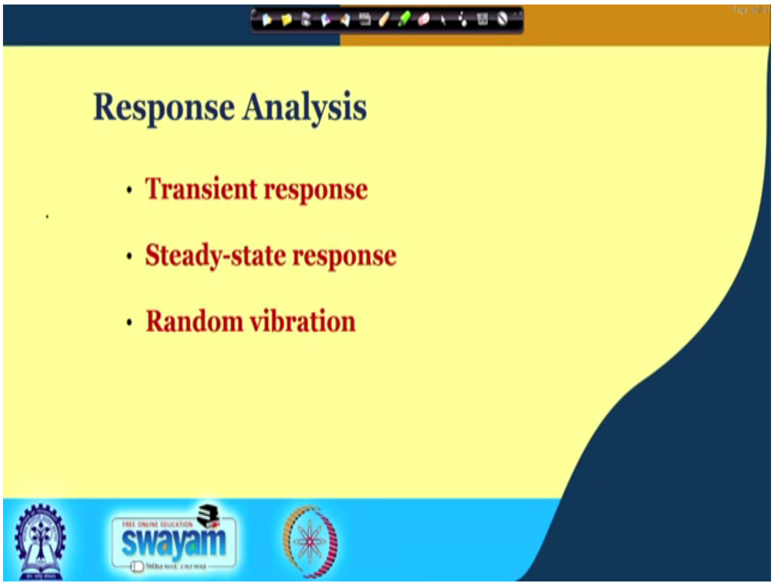
Electronic Packaging and Manufacturing
Prof. Goutam Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 32
Shock and Vibration – 2

Hello friends welcome to the course on Electronic Packaging and Manufacturing; this is Vibration analysis 2. In the last lecture you have learnt about the importance of vibration analysis namely the vibration testing. And, you have also known that the vibration testing is quite important from the reliability point of view. The because of vibration the electronic component specifically the joints may fail due to fatigue or due to excessive deformation or may be due to the excessive displacement leading to the collision of the components.

Now, in this lecture we are going to learn something about the basics of vibration analysis. So, here in this course you will in this lecture you will learn mainly these following points: the transient and steady state response, force transmissibility and motion transmissibility these are very important concepts which we will learn slowly.

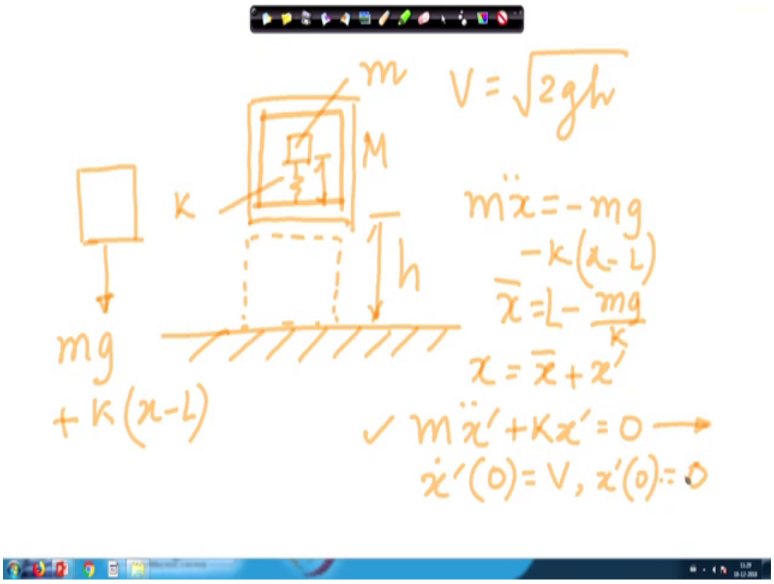
(Refer Slide Time: 01:24)



Now, first of all and in the response analysis we have a vibrating body and which is subjected to loading. Now, this loading can be from outside or that is from the force or from the displacement. So, in the first case we will call forced loading and in the second case we will call displacement or the motion loading.

Now, during this loading process the transient response also plays certain role as well as the steady state response. In the steady state response normally the excitation level can be different. And, in one type of excitation which is very important from the electronic packaging point of view that is the random vibration. Let us learn something about the response analysis.

(Refer Slide Time: 02:23)



So, let us first consider the response of an electronic package which is which is in the form of a spring and the mass. And, another casing which may have certain substantial mass; let us consider this mass to be capital M and this mass to be small m whereas, the spring stiffness is K. Now, suppose that this is getting dropped from a height h ok. So, in that case when the body falls then it acquires the velocity V which is equal to 2 g h. Now, after it touches the ground then it may it may rebound back or it can stay for some time.

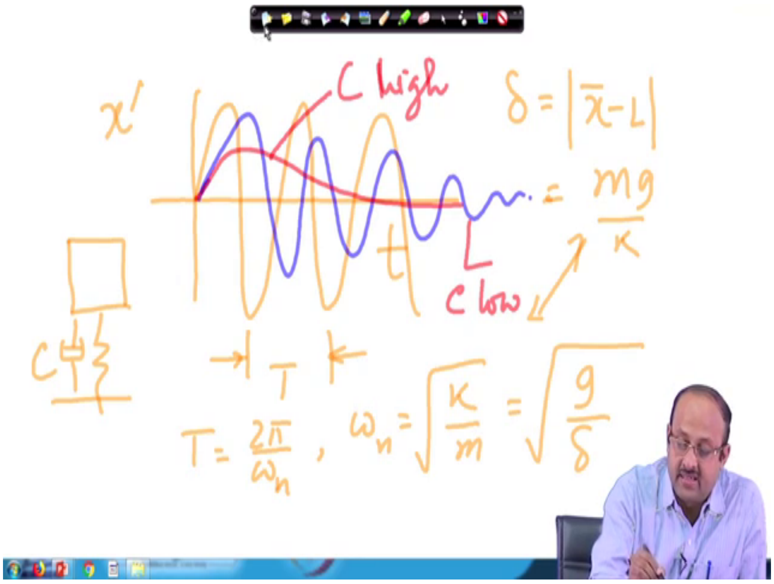
Now, as it so, this depends upon the nature of the collision; supposing that the box remains stationary when it comes to this position. Then the equation of motion of this particular mass will be written as m x double dot, this is the mass times acceleration. And, that is the equal to the total force and for this mass the total force will be equal to mg which is the weight and plus K and x minus L, where L is the free length of the spring. Now, so you write down this 1 mg minus K and x minus L.

So, it comes with a velocity V and then after that when it reaches the bottom most point, that is the box touches the ground then it starts vibrating. After that there is almost no force, but only the force due to gravity which is a static force. Now, you will see that for some value of x x dot and x double dot will be equal to 0 and that value will consider to be the equilibrium value. And, that is equal to minus mg this is a x equal to L minus mg divided by K. So, this is the value well where x double dot will be equal to 0 and this will consider to be the equilibrium position.

So, what will happen because of this self weight it will have in the equilibrium it will be displaced and the spring will have certain deformation. Here of course, I measure x as the distance from this bottom point to the to the mass. So, this is the equilibrium position and if you consider the displacement around the equilibrium position, that is this plus x dash say. Then the equation of motion around the equilibrium position could be written as m x dash double dot plus K x dash equal to 0. So, here you will see that this is the equation of motion of free vibration and it has its velocity.

So, velocity what will be the velocity? The initial velocity is x dash dot at t equal to 0 this is equal to V; the velocity with which it is touching the ground and, then the displacement x dash 0 is let us say its displacement will be equal to x dash. So, some displacement which I which I called to be let us say 0. Now, what is going to happen in that case; so, this becomes actually the equation of motion of vibrating system just single degree of freedom system with a mass and the stiffness K. And, this is the free vibration problem with the initial velocity V and the displacement 0.

(Refer Slide Time: 07:09)

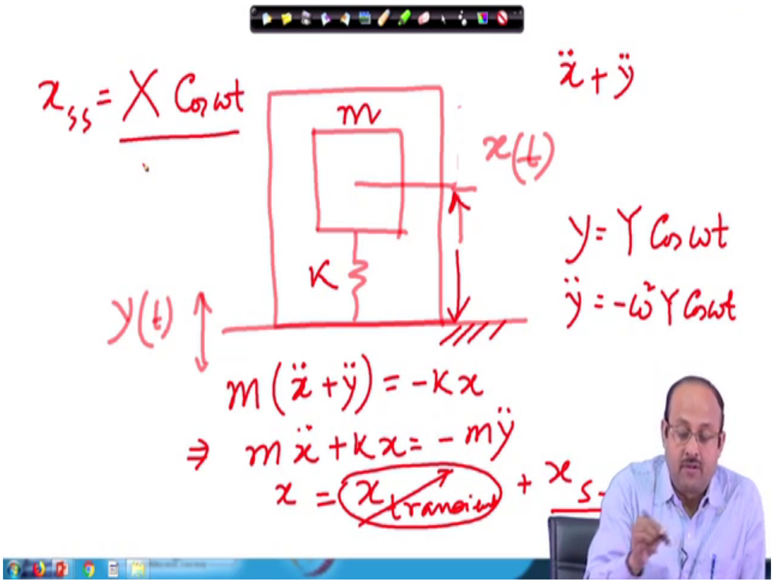


Now let us see what is going to happen to this. So, here so, if you plot x versus t x bar versus t then what is going to happen, because of its velocity it will start oscillating and it will keep on oscillating. So, this is the time period of oscillation which is T and that is T is equal to 2 pi by omega n where, omega n is the natural frequency under root K by m. And, you have seen that if this is the delta is the static displacement; then this is equal to actually the static displacement it was equal to x minus x bar minus L and which is equal to m g by K.

So, from these two results we can see that this will be equal to omega n is under root under root g by delta where, delta is the static displacement due to its own weight. Now, what will happen if there is a damping; remember that I took only the spring and the mass. Now, if I take another damping over there then depending upon the coefficient of the damping, then two things may happen. Either the damping, if the damping is low enough then what will happen is that this will keep on oscillating, but it will eventually die down.

So, this is the case for the under damped situation and if the damping is quite large. For example, here then it may so, happen that it will decay without oscillation; so this first this is the case of undamped system, this is this is the under damped that is C is low and this one is the over damped C is high. So, this is so far as the stationary, that is the transient response is concerned.

(Refer Slide Time: 09:45)



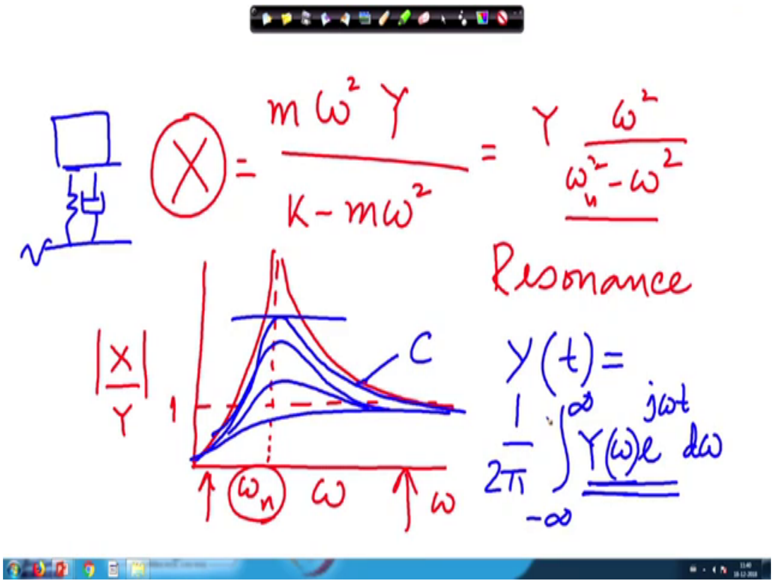
Now, when we have a steady state steady state motion; that means, let us consider now the situation where we have again similar box and this is actually the box. And, this is connected to a table and this table is actually getting vibrated with a certain motion y t, then what is going to happen. So, here if I consider x t and this x t suppose this x t I measured not from here, but let us say this is our x t that is the measure from the base.

Then the equation of motion equation of motion of this case will be equal to m. The absolute acceleration of this body absolute acceleration will be x double dot plus y double dot. Because, x is the relative x double dot is the relative acceleration with respect to this moving base and y is the displacement of the base. So therefore, the absolute acceleration will be double dot and that is balanced by the force minus K x and, this gives rise to this equation m x double dot plus K x equal to minus m y double dot.

So, this is the generic situation when we have the base moving with certain acceleration. Now let us consider the case the simplest case is y is equal to Y let us say cosine omega t; that means, that is we have the harmonic excitation given to the base. In this situation y double dot will be equal to minus omega square Y cosine omega t. And, then to find out this x actually x the response will have two parts: one will be the transient response and another will be the steady state response. Now, eventually what will happen that this transient response will go to 0 because, there will be always certain damping present.

Even if the damping is very small so, that the steady state response is not sensibly affected by it, but there will be always certain damping and that will lead the x transient to go to 0. What remains is the steady state solution. Now, the steady state solution we express the steady state s as let us say, X cosine omega t in this particular form. Why this cosine omega t, that is because since there is no damping. So, these two motion that is y and x they will be either in phase or out of phase. Now, it is a very trivial exercise to substitute this into the equation of motion and find out the value of x.

(Refer Slide Time: 13:12)



Now, what will happen to this is this X will be equal to then omega square Y m omega square Y and this divided by K minus m omega square. So, this after writing it will be Y times omega square divided by omega n square minus omega square. So, now you see that the X this one will behave something like this, if you plot mod X. There is a possibility that the denominator here becomes negative.

So, in order to avoid that we consider only the modulus when it is negative it is actually out of phase motion. And, when it is positive then Y and X these are in phase motion; now what will happen when omega is very small. So, if I place this with omega then what is going to happen when. So, when omega is very small then it is approximately 0 whereas, when omega is quite high then it reaches this value 1, this actually this divided by Y let me consider.

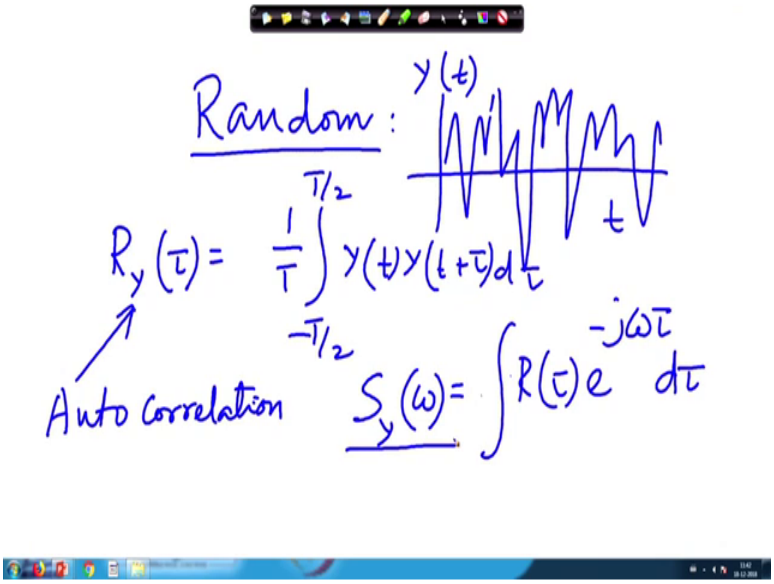
So, you will now see that when the excitation frequency is near the omega then omega n, then actually the value of X becomes infinitely large. Actually what happens then there it becomes their resonance. So, there is a phenomena resonance which is very very important from the vibration point of view. And, during resonance what will happen the response will build up to infinity. Otherwise the response will be bounded and we have seen that when here when excitation frequency, let us say in this region the X value becomes quite small whereas, if here it is omega then X is of the same order of magnitude as Y. In fact, when omega tends to infinity then X and Y becomes same. So therefore, entire vibration is transmitted to the to the body.

Now, what we will see later on that this X has something to do with the force which is being experienced by the spring. And, in electronic packaging this spring will come in the form of the lead or other connections of the equipments to the packaging level. So, so that will play a certain significant role. Now, what will happen when the damping is included, then the damping in is included then actually this becomes changed. And, we have in that situation if we have small value of damping then the response will be something like this it will be bounded.

So, it is not unbounded like this; so, we have certain value of damping and if the damping is quite high then it becomes like this. So, this is the high value of damping intermediate there will be certain situations like this. So, with if you increase the value of the damping here then damping means that, in this case I had the system. And here there is a damper and this base is excited. Now that is so, far as the steady state motion is considered with the harmonic forcing.

Now, if you have any arbitrary forcing let us say y t which is continuous, then we know that according to Fourier transformation then y t can always be expressed as Y omega and e to the power j omega t d omega. So, in this particular form Y omega is the is the Fourier let us say Fourier transform of the function y t. So, if the system is linear and we have already considered this part that is the harmonic response due to that. So, using the principle of superposition we can always find out the response due to any arbitrary loading; so, far as this is continuous, but a different situation happens when we have a random loading.

(Refer Slide Time: 17:58)

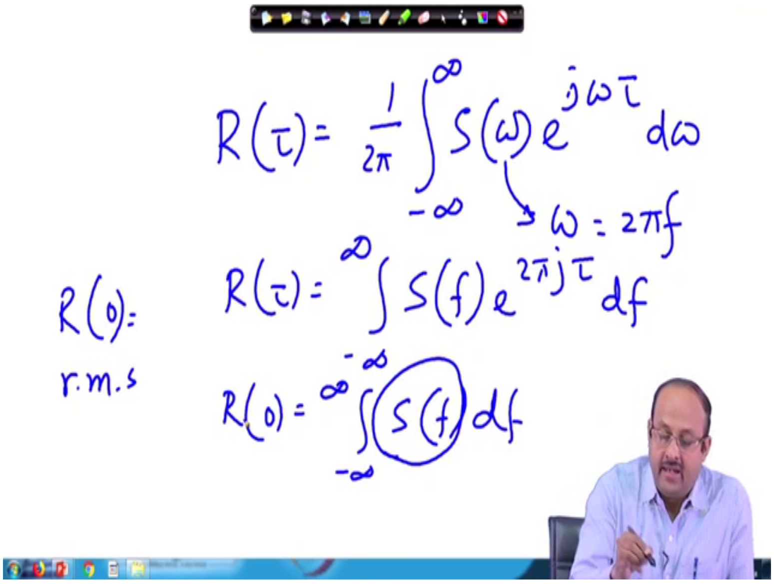


That is if we have the random signal, if you have random signal then what happens this consideration does not work because, the Fourier transformation of the random signal does not exist. And then so, in this case what we are considering a signal y t which is completely random and we do not know how does it vary. But, there is a technique of extracting certain continuous functions out of this random signal. What will happen that if we consider let us say a function y which is known as the autocorrelation function.

And, this autocorrelation function is defined this way minus T by let us say y t y t plus tau d tau. And this is the autocorrelation function autocorrelation function. So, this autocorrelation is a very well behaved function. If we consider the Fourier transformation of the autocorrelation function then actually we will get S and this, this will be equal to 1 by the Fourier transformation. So therefore, this will be this 1 by 2 pi will not be there. So, Fourier transformation of this signal this will be equal to R tau e to the power minus j omega tau d tau.

So, this is actually known as the power spectral density. So, power spectral density; if we take the inverse Fourier transformation then that becomes R tau.

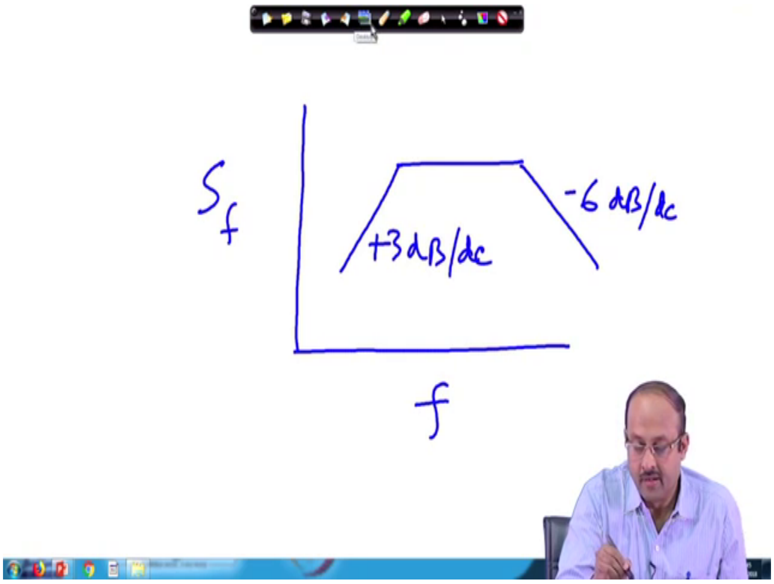
(Refer Slide Time: 20:03)



This is equal to 1 by 2 pi and S omega e to the power j omega tau d omega minus infinity to infinity. Now, sometimes instead of omega which is the circular frequency we write down in terms of the frequency hertz. Then in that situation R tau will be equal to minus infinity to infinity S is a function of f and e to the power 2 pi j j tau and d f.

Now, R at 0 if you refer to the original definition R at 0 this is actually the rms value. So, rms value of this function this is nothing, but minus infinity to infinity S f and d f. So, this part is actually the power spectral density function. And, the importance of the power spectral density function is that as the; if you integrate the power spectral density function over the frequency then you will get the rms value of the function.

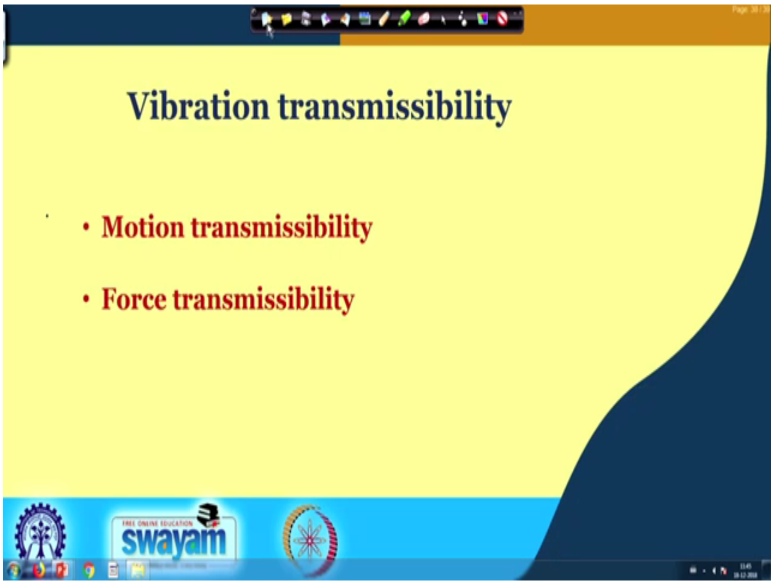
(Refer Slide Time: 21:26)



Now, in many situations when we have rms value that is in many cases the y is specified in terms of its rms value. If we have f then the rms value of this thing; that means, we consider S f. And in many cases the function is something like this for example, when we in a in a standard testing protocol will have S f function is kind of this is increment is 3 dB maybe this is minus 6 dB or so.

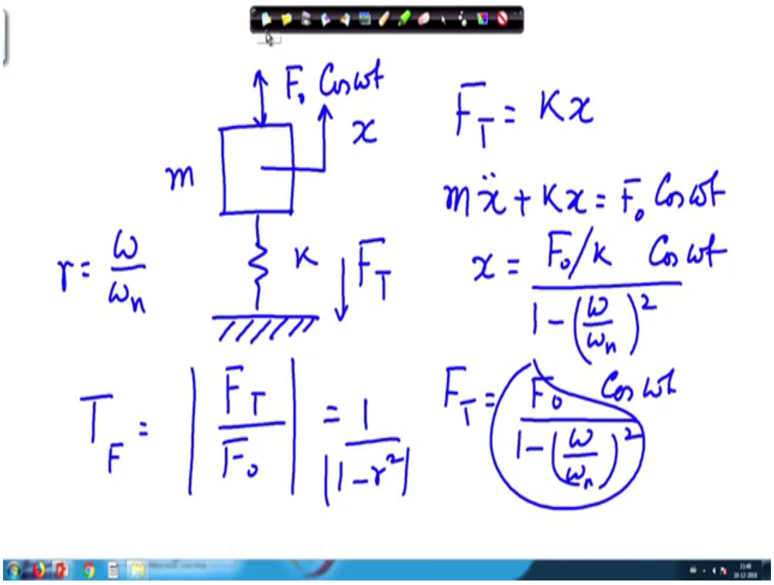
And, dB per decade or decade and this is a constant function. So, it is a typical S f curve; knowing this S f functions we can find out the response using the standard theory of random vibration which is of course, something which is quite deep and I am not going to go into those details. Now, coming back coming back then to this analysis. So, we have considered transient response, steady state response and the random vibration.

(Refer Slide Time: 22:36)



Now, then comes the vibration transmissibility. Now, what is vibration transmissibility? That is from the name it suggests that it is something which is transmitted. Now, there are two kinds of transmissibility: one is motion transmissibility and other is force transmissibility. These two are quite important, let us now learn them one by one.

(Refer Slide Time: 23:03)

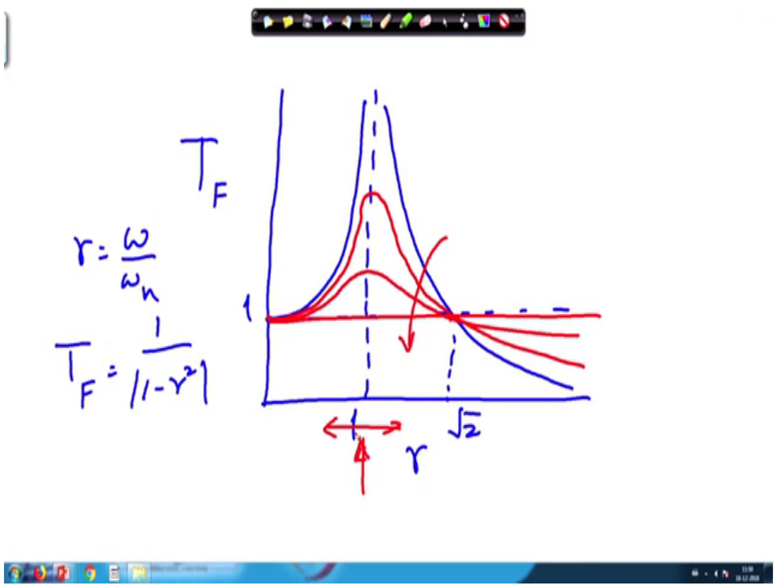


So, let us first consider the force transmissibility. We have a vibrating system and here there is the spring and the mass. And, this is supposed excited by harmonic force F naught cosine omega t. Now, if you write down the force transmitted; that means, what will happen that during the vibration this spring will be deformed. And, this spring deformation will transmit certain force to the ground. This is quite important from the electronic pocket packaging point of view. Because, when the component vibrates it transmits certain force to the ground.

And, these force is actually essential for development of the stress and that stress if it is quite large then it may lead to the breakage of the component. Now, when you have the transmissibility that is the force transmitted here; now, the force transmitted in this case actually we can find out the force transmitted F T as K x and x you can find out from solving this equation m x double dot plus K x equal to F naught cosine omega t. And, here you have x t one can find out this as F naught divided by K. And, that is equal to 1 minus omega by omega n square cosine omega t.

So, here in this case you can find out this F transmission F transmissibility this will be equal to F naught cosine omega t divided by 1 minus omega by omega n square. Now, the transmissibility T F the force transmissibility is defined as the magnitude; that is the amplitude of the force transmission divided by this force F naught. And, which is seen to be equal to 1 divided by 1 minus r square. So, this is where r is equal to omega by omega m, it is called the frequency ratio.

(Refer Slide Time: 25:28)

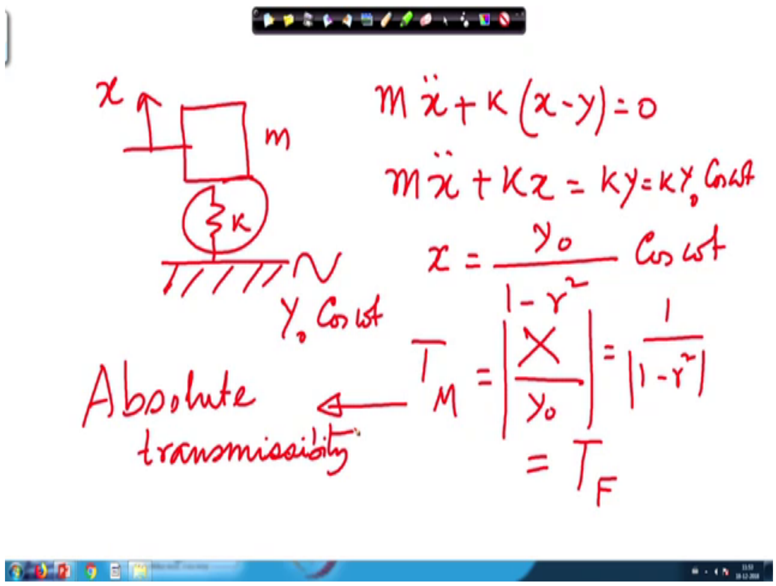


Now, let us see how the transmissibility behaves. So, if I plot omega by omega n r equal to omega by omega n and here it is transmissibility T F. So, transmissibility is 1 by 1 minus r square. What will happen when r is approximately 0? Then it will start from 1 and when r is equal to 1, it will go to 0. Interesting thing is that when r is equal to root 2 then also it becomes equal to 1. So, the curve will be like this and so on. So, this is the point root 2 and that is the point 1.

So, you see that at the high frequency excitation the transmissibility becomes low whereas, during the resonance the transmissibility becomes quite high. Now, what will happen if damping is incorporated? Then the damping is incorporated things may change, you will see when the damping is quite large then damping is quite small then what will happen that this transmissibility will be like this. So, again it is interesting to note that for any value of the damping the transmissibility will be equal to 1 when r is equal to root 2. If the damping is higher then something like this will happen, if the transmissibility is infinite then it will be 1.

So, if damping is increased then we see two things that is the transmissibility decreases in the resonance whereas, in the high frequency excitation the transmissibility increases. So, to reduce the transmissibility that is the force transmitted to the ground we need to we need to have low damping; if the frequency is very large compared to the natural frequency. Whereas, we have to have a very high damping if the frequency of excitation is very near to the resonance condition. So, this is important thing. Second we come to the motion transmissibility, now what is that?

(Refer Slide Time: 27:43)

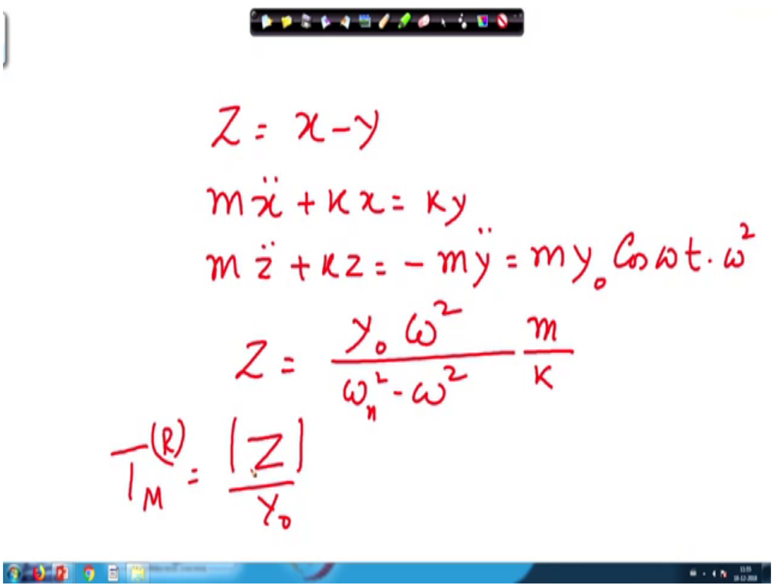


Here we have similar situation that is we have the mass, we have the spring and then this is the moving part which is excited cosine omega t. The equation of motion at this point we consider x to be the absolute displacement of the of the body with respect to certain inertial reference frame. Now, in that case the equation of motion will be m x double dot plus K x minus y that is equal to 0 and in that situation x minus y so, the so, the x minus y equal to 0. So, m x double dot so, in this situation actually you will get again the same thing m x double dot plus K x equal to K y which is equal to K y naught cosine omega t.

And in that case the steady state response once again you can find out that this is to be equal to y naught divided by 1 minus r square cosine omega t. So, here the motion transmissibility T M motion transmissibility will be equal to the response amplitude. That means, this part that is the X divided by y naught and that and this is equal to 1 by 1 minus r square.

And so; so now we have seen that this is the same thing as the force transmissibility that is a it is not very coincidence, but it is true only for the linear system. If you have a spring non-linear spring then for example, or if there is any non-linear damper which may be always present in that case the T M and T F will be totally different. This is here we call this absolute motion transmissibility.

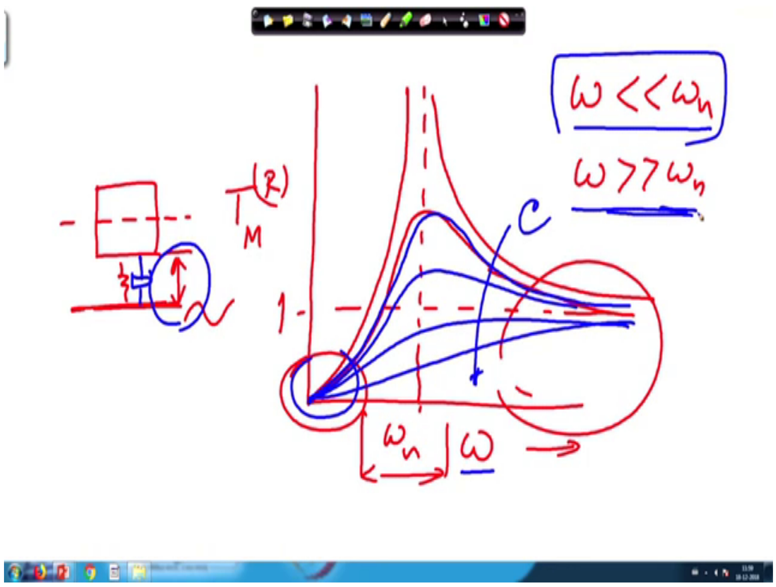
(Refer Slide Time: 30:18)



We can have also a relative transmissibility, relative transmissibility is like this there; we have the relative displacement x minus y. And, the equation of motion which is the original one that is K x equal to K y we can write down this way m z double dot plus K z equal to minus is equal to minus m y double dot. So, this is the way we can always write down and then this ones become y naught omega t and omega square. So, there is an omega square term which will be appearing.

In that case the z that is equal to z will be equal to m and the same will cancel out. So therefore, this will be y naught omega square; this is equal to 1 omega n square minus omega square. So, that is what we have here; now again we now see that the motion transmissibility and of course, will have in that case m by K. So, this part so the motion transmissibility will be now the z by y naught that is equal to the motion transmissibility, but this is the relative motion transmissibility.

(Refer Slide Time: 31:58)



If we plot the relative motion transmissibility then this becomes like this in the low frequency region it goes like this and in the high frequency region it will be going like this way. So therefore, this is T motion with the relative transmissibility and here this is omega. Once again we see that at the resonance condition the; that means, that when omega is equal to omega n in this region the relative transmissibility is quite high and very high. In fact, in if damping is not there it is actually infinite.

In the low frequency region T M is actually going to 0; that means, we will have a favorable situation in the low frequency condition. And, remember that this frequency condition it is a relative one; that means, it is it means that omega must be less than omega very much less than omega n that is in this zone. If omega is quite large compared to omega n, that is in this particular region then we have a different scenario. We have that the transmissibility relative transmissibilities approximately going to 1; that means, the base one the base and the mass these two are having the base motion and this motion is actually having the same thing.

That means whatever motion is being there this vibration is actually of the same; that means, this is not moving at all. There is a phase difference, but now let us consider the damping. If damping is present if damping is present then again similar thing may happen. So, damping is present this will happen and similarly like this if for a very high damping will have this situation and so on. So, this is in the direction of the increasing damping; that means, here I have introduced one damping with certain coefficient.

Now, from the vibration point of view transmissibility is very important concept because we have to design the component or design the packaging or the design the connection in such a way that the motion that is generated in the base should not be transmitted to the equipment. Or, what is also important is that the force generated in this in this place will be small and that will depend upon the relative magnitude of omega and omega n. So, suppose we want that the transmitter there is a motion and we want that this vibration should not be transmitted here. Then we can consider two things, that is if the relative motion we want to reduce then we have to design the system such a way that omega n must be much much much omega n should be much high compared to omega.

So, that it is in this region whereas, if omega when the when in the other situation what will happen, that this will become the vibration will be very large. Whereas, when we want to reduce the absolute motion of the body that may sometimes lead to the lead to the collision of the systems; then we will have to design in a different scenario. So, here you have learnt then the motion transmissibility and the force transmissibility vibration and so on. So, in today's lecture you have learned how to analyze the vibration of a system with the with the force, without force with random forcing.

And, you know the important concept of transmissibility. And, in next lecture we are going to use this particular concept and see how this can lead to a better design; so this part for today.

Thank you for your attention.