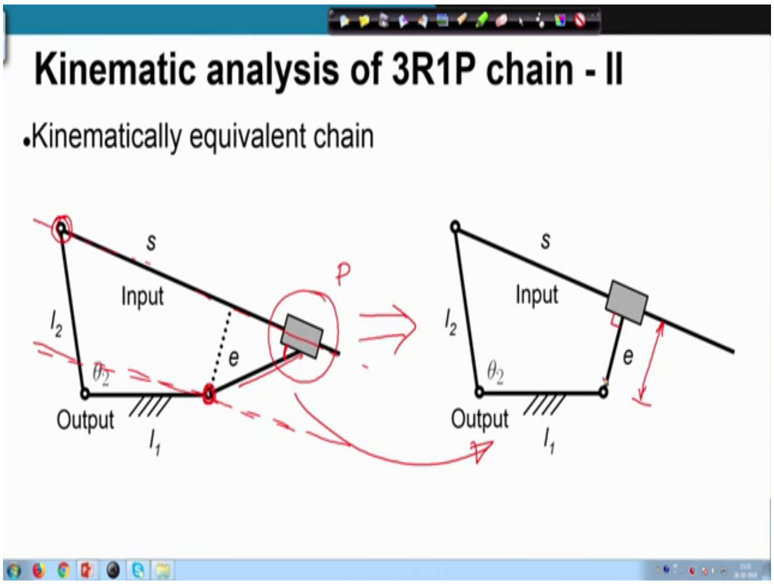

Then, we go over to the 3R1P chain of type 2. Here, I have shown 2 kinematically equivalent chains. If I am given the chain on the left where this angle is an arbitrary angle, then I can convert to a kinematic a 3R1P type 2 chain in which this angle is 90 degree and this distance is e. This distance is e the offset of this 3R1P chain. You remember that the offset is the perpendicular distance between the 2 lines which pass through the 2 hinges of the links which are connected by the P pair. So, this is the P pair. This is one link, this is the hinge of that link and the hinge of the other link is here. So, the perpendicular distance between these 2 lines gives us the offset.

I can therefore, convert this chain to this equivalent chain, solve the kinematics of the equivalent chain and later on go back to the original chain.

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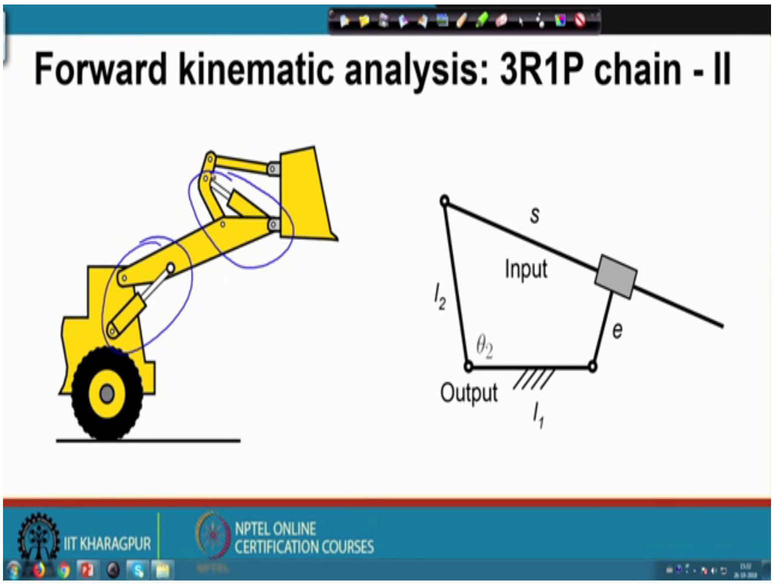

This kind of chain is found in a number of applications. Here I have shown you one application. You can see this 3R1P chain or even this 3R1P chain which corresponds to this chain of type 2.

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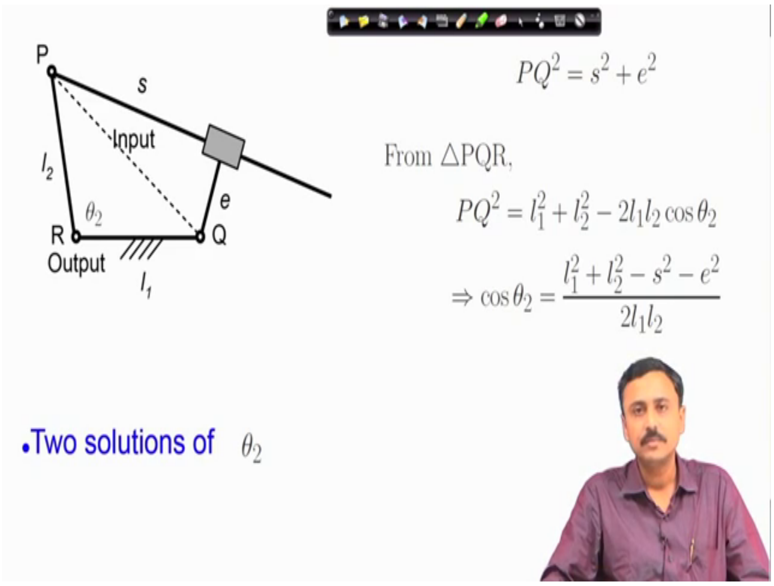

First, we will discuss the Forward kinematics problem. In this problem, we are given the input. Input is usually as you have seen input is usually the hydraulic actuator. Therefore, this is given s is given which is the input and we have to find out this angle theta 2 which is the output.

I have made a construction; I have joined P and Q by a line like this. You can very easily write P Q square as a square plus s square. This is s this is e and since this is 90 degree. Therefore, P Q square is s square plus e square; now, from triangle P Q R. So, this triangle, you look at this triangle; then we can write P Q square using the cosine rule we can write P Q square is l 1 square plus l 2 square minus twice l one into l 2 cosine of the included angle theta 2. Therefore, cosine theta 2 is given by this expression.

Now here, I know the link length parameters and I am given s; therefore, I know cosine theta 2. It only remains to determine theta 2 from this equation and naturally there are two solutions.

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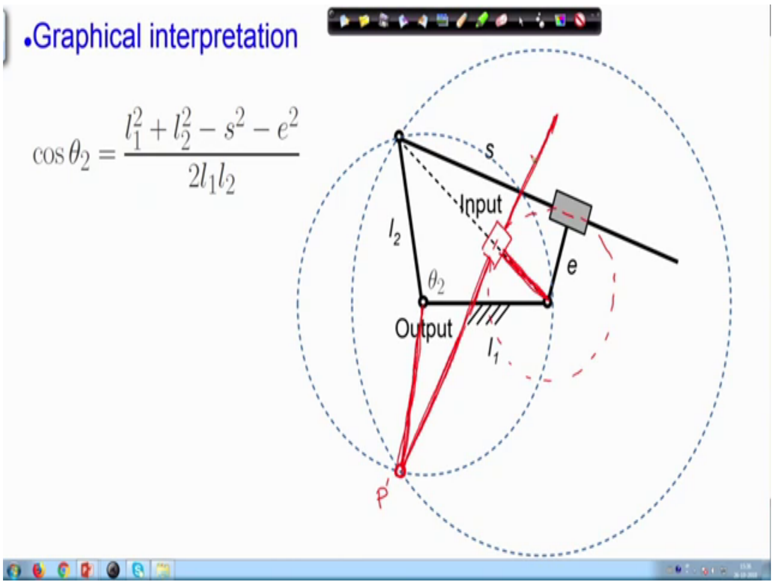

Let us look at these two solutions graphically. Remember that s is specified. If s is specified, then this P Q length is fixed. This is P Q. This length is fixed, but what is not fixed are this angle theta 2 and also this angle. Therefore, when I change theta 2, the point P will move on this circle; when I change theta 2, the point P will move on this circle and when I change this angle, then point P will move on this circle.

Therefore, the mechanism can be assembled only when the location of P on both circles coincide which means they are the intersection points; one intersection point is already shown, I will show you the second intersection point. The configuration the first intersection point is shown in black, this is the second intersection point of P. Now here, the configuration will look like this the link l 2 will move like this and to locate the other part of the mechanism, we consider tangent to this small circle. This is the slider block and this is the other link. So, this is the red configuration is the other configuration of the mechanism.

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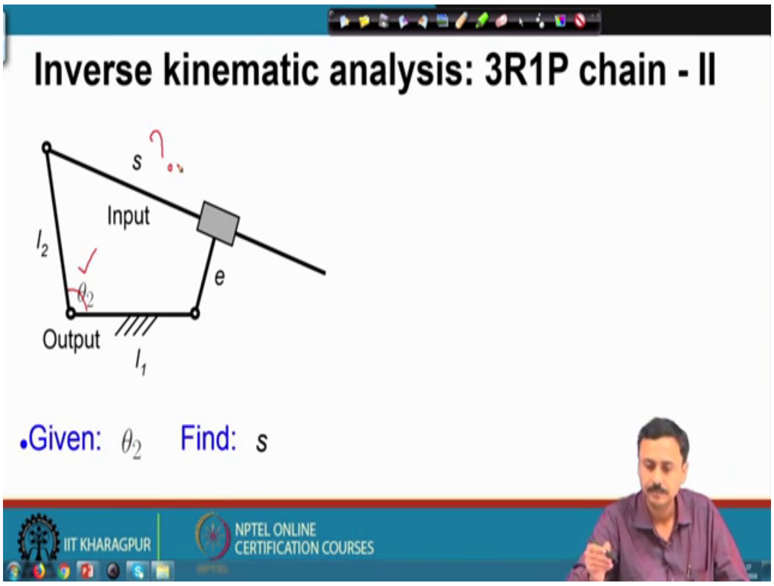

Now, we go over to the Inverse kinematic analysis of 3R1P chain type 2. Here, we are given theta 2. So, this theta 2 is given we have to find out s. Let us look at this triangle P Q R.

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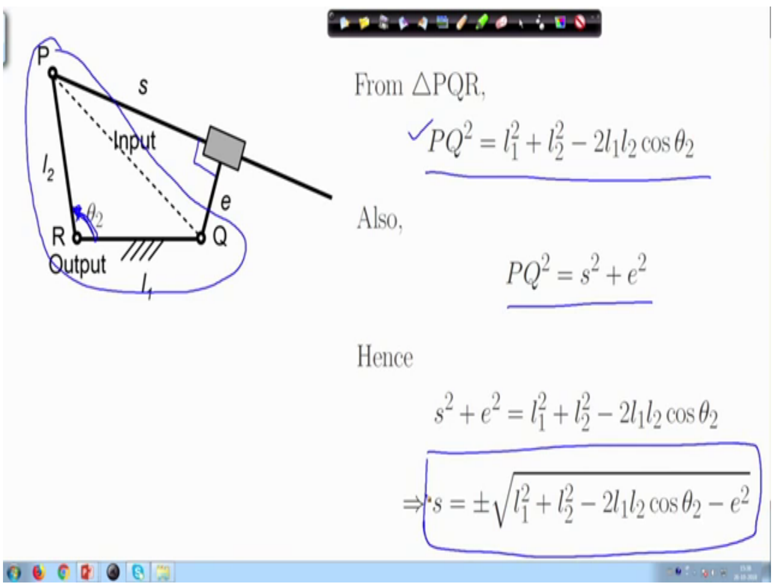

And the cosine rule, I can very easily write P Q square as l 1 square plus l 2 square minus 2 l 1 l 2 cosine theta 2 theta 2. Remember that theta 2 is specified. Therefore, I can find out P Q. I know P Q. Now P Q square is s square plus C square. This being 90 degree. Therefore, I have 2 solutions of s from here given by plus or minus. Now let us look at the interpretation of these 2 solutions.

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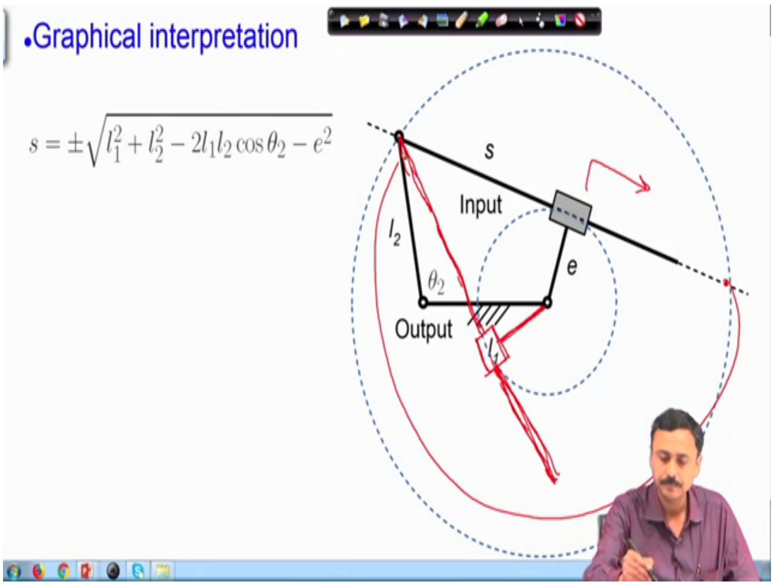

So, I have written out the solution of s plus plus or minus this quantity. Now, plus is interpreted as a displacement in this direction. This is positive s and of course, this is negative s.

Now, what is specified is theta 2? So, this is fixed. Therefore, this point is fixed. What we need to find out is the configuration of this link so that we can assemble the mechanism. Now, this can be assembled in 2 ways; one is already shown in black, the other configuration in order to find out the other configuration. Again we think of the other tangent to this circle and we bring in the slider on this side. So, this is the other configuration of the mechanism. But then, you must realize that this can happen this configuration, the red configuration can happen only on the negative side of s; that means, some point here can be brought and connected to the hinge at P.

Therefore, this configuration if you start from the black configuration, you cannot reach the other configuration. So, this has to pass through the slider on the other side and go over to the other side. So, these are the 2 different modes assembly modes of this mechanism corresponding to these two solutions. So, you interpret the negative value of s as the value of s on the other side of the slider.

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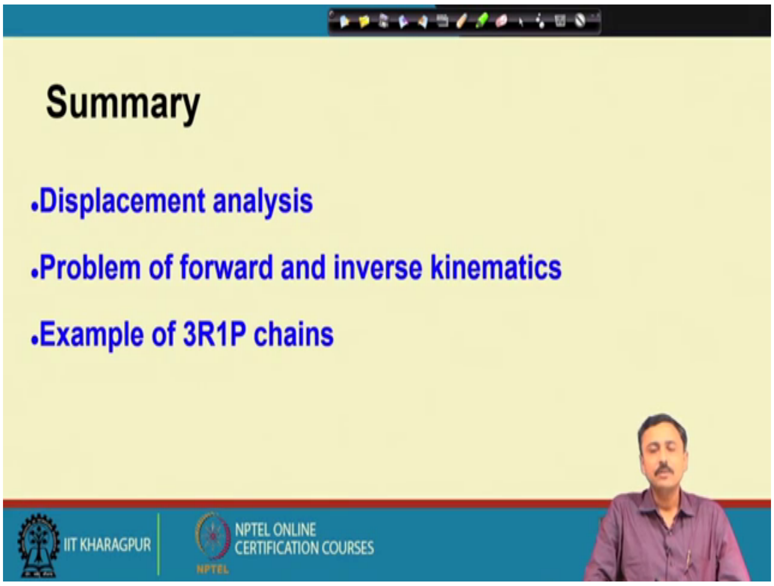

So, let me summarize. We have looked at the Displacement Analysis problem of the 3R1P chain. We have considered 2 kinds of chain; type 1 and type 2 which are found in a number of applications. With this I will conclude this lecture.