Kinematics of Mechanisms and Machines
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Lecture - 10
Displacement Analysis - I

In this lecture, I am going to discuss Displacement Analysis of Kinematic Chains.

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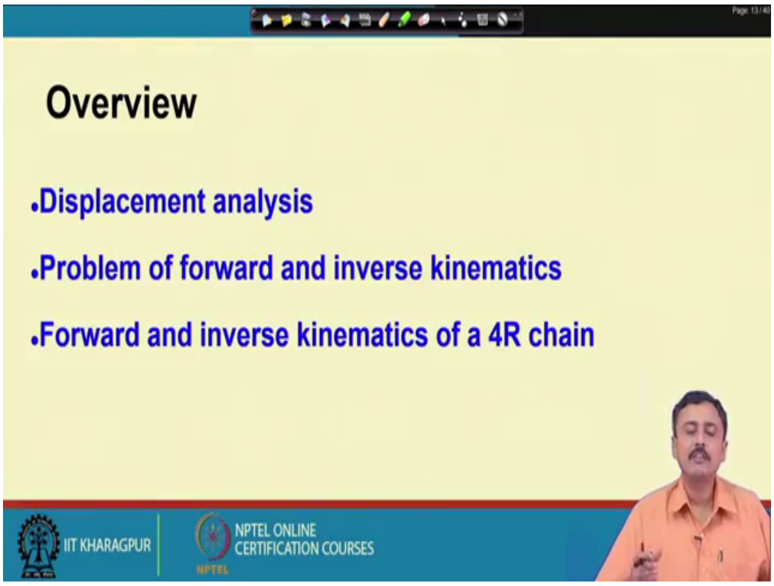

As I have mentioned before, Displacement is quantified in terms of position, orientation etcetera and as we will see that it also has implications for the path that the mechanism can take. So, the overview of this lecture is as follows. I will introduce the problem of Displacement Analysis. I will discuss the problem of forward and inverse kinematics in the context of kinematic chains and I will discuss, I will calculate the forward and inverse kinematics of a four r kinematic chain.

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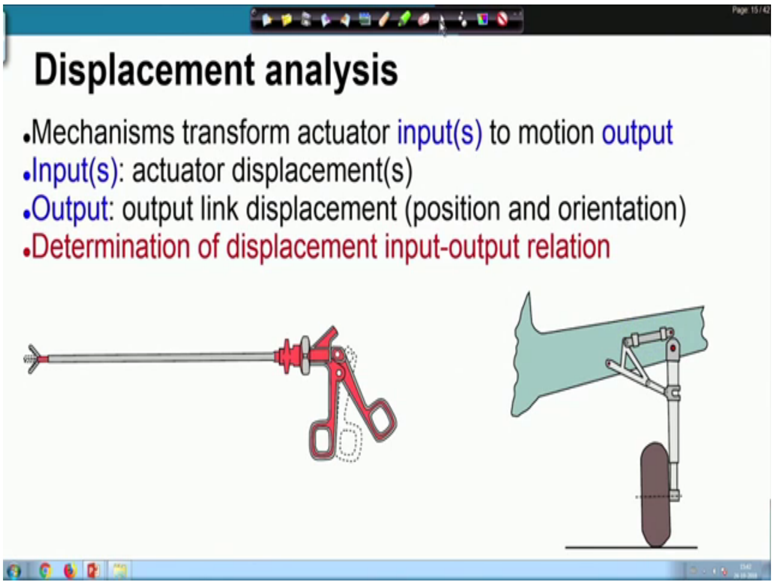

So, what is Displacement Analysis and why do we require Displacement Analysis? As you know the mechanism; mechanism transforms and transfers motion from certain inputs to an output. Now, these inputs are some actuator displacements and output is the output link displacement which is where the useful work is going to be done. This determination of the input output relation from the actuators to the output link is the problem of Displacement Analysis. We have to determine the input output relation. So, given the input, what will be the output or given the output what should be the input?

Let us look at certain examples in which this is very relevant. In this surgical tool, I would like to know for example, how much should be the input angle of this finger grip so as to open the end effectors of the surgical tool to a certain angle. Here therefore, we have a relation between the angle of motion of the finger grip to the angle of opening of the end effector. In the case of this landing gear, how much should the prismatic actuator? Here, we have an actuator. How much this actuator should expand in order to fold the wheel in? This is a problem of Displacement Analysis.

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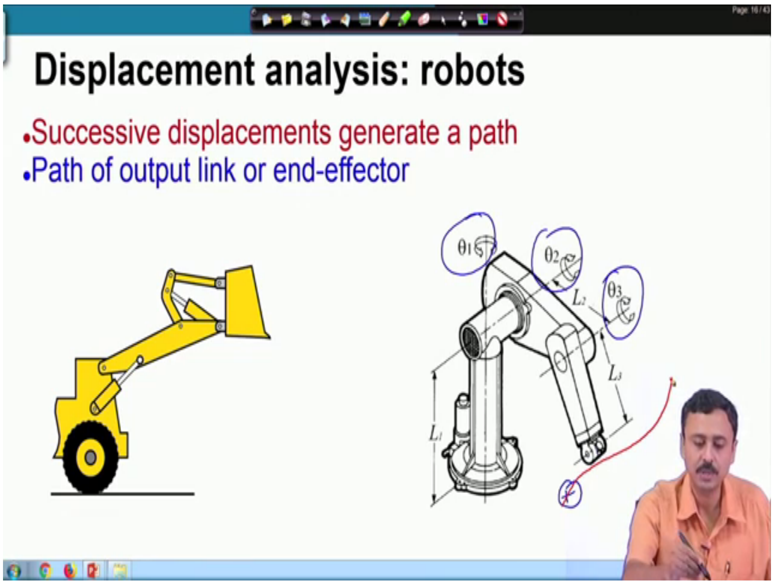

In the case of robots, where we have degree of freedom two or more the inputs which are given by the actuators; they are they can be given independently driven and the output link can have an arbitrary configuration. Now as you keep moving the actuators are actuating, if you change the angle or the expansion of the actuator; then this output link is going to move that is going to generate a path. Therefore, successive displacements of the output link in response to the successive motion of the inputs will generate a path for the robot that is also a part of Displacement Analysis.

Let us look at this example of this excavator. This bin at the current position for a given actuator throw is here. Suppose, I change the actuator throw, where will this bin move or I can also ask this question if I want my bin here, if I want my bin here; what should be the actuator throws, what should be the expansion of the actuator? This is again a problem of Displacement Analysis.

Similarly, for this robot if I want to move it to this location, how should these angles theta 1, theta 2, theta 3 be relocated; what should be these angles so as to put this end effector at this location and successive displacement. Suppose I want this end effector to move along a certain path; what should be the successive angles theta 1, theta 2, theta 3. This is the problem of Displacement Analysis. There are two kinds of problems under Displacement Analysis that is that we are going to discuss.

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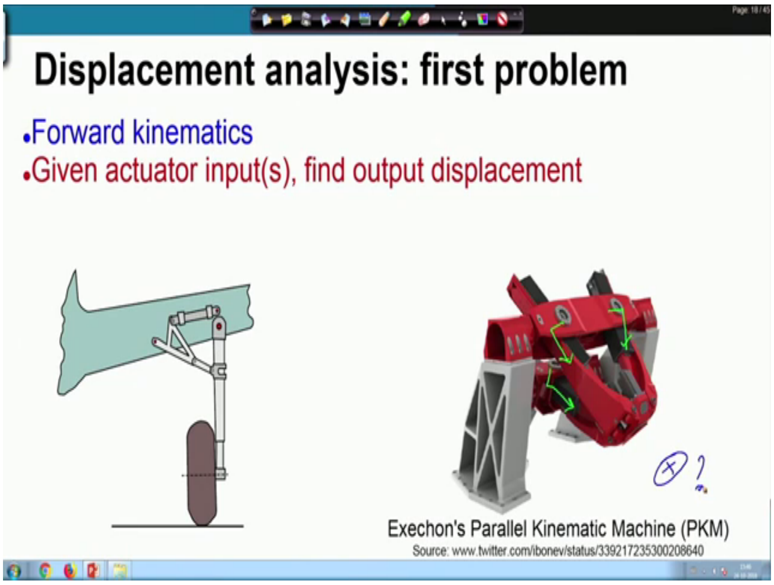

The first problem is known as the Forward kinematics problem. In the Forward kinematics problem, we are given the actuator inputs. For example, what should be the, so what are the angles the joint angles of the robot? And given these joint angles, what is the location of the end effector? So, we have to find out the output displacement.

In this a landing here, I am given let us say this through of this actuator, it expands by a certain amount; what will be the angle of the wheel assembly? Or for example, in this parallel kinematic machine, if I give the expansion of these actuators, where is the end effector of the manipulator of the machine? This is known as the Forward kinematics problem. So, given the actuator inputs, where is the end effector or the output link?

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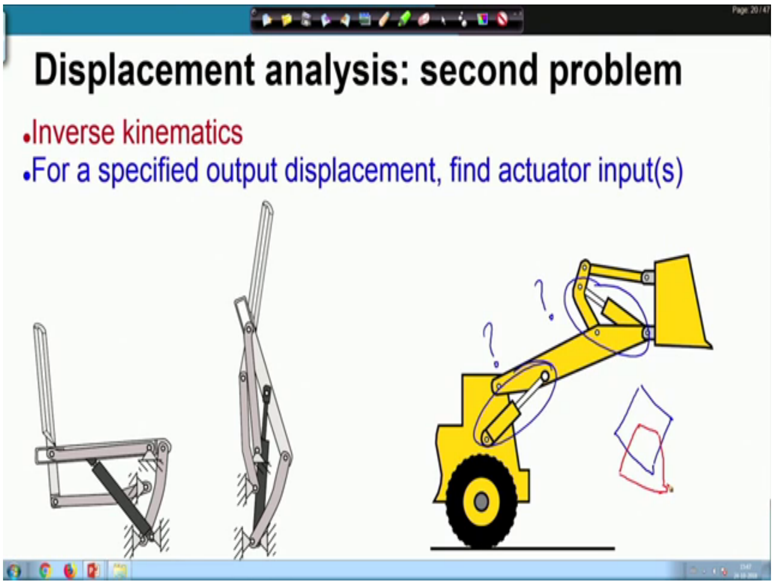

The second problem therefore is the reverse of this, is called the Inverse kinematics in which the outputs output is specified. And I have to calculate what should be the actuator throw or what should be the angle joint angle to reach a certain point.

Here, I have an example of the transfer device. I want to go from the sitting position to the standing position; what should be the throw of this actuator? How much should this actuator expand to take me from this sitting position to this standing position? Once again, the excavator example if I want the bin, to be in this position what should be the throw of that these two actuators? This will also help me if I know successive positions, if I know how to solve this inverse kinematics problem. Then if I am given successive positions of the bin, I will know how the actuator should move in a coordinated fashion so that this path is generated.

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Let us plan our discussion on Displacement Analysis. In this manner will first take up Constraint Mechanisms. Later on, I will discuss open chain robots and finally, closed chain robots.

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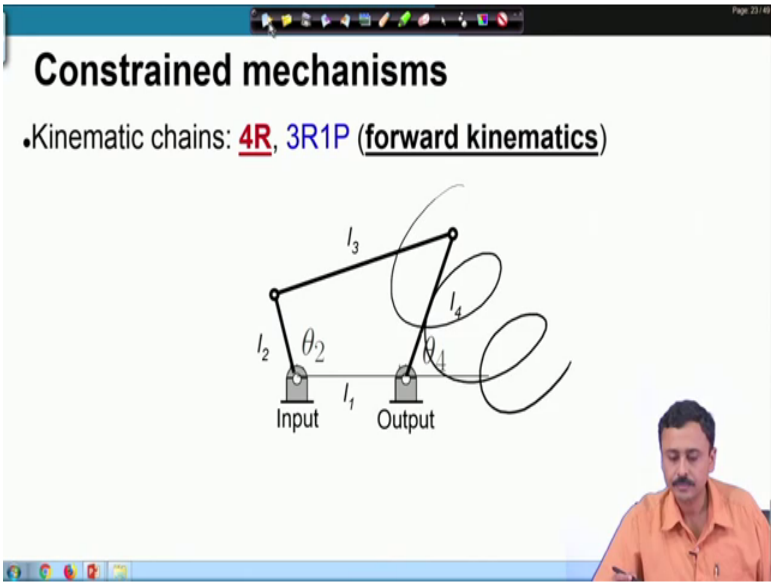

So, we will start with constraint mechanisms and the simplest constraint mechanism is a 4R kinematic chain. I will discuss 3R1P kinematic chain subsequently. The first problem is the Forward kinematics problem. Here, in this chain I have marked out, what is input and what is output. The forward kinematics tells me or asks the question given the input; that means, given theta 2 what should be theta 4?

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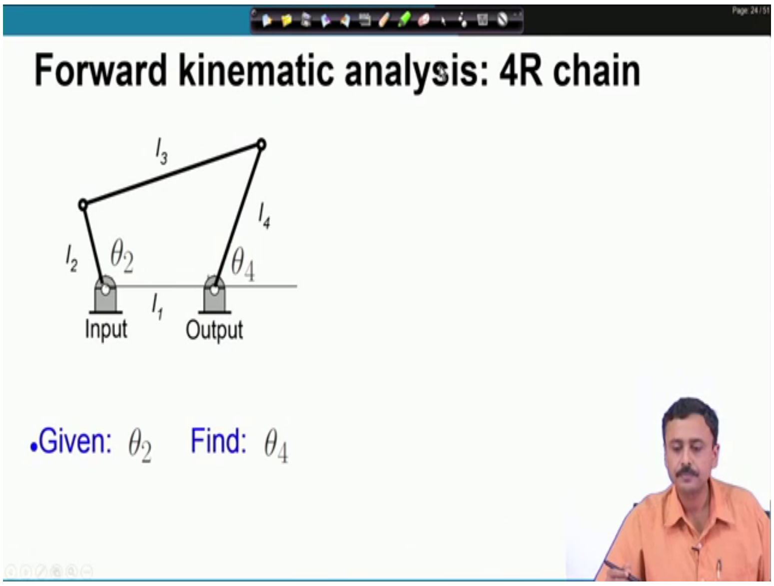

So, given theta 2; theta 2 is given what is theta 4? This is the problem of forward kinematics. So, given the input what is the output? In this case the output angle.

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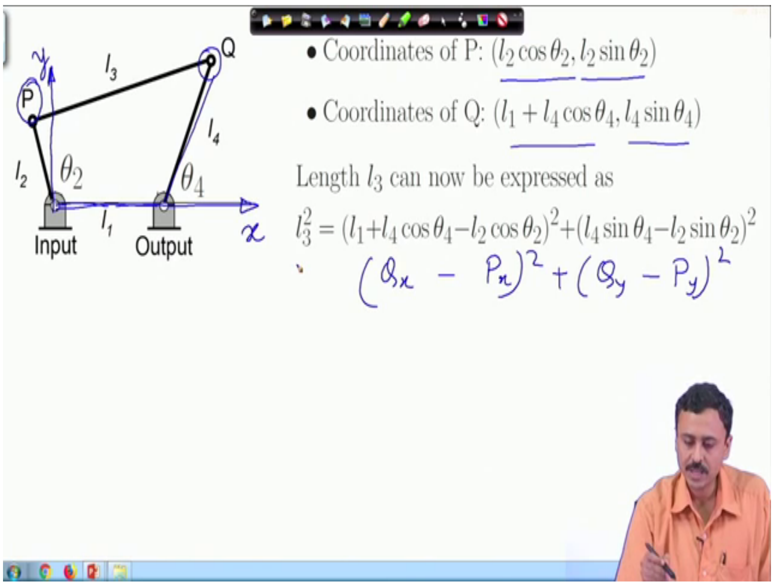

Now, these steps are straightforward. First, we write down the coordinates. For that, I will require the coordinate system. Let us say this is the x axis and this is the y axis. I have written out the coordinates of point P which is one of the revolute pairs. This is very simple to write it out. So, this is l 2 cos theta 2 is Px and l 2 sin theta 2 is Py. Similarly, the coordinates of point Q, it is l 1 plus l 4 cos theta 4 and l 4 sin theta 4.

Now, if I express these lengths l 3; l 3 square. So, l 3 square is this is Qx minus Px whole square plus Qy minus Py whole square. So, that is how I get l 3 square.

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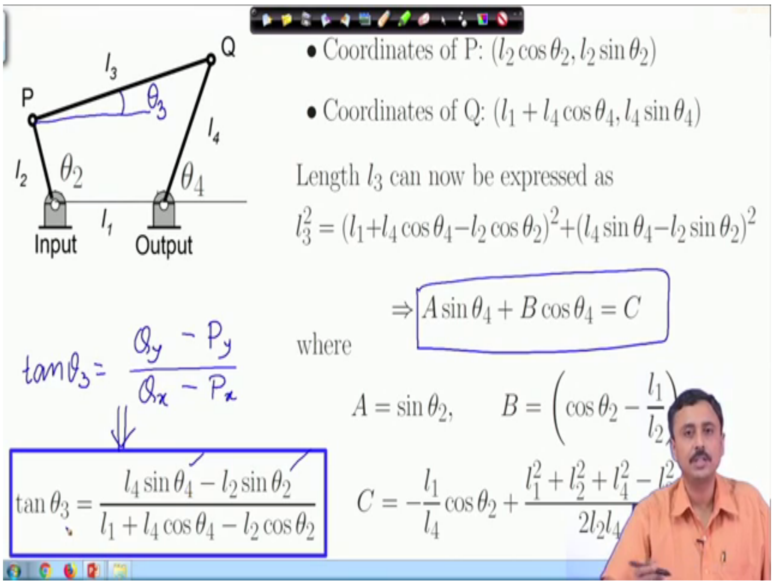

Once I have done this, I can simplify this expression. I can open up the brackets and I simplify the expression. Remember that I am given theta 2. So, I am given theta 2. I have to find out theta 4. Therefore, I have rearranged this expression in such a way that theta 4 is the unknown and these coefficients A B and C are in terms of theta 2 and the other link lengths which are known.

Therefore, A B C are in terms of theta 2 which is known. Therefore, A B C are known. We have to solve for theta 4 from this equation. Now, this equation can be solved in a number of ways, but if you have to find out the correct quadrant in which theta 4 lies. Then, we have to follow a certain procedure which I am going to show you now, and before that let me also discuss about determination of this angle theta 3. So, this is theta 3. So, tangent of theta three is nothing but this is Qy minus Py by Qx minus Px from there we will get this expression. So, that is tangent of theta 3.

So, if I can determine theta 4, I am I have been given theta 2; I can determine theta 4 from this equation. If I can find out theta 4 from this equation, then I will substitute theta 4 here, theta 2 is known and then, I can find out theta 3 as well.

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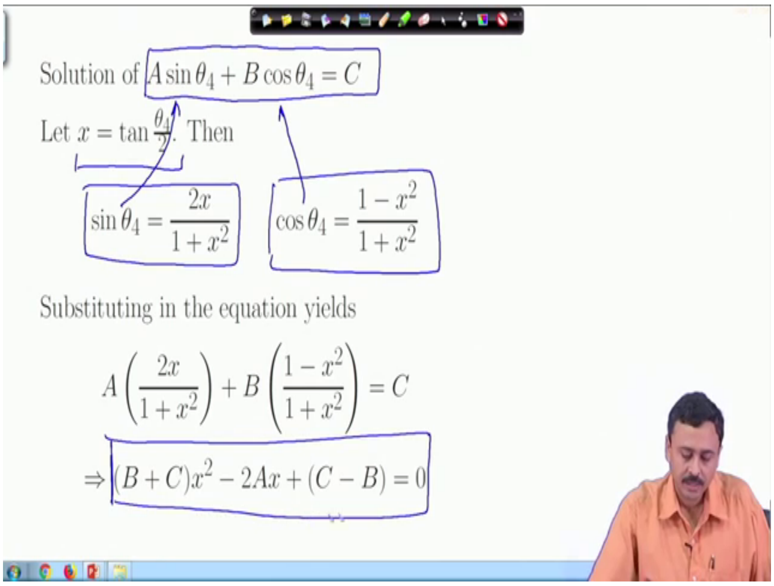

Now, how to solve this equation A sin theta 4 plus B cos co sin theta 4 equal to C.

We make a substitution, x equal to tan theta 4 by 2. In the if I consider x in this form, then sin theta 4 is 2 x divided by 1 plus x square plus co sin theta 4 is 1 minus x square by 1 plus x square. This is straightforward trigonometry. When we substitute these expressions here, you will get this equation which finally, simplifies to this quadratic equation in x. So, we have this quadratic equation in x. Why I are we going through this path, because, we want to get the quadrant of theta 4 correct. This is specially required when you are solving this on a computer.

So, we have a quadratic equation, equation whose roots are to be determined which is very straightforward.

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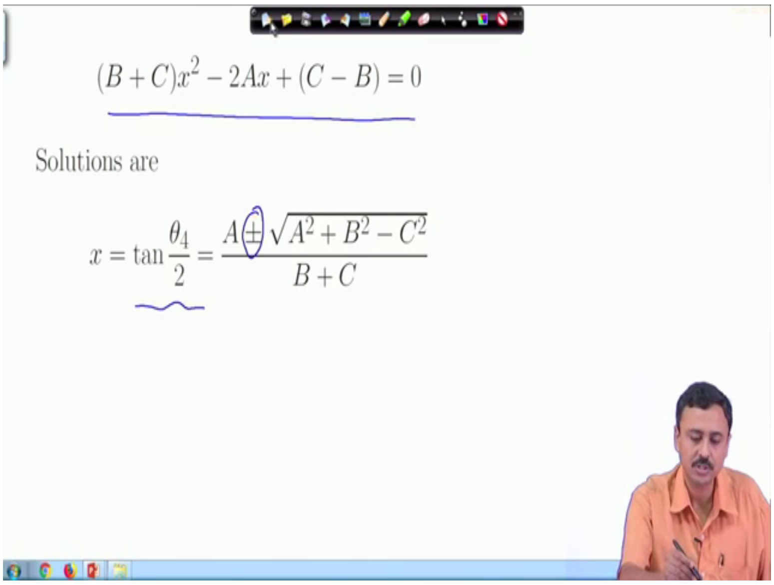

Roots are given by A plus or minus under root A square plus B square minus C square whole divided by B plus C. That follows from this quadratic equation and x is nothing but tan theta 4 by 2. Therefore, I can now have two solutions, because we have this plus or minus, we will get two solutions of theta 4.

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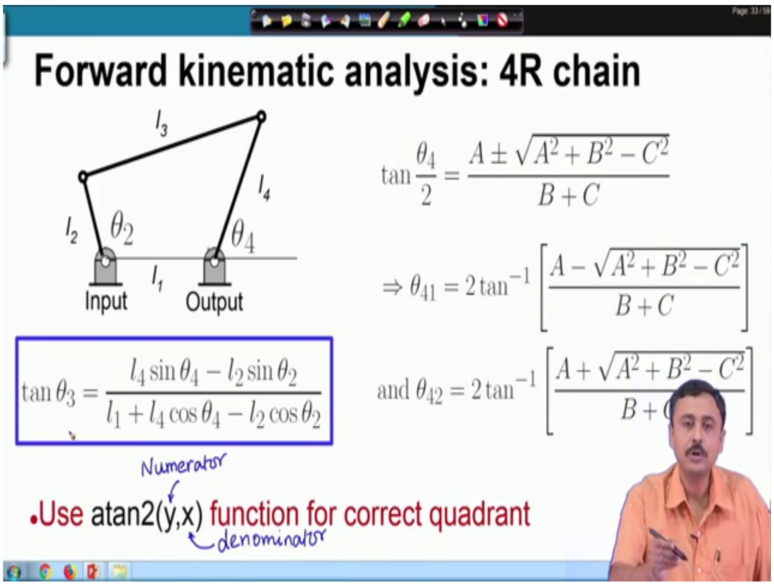

These two solutions are obtained by taking the negative and the positive signs as I have shown here. Once I have this theta 4 solution, there are two solutions theta 41 and theta

42. Therefore, we will have two solutions for theta 3 corresponding to this theta 41 and theta 42.

Now here, I would like to mention, we require this tan inverse. How do you calculate this tan inverse; how do you find out the angle in the correct quadrant? So, for that you need to use the atan2 function. This is a standard function in lets a mat lab or in C programming languages or other programming languages, in certain software's the format might be different, but there is function corresponding to atan2 which takes the numerator and denominator separately, because we have a ratio here tan inverse a ratio, we have to give the numerator and denominator separately.

So, this is our numerator and this is the denominator and this atan2 function will look at the sign of this numerator and denominator and decide the quadrant. So, it gives you the quadrant in one shot. Therefore, you will get 2 solutions; theta for 1 and theta for 2 as written here and corresponding to that you will get two solutions of theta 3.

So, you have to use this atan2 function to determine the correct quadrant.

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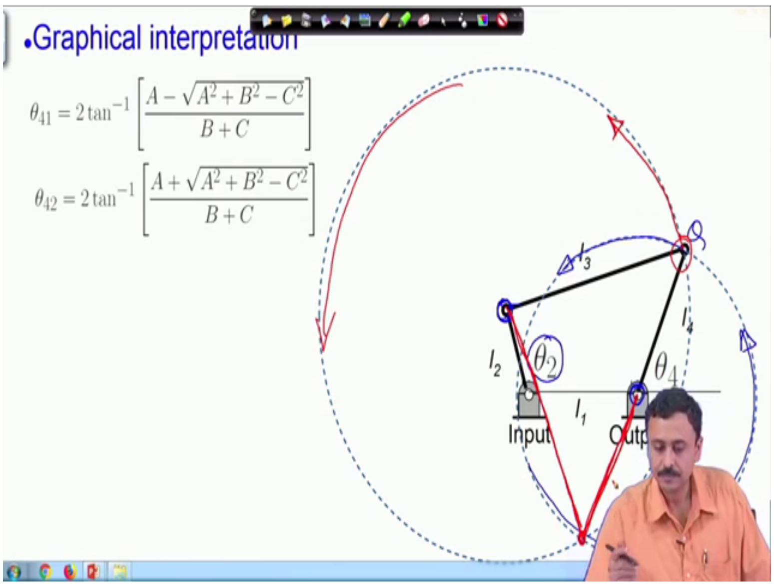

So, we have solved this problem. Now how do we understand the solution graphically? So, here I have shown the two solutions, theta 41 and theta 42 and now, I will explain the physical significance of these 2 solutions. Remember that we are given theta 2. So, this angle theta 2 is fixed; this angle theta 2 is fixed. Therefore, location of this is fixed and this ground hinge is also fixed. So, the location of this down hinge now between these two hinges therefore, I have to attach the link l 3 and the link l 4. Now, it is possible in two ways. I consider that this hinge which was Q, if I consider link l 4, then Q moves on this circle.

If I consider link l 4, Q moves on this circle and if I consider the link l 3, Q moves on this larger circle. If I consider the link l 3, Q moves on this larger circle. Therefore, that mechanism can be assembled only when these two positions of Q coincide. Now I can coincide to a two positions; one is what is shown here, the other position is this. So, wherever these two circles intersect and that gives us the two solutions. So, one configuration is shown in black; the other configuration, I will show here in red.

So, the red configuration is one assembly mode of the mechanism. The black configuration is the other assembly mode of the mechanism. Now if this is a grash of chain as we have discussed, these two assembly modes will be different. If it is not grash of chain then, they are same here you can and from this schematic diagram it seems that this link has crossed the line of frame. So, these two are not two distinct assembly words, but they are too distant configurations. As a configuration, they are two distinct configurations. So, this is the physical implication or physical significance of the two solutions. So, there are two ways of assembling the mechanism which is given by the two solutions.

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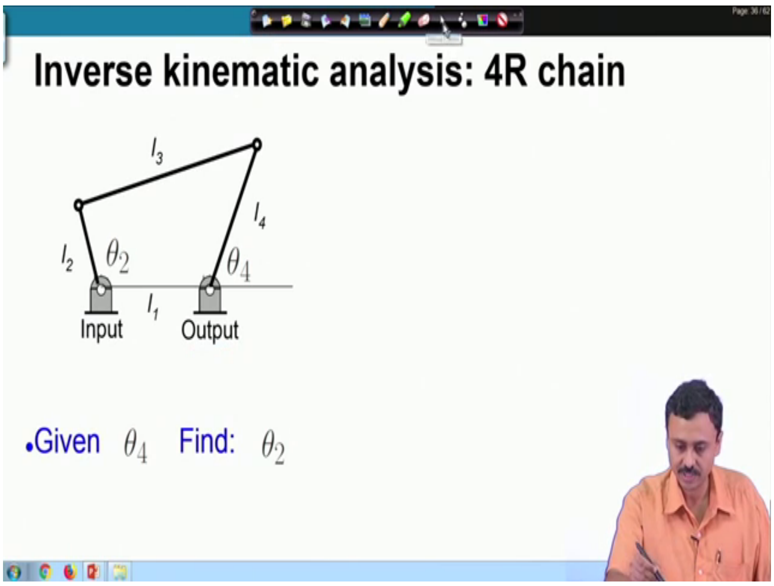

Now let us look at the inverse kinematic problem for the 4R chain which is absolutely straight forward. Here, we are given theta 4 which is the output. So, this is given we have to find out theta 2. We are given theta 4, we have to find out theta 2; theta 2. Now, this problem is absolutely similar to the direct kinetics of the forward kinematics problem of the 4R chain that we have discussed; only thing the angles that we are going to determine they have changed, but the starting equations they remain the same.

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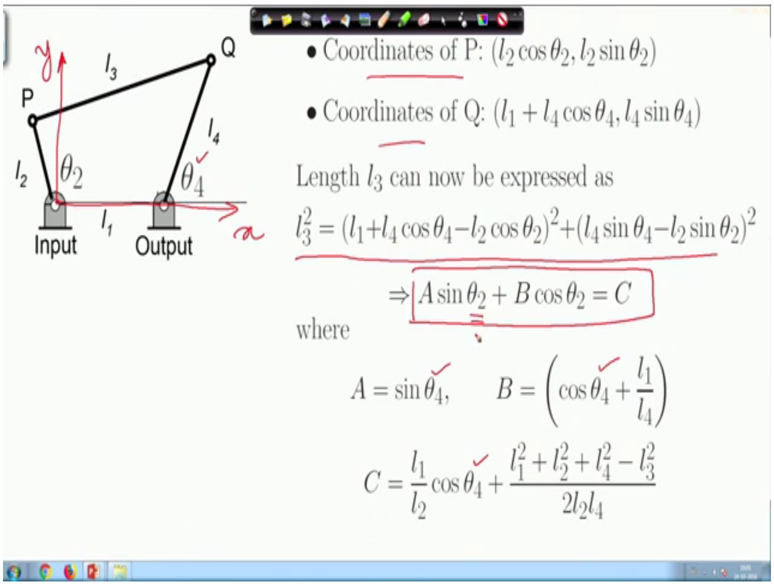

Let us go through this quickly. We had this coordinates of P and Q corresponding to this coordinate system and we had written out this l 3 square in terms of the coordinates of P and Q. Now, we are given theta 4 and we have to solve for theta 2. Therefore, we rearrange this equation keeping theta 2 as unknown and theta 4 as known. So, A B and C are now completely known because theta 4 is now given and the link lengths are given. Therefore, we know A B and C, we have to solve for theta 2 from this equation. And we follow the procedure as we are discussed.

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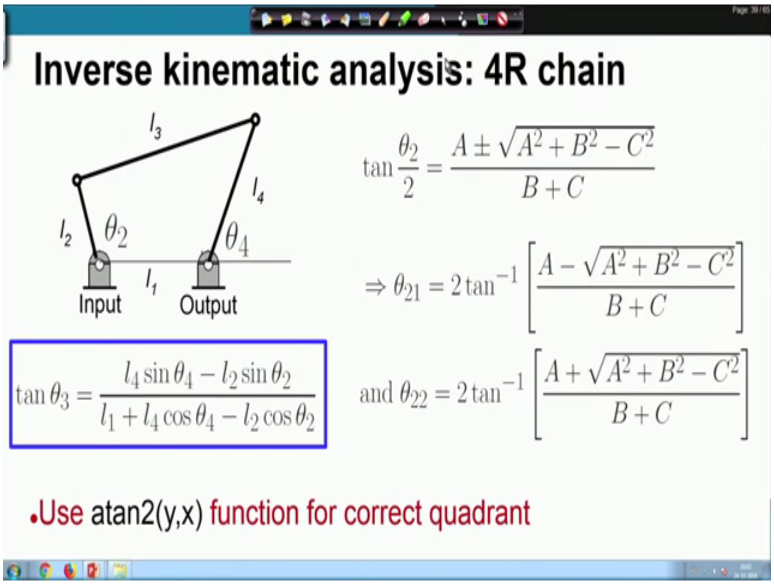

There are these are the two solutions of theta 2. Now theta 21 and theta 22 depending on the sign that you choose plus or minus and corresponding to theta 2, 1 or 2 we have theta 3; we can solve for theta 3. Once again, you have to remember that in finding out there is tangent inverse, you have to use the atan2 function with numerator and denominator given separately under atan2. That will give you the quadrant correctly.

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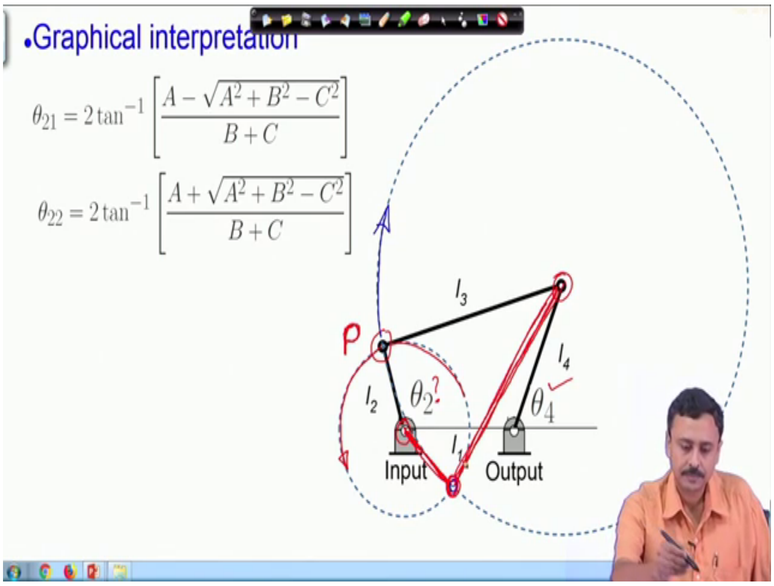

So once again, the graphical interpretation of the two solutions that we have; what is given? Theta 4 is given and we have to find out theta 2 because theta 4 is given this is fixed and of course, this ground hinge the location of this ground hinge is also known. We have to find out the location of this hinge which was marked as P. If you consider the link l 2, then P moves on this circle, the smaller circle. If you consider the link l 3, then P moves on the larger circle. Therefore, the mechanism can be assembled only when these two circles intersect, only where that two circles intersect. So, this is one the other intersection is here. Therefore, the other configuration at which we can assemble the mechanism is this.

So, these are the two assembly modes of the kinematic chain. So, these two solutions signify the two assembly modes, the modes in which this mechanism can be assembled.

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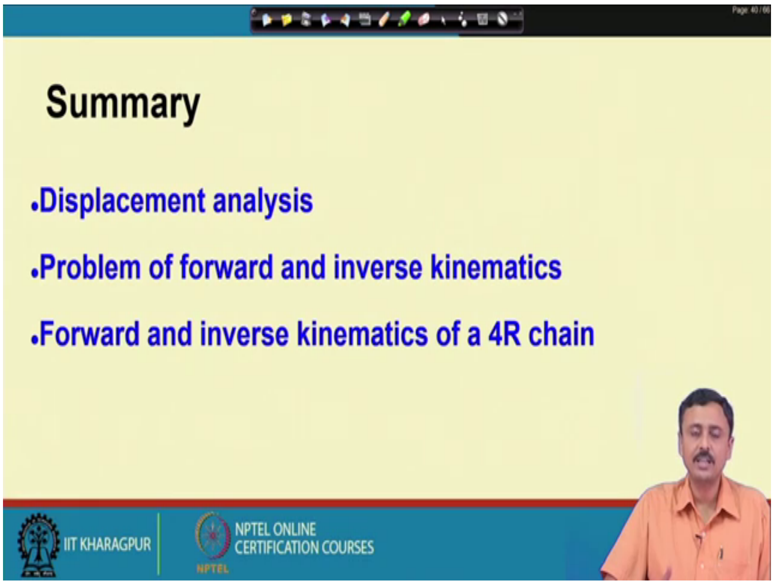

So, to summarize, we have looked at the Displacement Analysis problem, we have defined the Forward and Inverse kinematics problem for mechanisms and we have looked at this example of a 4R chain which is a constrained mechanism. We have looked at the Forward and Inverse kinematics of the 4R chain. So, with that I will close this lecture.