Electronic Packaging and Manufacturing
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Lecture – 33
Shock and Vibration – 3

Hello friends welcome to the course on Electronic Packaging and Manufacturing. This course we will deal with the vibration, this lecture will be with Vibration analysis and this is the part 3. In earlier classes you have learnt about the vibration analysis, you have seen how the vibration is important, how testing is to be done and in last lecture you have learnt how to analyze a simple vibrating system. Now in today's lecture, we are going to learn something real about the vibration of the electronic package system.

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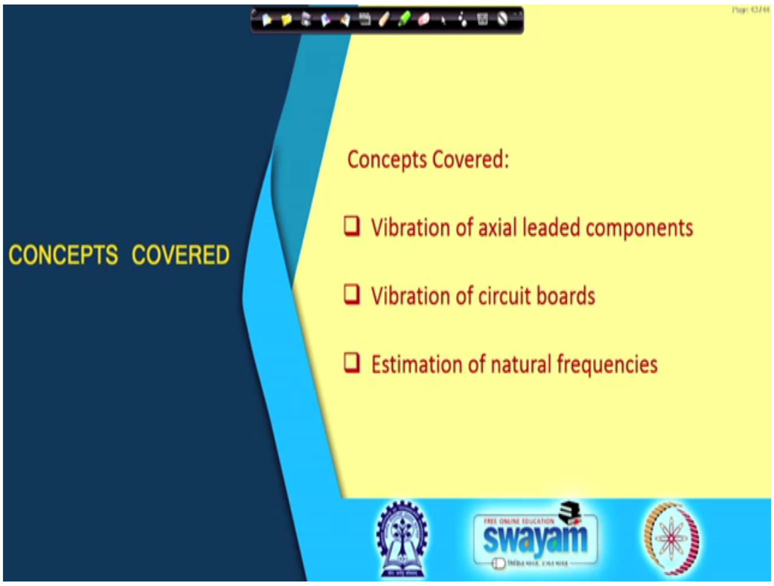

So, here in this class, you will learn this concept that is vibration; we will consider vibration of axial leaded components, vibration of the circuit boards and a very important part is the estimation of the natural frequencies. In last lecture, we have talked about the importance of the natural frequencies and we have said that depending upon the location of the natural frequencies with respect to the external frequencies, the vibration can be reduced or can be enhanced. So, of course, our objective is to reduce the level of vibration.

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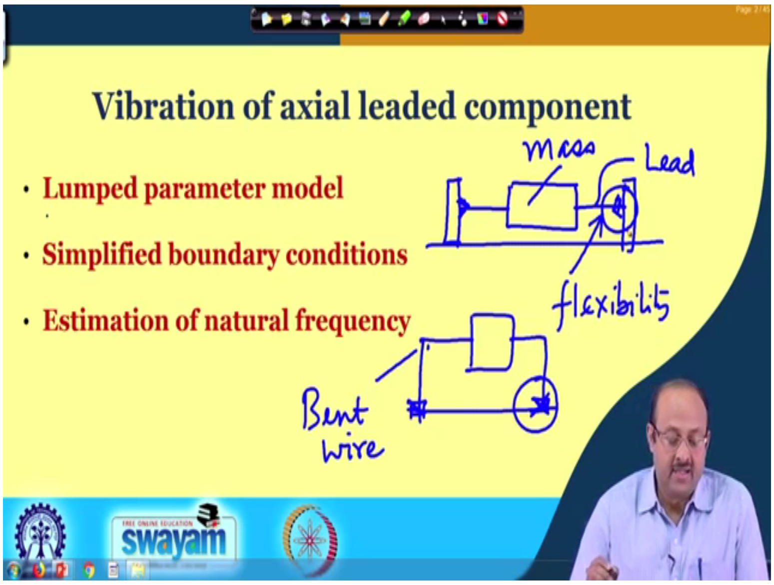

So, now vibration of axial leaded component. Now let us consider a simple system where we have so this is the system we have and there may be two situations. So, this is the circuit board and this is the post and here this is soldered and these are the lead and this is the capacitance or the resistance whatever it may be. Now in analyzing the vibrating vibration of this relatively complicated system, we have to have certain simplification. And what are those simplifications? That is we first consider the lumped parameter model; that means, that here the resistance or the capacitance, it has large mass compared to the mass of the lead wire.

Now, the lead wire this is this is very thin wire a very small diameter and of course, certain length; it has flexibility. So, we will consider this to be the mass and this part it will generate the flexibility. There may be other situations like we have the PCB and this is actually similar thing and the soldering is done over there. So, this is the bent wire bent wire or bent lead. Two things so, the analysis of this one will be much more complicated compared to this.

Second thing we need to know what is the boundary conditions because we have the shouldering here, here. So, the boundary conditions are very important for finding out the natural frequencies because the flexibility will depend upon the boundary conditions here. We will assume a very simple boundary conditions namely the clamped boundary condition here where at this point there would not be any angular deformation as well as any transverse deflection or any axial movement.

So, with this let us try to go to the vibration analysis of the; let us go to the vibration analysis of the system.

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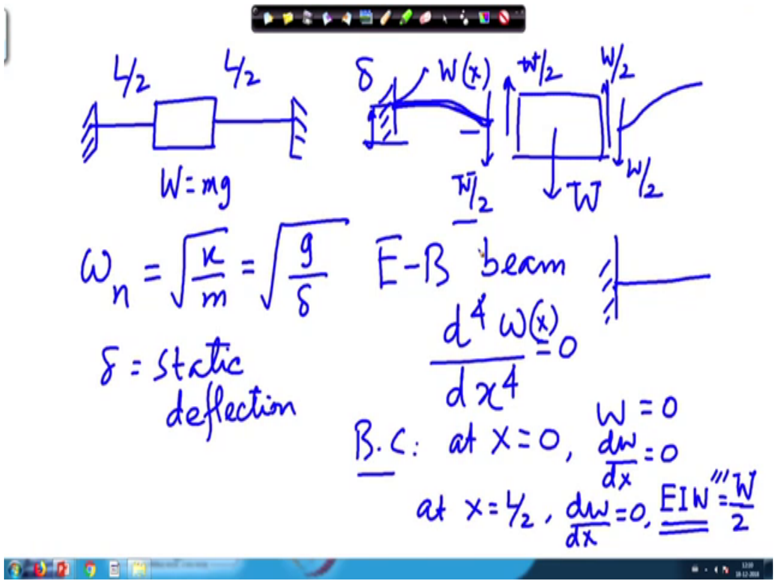

We have here again the simple situation. We have lead wire with let us say the length L by 2 and L by 2 and this has the weight W. So, this is equal to m g. Now when this vibrates we need to know the natural frequency of the system. So, omega n will be equal to k by m where m is the mass here and the K is the stiffness of the system. And we have learnt from the earlier lecture that this is equal to g by delta where delta is the static deflection; static deflection due to its own weight.

Now, to find out the static deflection; that means, here what we do? We have the system here and so this is a static deflection. Now so, it will have the weight W, this will be restricted by W by 2 and W by 2. So, this portion will have W by 2 weight and W by 2 weight there. Now because of the symmetry you see that this cannot have any rotation because if it rotates in which so this will have the same opposite rotation and because of the symmetry, it will definitely become 0.

So, we this we have a say additional boundary condition coming from the symmetry. Now we know the so to find out the deflection that is delta because of the weight, we have to use the principles of the theory of elasticity. Now what we consider? These we will consider as a very as a beam which is modeled as an Euler-Bernoulli beam Euler-Bernoulli beam theory. And according to Euler-Bernoulli beam theory, the displacement will be will be according to since there is no other load acting transverse to this beam. So, therefore, the displacement will be according to this form that is where W is a function of we; do not confuse this W with the frequency of excitation this W is the displacement of this beam measured in the upward direction.

So, we have this and then the boundary conditions boundary conditions we will consider this as X at X equal to 0. Let me consider only one half of it; that means, this part at X equal to 0 W will be equal to 0 dW dX will be equal to 0 and at X equal to L by 2 X equal to L by 2 we have the 2 other boundary conditions that is at X equal to L by 2 we have the slope 0; that means, dW dX equal to 0 and another boundary condition comes in the form of the shear force and here it is that EI W equal to the weight of this one.

So, this is the third derivative of W and this is the shear force and we have seen that the shear force is equal to W by 2 which is the weight acting. This W by 2 this is the weight acting at this point.

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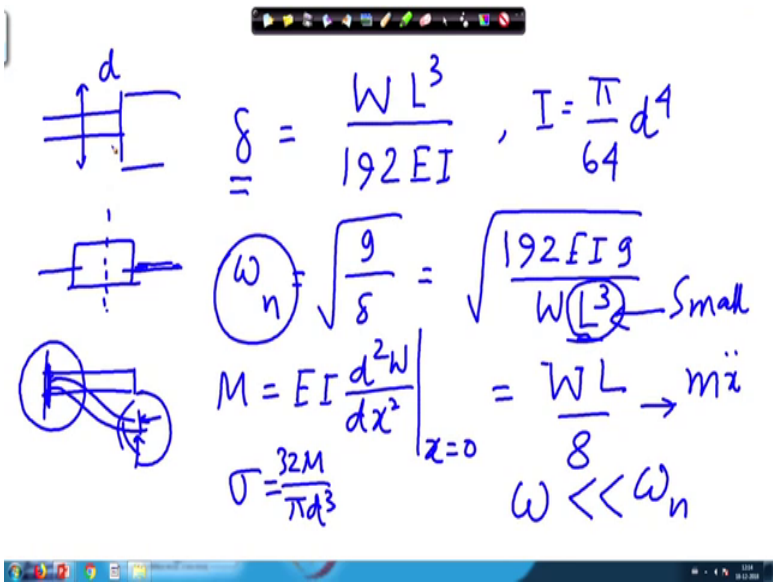

Now, with all this what you get is the delta comes out to be; if you solve this equation you will get W L cube divided by 192 E I where I is the second moment of area and that is equal to pi by 64 d to the power 4 where d is the diameter of the. So, d is the diameter of the lead.

Now, if you know this, this value delta, then we can easily find out the natural frequency omega n which is equal to g divided by delta and that is equal to g divided by delta and that will be what we get here that is 192 EI g divided by W L cube. So, one thing we see from here is that if we increase the value of L, then W n goes down whereas, if we increase the decrease the value of L, then W n will go up and if you remember that in this situation if we increase the displacement that is if you increase the so here the stress developed. Now let us consider now the, what will be the stress developed here.

Now, if this is the beam and here this is deflected like this deflection is like this. So, the stress developed if we again use the use the use the same principle the of strength of material, the bending moment here bending moment will be equal to E I del 2 W del X square and we evaluate at X equal to 0 because there the stress the shoulder joint is there. So, at X equal to 0 it develop and then this becomes you can easily calculate from the earlier analysis that this becomes equal to W L by 8.

So, now this is the bending moment here and according to the Euler-Bernoulli theory what we have that the sigma will be sigma maximum will be equal to m by 32 M divided by pi d cube. So, you see now that the stress will depend upon the bending moment developed over there and this bending moment is actually also dependent upon this W. And this is for the load static load, but when it is removing dynamically, then this W is actually replaced by the dynamic loading; that means, mx double dot.

So in fact, here the stress developed in this portion will be high if the acceleration or the displacement of this mass becomes quite large. And this leads us to consider the fact that the if we consider omega n quite large, then for a given value of omega that is the external excitation frequency the response will be quite low; if omega is actually very small compared to omega n. So, in that case what we are going to see that we have to have a design such that omega n is quite large.

So, how to make it quite large; that means, we can make these as L cube these as small. But if you remember that L cannot be made small to any great extent because what will happen when we are shouldering this joint, then the heat will be generated and this heat will be transferred through this part and ultimately it will go to the to the to the chip or the to the equipment here. And that so much of heat should not be allowed because it will then destroy this equipment.

So, minimum value of L has to be maintained, but maintaining this value minimum value we can always find out certain combination of this parameter. So, that omega n becomes large. Now this was a very simple situation when we have the lead wire in the straight form.

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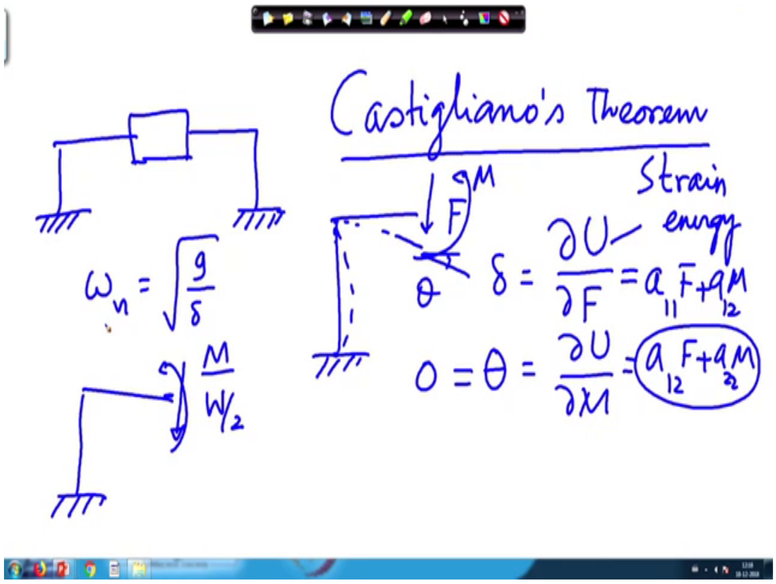

But when you have a bent wire like this bent wire where this is the, then how to get the natural frequency? Natural frequency will be again we can find out this as g by delta where the static displacement, but what is how to get the static displacement. Here of course, we see that when we take only this portion, then there will be a force W by 2 and there will be a moment acting M. Then this what is this M? This M will be such that the displaced the slope at this point will be equal to 0. So, M and W these are not unrelated, but they will be related according to certain consideration certain formula.

Now, to get that we again use the principle of the elasticity and there is one important theorem known as the Castiglianos theorem. Castiglianos theorem which is very useful theorem in finding out the displacement of the function of the deformation of a structure knowing its strain energy. So, here in according to this theorem actually what will happen is that the force developed; if we apply a force F and if we apply a moment M, there then the displacement in this direction vertical displacement that is it will be displacing this way. So, the vertical displacement delta this is the where this is the strain energy

Similarly, the angular deformation angular deformation will be del U del M, the partial derivative of strain energy with respect to M. Now delta this is actually if this is a linear system which it is then in that case, it will be a linear combination of F and M. So, a 11 F plus a 12 M and similarly this delta theta will be also a linear function of M and this will be a 1 2 f plus a 2 2 M.

Now, as I said that because this is equal to 0 so, it gives a relationship between F and M. Now with this relationship we can find out delta. So, Castiglianos theorem can be used not only for this kind of situation, but for very complicated situations and ultimately we can get the get the value of delta here. So, while knowing the value of the delta we can find out omega n ok.

So, so, we have we have now seen the vibration of axially loaded component with lumped parameter model with the simplified boundary condition that was very important and we have estimated to some extent the natural frequency of the system.

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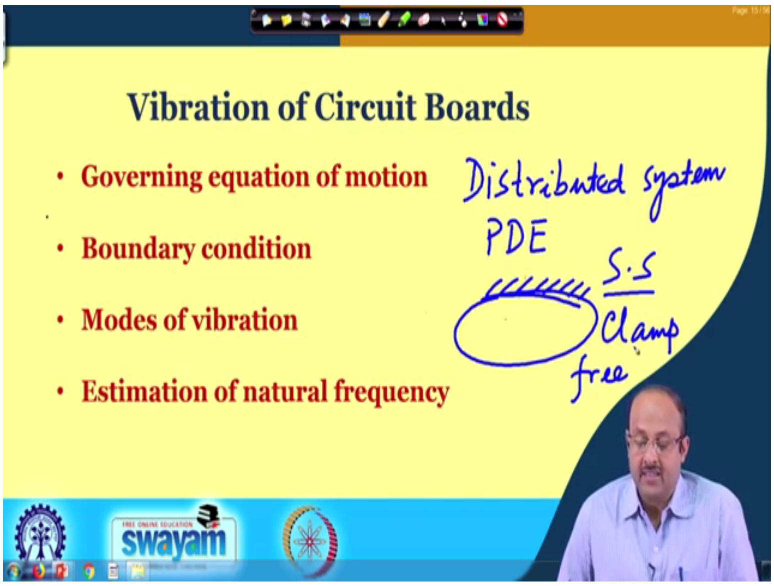

Now, let us now let us go to the vibration of the circuit board which is a very important component of a an electronic equipment. And the circuit board just like a continuous system it its vibration characteristics can be quite complicated.

First of all circuit board is actually a distributed parameter systems. Unlike the axial leaded component which we have considered earlier where the mass is concentrated at one particular point and the other member that is the wire or the lead was considered to be a massless beam whereas, a circuit board is actually a distributed mass system. So, the circuit board here the governing equation of motion.

So, we can consider this to be; so, actually it is a. So, you have a distributed system distributive system and for the distributed system the governing equations of motion, it is a PDE that is Partial Differential Equations. The other type of systems the lumped parameter systems, the equation of motion was ODE, but now this is partial differential equation which is quite complicated one and the solution is also very complicated.

The boundary condition is the most important part here. The boundary conditions can be different because if we have a continuous systems. So, if the boundary of this one can be many things many like here, this may be a simple supported SS boundary condition, Simply Supported boundary condition. It can have a clamped boundary condition or it can have a free boundary condition.

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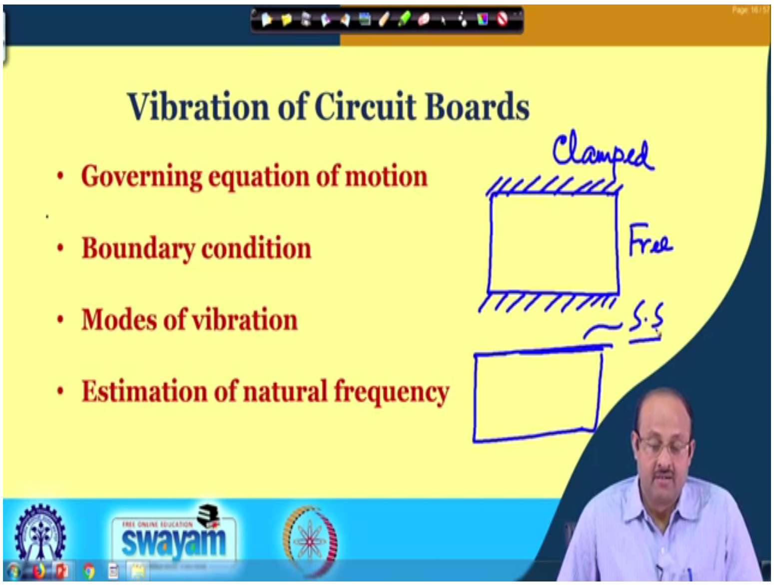

So, in a PCB actually you have pcb you have the different edge connectors. So, a PCB can have 2 sides clamped or may be simply supported this side may be free. So, this is free this is clamped this is free if there may be a simply supported condition simply supported condition.

So, boundary conditions are quite different. The third aspect of this is that it is actually continuous system. So, it will have a number of modes and number of natural frequencies in the single degree of freedom system which we have considered earlier, there was only a single natural frequency because there is a single mass and a single spring.

Now here it is a distributive system. So, we can consider to be a system comprising of infinite number of masses and infinite number of springs. So, these are infinitely dimensional infinite dimensional system and it will have different modes of vibration. Now what is modes of vibration? Modes of vibrations are actually a specific configurations of the vibrations in which if you allow the system to vibrate it will not go to the other configuration. So, it is very characteristics of the system that is if left to itself it will not go to other modes.

Again we are going to consider the natural frequencies or we are going to estimate the natural frequencies of the system because it is very important from our design point of view. We are now going to see the vibrational aspect of the PCB or the circuit board.

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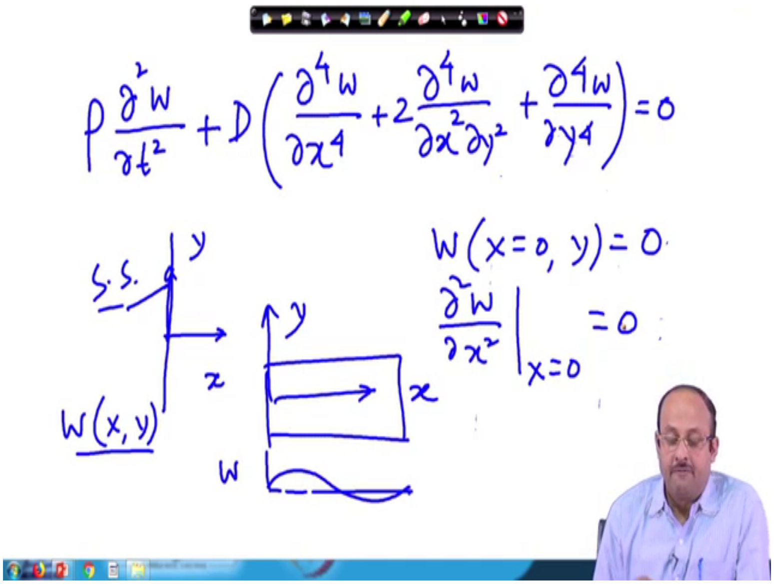

Now, the governing differential equation is actually a partial differential equation and the partial differential equation can be written this way that is the rho del 2 W del t square this plus d del 4 W del X 4 plus 2 del 4 W del x square del y square plus del 4 W del y 4, this is equal to 0 for the free vibration problem. And then the solution of this problem is quite difficult.

The boundary conditions are again quite important. Boundary conditions for the simply supported boundary conditions for example, if we have a simply supported edge. So, if this is a simply supported boundary condition you see that according along this direction and along this direction suppose this is x and that is y direction so, here if we consider W as a function of X and Y where W is the transverse deflection of the plate. So, if we consider the who are there and this is the first traverse deflection W. So, this is x that is y and so on.

So, in the simply supported boundary condition we have W at X equal to 0 for any Y that is equal to 0 and del 2 W del x square at X equal to 0 this is equal to 0.

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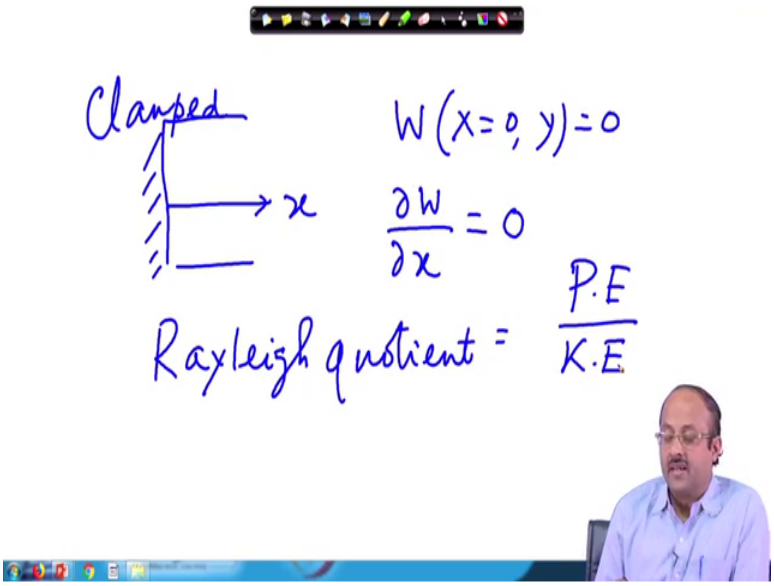

If we have another other type of boundary condition. So, if you consider clamped boundary condition clamped boundary condition then we can check that the boundary conditions will lead to this W X equal to 0 Y this is equal to 0 naturally because there is no displacement along that line. And then the slope along this direction will be 0 because there is so, slope along this direction will be equal to 0.

So, these boundary conditions are to be considered. Now with this boundary conditions and the governing equation, we have to find out the natural frequency. Now getting the natural frequency is a difficult task, but there is certain estimation procedure. One estimation is done to the Rayleigh quotient which is equal to the potential energy which is the strain energy and this divided by the kinetic energy.

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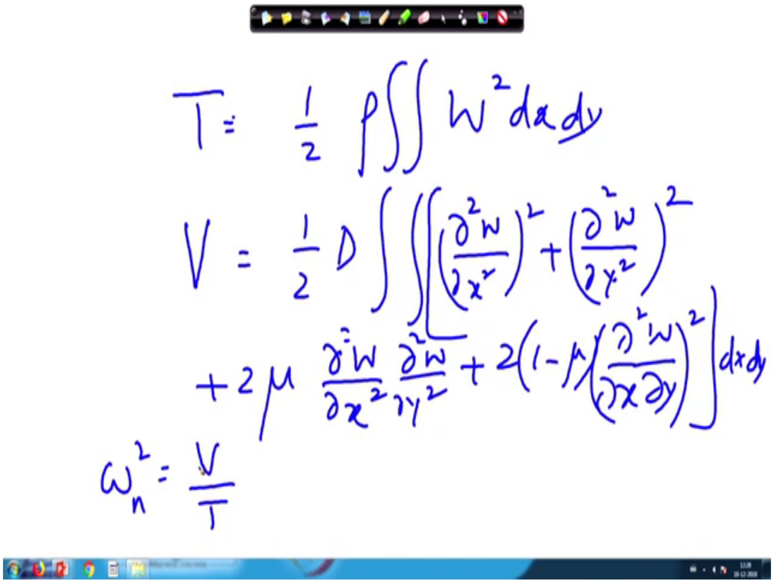

Now, for this case the kinetic energy kinetic energy T this will be equal to rho half rho and W square dx dy double integral and the potential energy this will be equal to half D and this is the complicated function which I am writing down del 2 W del x square square plus del 2 W del y square square plus twice mu which is the Poissons ratio del 2 W del x square del 2 W del y square plus 2 1 minus mu del 2 W del x del y square and dx dy.

So, knowing this expression and then by T we can find out the natural frequency. So, the natural frequency definitely depend upon the, it depend upon depend upon the boundary conditions. So, this is how it the control the vibration of the circuit board can be can be can be known and calculated.

So, here in this lecture, you have find out the, you have seen the how to calculate the vibration how to how to calculate the natural frequency of the system and for the printed circuit board that is continuous system and for axial loaded member which is the lumped parameter system. In the next class we are going to learn how to control the level of the vibration of the electronic package system and that will be the end of the particular module on the vibration analysis.

Thank you very much.