

Electronic Packaging and Manufacturing
Prof. Anandaroop Bhattacharya
Department Mechanical Engineering
Indian Institutes of Technology, Kharagpur

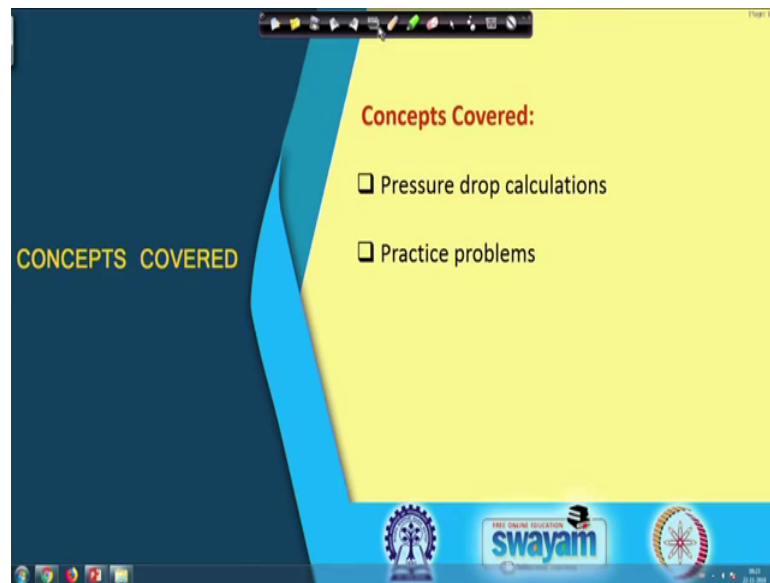
Lecture – 28
Thermal Management 7: Practice Problems

Welcome back this is a course on Electronic Packaging and Manufacturing and we will continue our discussion on Thermal ok, this will probably go on for another maybe three lectures. The first one today what we will talk about is pressure drop calculations because remember last in the last class we looked at various correlations and this was all about finding the heat transfer coefficient especially for convective heat transfer ok. So, we had correlations for heat transfer coefficient in the non-dimensionalized form which we call Nusselt number.

So, we had Nusselt number correlations for flow over flat plate, flow through a tube, flow over a cylinder natural convection. We also talked about you know parallel plate heat sinks especially natural convection why there should be an optimal fin spacing ok. Now in all of these which I said before when I first talked about you remember the fan curve and system impedance curve, we said that you know what happens is if you keep on increasing number of fins. In other words you make each of these channels smaller in order to accommodate more of them in a given cross sectional area the pressure drop does increase the flow resistance increases.

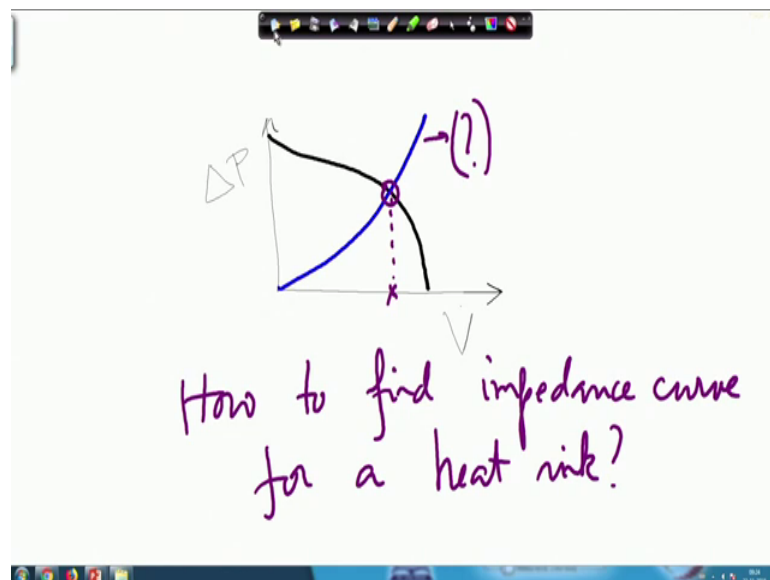
So, nothing comes for free ok, you pay a penalty in terms of pressure drop the pressure drop goes up which is a penalty, but then your thermal resistance comes down right. And we also said that when you have a fan curve and a system impedance curve the operating point corresponds to the point of intersection which for a system with higher impedance will correspond to a lower flow rate right. So, all these things we had discussed. Now, the next question is that if I give you a heat sink how do you find out the impedance the flow impedance ok, which you are going to plot on the fan curve to get that operating point right.

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When we first started we say that I have the fan curve, I have the system impedance curve and the point of intersection between the two is going to be the, is going to be the operating point remember we had drawn something like this right.

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This was my flow rate and this was my delta P. So, we said that the fan curve looks something like this and the system impedance curve will be something like this. And the operating point is the point of intersection and this will be the flow rate. Now the question is how do you find this system impedance curve? Especially, for a heat sink

with a fan how do I find it? So, how to find impedance curve for a heat sink ok? So, we will talk about that alright. So, let us go back. So, that for the first pressure drop calculations and then we will do a couple of practice problems alright.

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Pressure drop calculations

□ Fully Developed flow through a channel – pressure gradient along the flow direction (x)

$$\frac{dP}{dx} = \frac{f \rho v^2}{D}$$

f is the friction factor

$f = 64/Re_D$ for laminar flow ($Re_D < 2000$)

$f \rightarrow$ read from Moody chart for turbulent flow ($Re_D > 2000$)

$D = D_h$ for non-circular cross-section

$\Delta P = f \frac{L}{D} \frac{\rho v^2}{2}$

The diagram shows a pipe of length L with velocity v and pressure drop $\frac{dp}{dx}$. The Swayam logo is visible at the bottom.

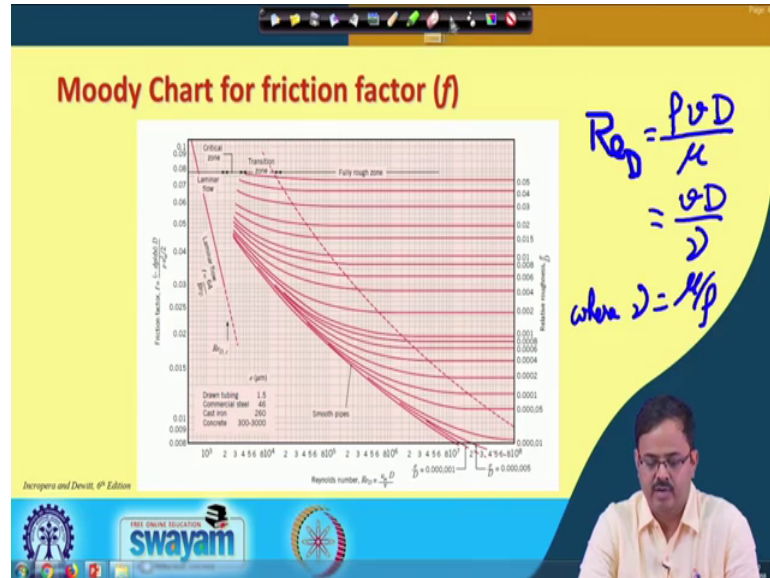
So, pressure drop calculations what is said is if you consider flow through a channel or a pipe or a duct the pressure gradient along the flow direction x is given as $\frac{dP}{dx}$ is f by D rho v square by 2 ok. So, again remember what is D ? D is the diameter if it is a circular cross section or otherwise it is a hydraulic diameter it is a if it is a non-circular cross section. So, D will be D_h for non-circular cross section agree and what is $\frac{dP}{dx}$ that is the pressure gradient.

So, therefore, if I have flow through a pipe in this manner and I have a velocity v sorry, then this pressure drop is given by that expression alright. So, if the length of this pipe is L then what will be the total pressure drop; the total pressure drop which is ΔP therefore, is going to be $f L$ by D or L by D rho v squared by 2 ok.

So, then the next question is that what is this f , what is this f ? So, f is known as the friction factor, f is this friction factor and we have got some expressions for that what are those if the flow is laminar that is Reynolds number is less than 2000. Remember Reynolds number u infinity rho u in rho vD over mu density times velocity times diameter over the viscosity ok. So, 64 over Reynolds number for laminar flow. So, as you can see that the friction factor actually goes down with velocity, but if its turbulent

then we have to take the help of something called Moody chart and what is Moody chart let us look at that.

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This is a Moody chart. So, this gives you f the y axis is f all right. So, for laminar flow which is up to you know around some people say 2000 some people say 2200, but that is what it is. So, for Reynolds number up to 2000 it is 64 over the Reynolds number, but there after if its turbulent then what you can see is you have to read the friction factor from these curves. And the other thing that you want to see over here or you can see over here is it is not just dependent on the Reynolds number, but it is also dependent on the surface roughness ok.

So, if e is the characteristic dimension of the roughness features on the surface inner surface of this tube or pipe or channel and D is the diameter then the friction factor at a given Reynolds number is also going to be a function of e by D ok. And definitely if the surface roughness is more because this is frictional loss at the surface because of frictional pressure drop the drag at the surface ok. So therefore, the more the surface roughness higher is your friction coefficient alright remember again, what was Reynolds number? Reynolds number was equal to $\rho v D$ by μ here in this curve it is known as is the same u is the velocity in the x direction and m stands for mean velocity ok.

So, $\rho v D$ by μ and what is that going to be equal to we can also write it as $v D$ by ν where ν is μ over ρ ok, this is also the ν is also known as a kinematic viscosity ν

is the dynamic viscosity ν is the kinematic viscosity all right. So therefore, what is it if I give you a flow rate if I give you a flow rate then the first thing for flow through a channel or a pipe or a duct first thing we have to do is we have to find out what is the mean velocity. And how do I find that? There is a flow rate divided by the cross sectional area, then the flow rate divided by the cross section area gives you the velocity from there we calculate Reynolds number.

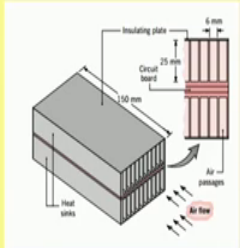
If its laminar you get the value of f if its turbulent you also have to ask for what is the surface roughness, if it is very smooth not a problem and as you meet to 0.0 is 0 and just take the lowest curve. But if there is a surface roughness provided calculate e by D and read out the frictional factor frictional friction factor from this Moody chart go back to the previous curve and find out the pressure gradient and therefore, the pressure drop clear all right. So, with that background what we will do now is sorry.

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Problem -1

An electronic circuit board dissipating 50 W is sandwiched between two ducted forced air cooled heat sinks. The sinks are 150-mm in length have 20 air passages of 6-mm x 25-mm. Atmospheric air at a volumetric flow rate of $0.06 \text{ m}^3/\text{s}$ and 300-K is drawn through the sinks by a blower. Estimate the operating temperature of the board and pressure drop across the heat sinks.

Assuming air properties to be $\nu = 2 \times 10^{-5} \text{ m}^2/\text{s}$, $Pr = 0.7$, $k = 0.03 \text{ W/m-K}$; $C_p = 1.78 \text{ kJ/kg-K}$,



The diagram shows a perspective view of a heat sink assembly. A central circuit board is sandwiched between two heat sinks. The heat sinks are connected by an insulating plate. The circuit board has a thickness of 25 mm. The heat sinks have a length of 150 mm. Each heat sink contains 20 parallel air passages, each with a width of 6 mm and a height of 25 mm. Air flow is indicated by arrows entering from the bottom and exiting from the top. A small inset shows a cross-section of one of the air passages.

We will look into a couple of problems and here what I will do I will not solve them I will just outline the steps and I will ask you to complete the calculations at your at your own time because you need to get that practice all right. So, the first problem is about force convection and electronic circuit board dissipating 50 watts is sandwiched between two ducted forced air cooled heat sinks ok, this is a problem from inked repair and divot. So, this is a heat sink consisting of parallel channels a lot of them as you can see and

there are two such heat sinks one above the motherboard or the circuit board one below the circuit board ok.

So, 2 heat sinks and the air is flowing in this direction as shown in this picture you can assume these to be insulated. So, that most of the heat is dissipated by convection the heat sinks are 150 millimeter in length as shown here and 6 mm by 25 mm is the cross section. 6 mm is the spacing between the fins 25 mm is the height; then what are what we are told is atmospheric air at a volumetric flow rate of 0.06 meter cube per second and at 300 Kelvin is drawn through the sinks by a blower ok.

So, somehow your fan or a blower which blows this air at this much flow rate and temperature why is the temperature important we will see why estimate the operating temperature of the board and pressure drop across the heat sinks. So, the heat transfer rate is given the heat that needs to be dissipated rate that is 50 watts and that is dissipated by convective heat transfer through these heat sinks by flowing air the properties of air is given or a r given. And, we will see what we want what we can do we have to solve the temperature of them of the circuit board and the pressure drop. So, the first thing what we do is we will just as I said I will outline the steps.

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1) Calculate velocity $v = \frac{\dot{V}}{A_c} = \frac{0.06}{20 \times (6 \times 25) \times 10^{-6}} \text{ m/s}$

2) Calculate $Re_D = \frac{v D_h}{\nu}$ $\rightarrow D_h = \frac{4 \times (25 \times 6)}{2(25 + 6)} \text{ mm}$
 - lamina or turb?

3) Find Nu_D \rightarrow find \bar{h}
 $Nu_D = 4.36$ (uniform flux at walls of channel)
 Dittus Boelter correlation $Nu_D = 0.023 Re_D^{0.8} Pr^{0.3}$

So, step number 1 is I have to find calculate I am sorry step number 1 is calculate velocity how will I do that ok. So, let us go back again to this problem I am sorry this let this writing remain there it is not going to hurt us. What is the total flow rate? 0.06 meter

cube per second. What is the cross section area of the flow? You have how many channels see? We have 20 air passages I believe 10 at the top 10 at the bottom each having 25 mm by 6 mm.

So, what is the total flow area? 25 by 6 by times 20 because; 25 mm by 6 mm is a cross section area per channel and we have 20 of them. So, the total flow that comes in at 0.06 meter cube per second will be equally divided into these 20 channels that is what we are going to assume in the absence of any other information that is a very fair assumption ok. So therefore, this is going to be velocity is going to be the volume flow rate divided by area of the cross section. So, which is going to be 0.06 meter cube per second remember. So, therefore, we have to convert the area of each cross section into meter squared 20 is the number of channels 6 into 25.

But these are all in millimeters. So, times 10 to the power minus 6 and you will get that number as meters per second. So, first step is calculate velocity, next calculate Reynolds number and what is that that is this v that I have calculated D by ν . Recall in this problem you have been given the kinematic viscosity 2×10^{-5} meter square per second we do not have ρ b do not have μ what does not matter because μ over ρ is given and that is what we need ok.

But however, this one is hydraulic diameter D_h and what is D_h ? You know 4 times area over wetted perimeter. So, its $4 \times 25 \times 6$ divided by $2 \times 25 + 6$ and millimeter, this is millimeter. So, to convert that into meters, you have to convert that to meters in order to find this the Reynolds number agree and next you have to make this judgment called laminar or turbulent clear ok.

So, if it is laminar you have to find the Nusselt number either way you have to find the Nusselt number ok. So, 3 find Nusselt number, if it is laminar then what is it if it is laminar then Nusselt number D is going to be I believe 4.36 why because this is 50 watts in the absence of anything we will assume that this is you 50 watts is uniformly distributed. So, it is a uniform flux at the walls of channel. Or, if it is turbulent use the Dittus Boelter correlation that is Nusselt D would be $0.023 \text{ Reynolds } D$ to the power 0.8 and parental to the power 0.3 clear. So, you find the Nusselt number D and from there sorry let me go back to the original color find heat transfer coefficient h bar because Nusselt is $h D$ by the thermal conductivity k of the fluid all right.

So, what have I found the heat transfer coefficient average heat transfer coefficient clear. So now, let us go to a new page now. So, I have found the heat transfer coefficient what was the last number 3.

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Handwritten mathematical derivations for heat transfer calculations:

- 4) Calculate $q'' = \frac{50 \text{ W}}{150 \times (6 \times 10) \times 2 \times 10^{-6} \text{ m}^2}$
- 5) $T_b = T_f + \frac{q''}{h}$ $q'' = h(T_b - T_f)$
- 6) $T_{f|_{\text{exit}}} = T_{f|i} + \frac{q}{m c_p}$
 $\Rightarrow T_b|_{\text{exit}} = T_{f|_{\text{exit}}} + \frac{q''}{h}$
- 7) $T_{f|m} = \frac{T_{f|i} + T_{f|e}}{2}$

A diagram on the right shows a coordinate system with temperature T on the vertical axis and distance x on the horizontal axis. A line represents the temperature profile, starting at T_f at $x=0$ and ending at T_b at $x=L$. A point on the line is labeled $30\%h$. Below the diagram, the mass flow rate is given as $m = \rho V$.

So, at 4 therefore, is so now, calculate sorry the heat flux q double prime what is that that will be 50 watts divided by the area of the circuit board. And what was the area of the circuit board? Let us go back it is 115 this direction and over here it is not given, but you can calculate right it is 6 mm and there are 10 such passages. And if we neglect the fin thickness then you get 6 into 10 60 mm you can also assume and state any fin thickness because that will be there. So, 50 watts divided by 150 times 6 into 10 now there are two such surfaces there is a table there is a bottom. So, therefore, times 2 and then this is in millimeter.

So, therefore, 10 to the power minus 6, this is in millimeter this is in millimeter. So, millimeter squared 150 and 6 are both in millimeter. So, millimeter squared and that will give you the heat flux, why is heat flux required because T of the board is going to be T of the fluid or air in this case plus q double prime over h why because q double prime is $h(T_{\text{board}} - T_{\text{fluid}})$ q double prime is $h \Delta T$ clear.

So, I know q double prime I know h I have to find board what is the fluid temperature? Remember and this is where your what do I say that is where your judgment comes in. If I consider flow over through this heat sink what will happen is and this is let us say this

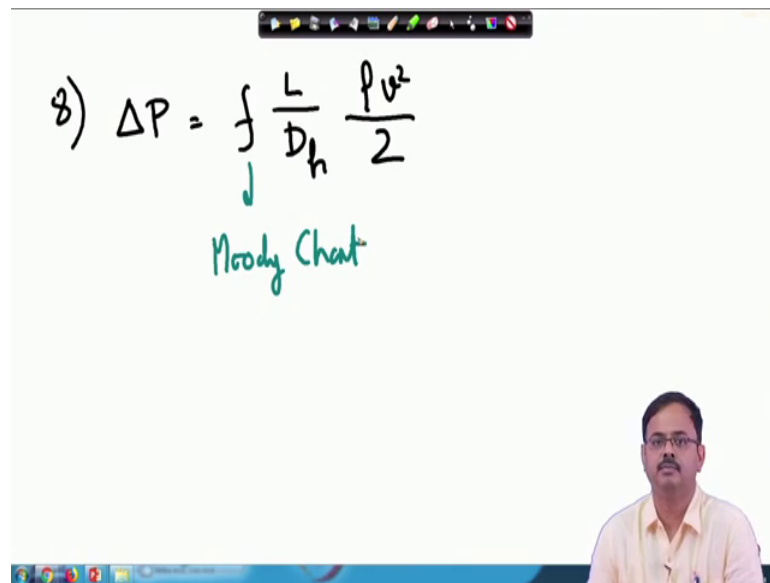
is the direction x in the direction of the fin. Then if I plot the temperature the fluid temperature is going to go up linearly and therefore, the board temperature will also be the fluid temperature plus this is constant.

So, this is going to be the board temperature and the ΔT the difference is going to be q'' by h right. This difference what happens is that the beginning why I stopped here is at the beginning there is something called a developing length, but let us forget all that this is all fully developed flow. So, therefore, this value is given to me as 300 K sorry. So, the inlet temperature is given to me as 300 K, but the maximum temperature of the board will be at the exit.

So, therefore, $T_{f, \text{exit}}$ what is that going to be this is easy because $T_{f, \text{exit}}$ is going to be $T_{f, \text{inlet}}$ plus total q which is 50 watts divided by the mass flow rate times the specific heat we know q is $m \dot{C}_p \Delta T$ and this $m \dot{C}_p$ is going to be again $\rho \dot{V} C_p$. So, therefore, and $T_{\text{board, exit}}$ which is also $T_{\text{board, max}}$ is going to be $T_{f, \text{exit}}$ plus q'' over h agree. The other thing you can do is what is the mean temperature? $T_{f, \text{mean}}$ is going to be $T_{f, \text{inlet}}$ plus $T_{f, \text{exit}}$ divided by 2 and therefore, $T_{\text{board, mean}}$ is going to be $T_{f, \text{mean}}$ plus q'' over h ok.

So, I found the motherboard temperature circuit board temperature whether it is the maximum temperature of the mean temperature we can find that out or at the inlet temperature inlet temperature 300 plus q'' over h . So, this problem that way is not fully defined and I intentionally kept it like that because, I wanted to tell you that it is subject to interpretations alright ok.

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The image shows a whiteboard with the following handwritten content:

$$8) \Delta P = f \frac{L}{D_h} \frac{\rho v^2}{2}$$

A green arrow points from the friction factor f to the text "Moody Chart" written below it.

In the bottom right corner of the whiteboard, there is a small video feed of a man with glasses and a mustache, wearing a light-colored shirt.

The final thing was pressure drop $f L$ by D_h ρv squared by 2 you know everything f we can find from Moody chart ok. So, let us write that, if it is laminar it is easy 64 over 9 ; somebody if its turbulent assume it to be a smooth flight smooth the wall and then find read it out from the Moody chart. L we know 150 mm D_h we have calculated ρv squared by 2 everything is known and that also tells me that the density of air should have been given ok.

So, anyway in a real problem there it is given otherwise you know typical in textbook problems what happens is some of the values will be given some of them, because there are charts at the end of the textbook the person who is solving the problem is expected to go back to those standard property charts and take whatever is necessary alright. So, that is problem number 1 I think we took a long time on this one. But its, I hope you went through all the steps and understood.

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Problem -2

A vertical array of printed circuit boards is immersed in quiescent ambient air at 17°C . Although the components on the board protrude from their substrates on the circuit boards, it is reasonable, as a first approximation, to assume them as flat plates with uniform surface heat flux. Consider boards of length and width $L = W = 0.4\text{m}$ and spacing $S = 25\text{mm}$. If the maximum allowable temperature of the board is 77°C , what is the maximum allowable power dissipation per board?

The diagram shows three vertical boards of length L and spacing S . A temperature profile T_b is shown, with the maximum temperature $T_b \text{ max}$ at the top of the boards. Gravity g is indicated by a downward arrow.

Problem number 2: we will keep it short a vertical array of printed circuit boards is immersed in quiescent ambient air quiescent means it does not move ambient air. So, therefore, this is natural convection problem there is no fan, no air current nothing. Although the components on the board protrude from the substrates and the circuit boards it is reasonable as a first approximation to assume them as flat plates with uniform surface heat flux fair enough. So, we know we have seen that there on the motherboard components are on the circuit board components are placed, but we are assuming that there we will neglect those and assume these to be uniform flat surfaces sorry flat surfaces with uniform surface heat flux the consider the boards of length and width L and W .

So, L is given and the W is in the third direction which is shown here and spacing, S is 25mm , if the maximum allowable temperature the board is 77 degrees what is the maximum allowable power dissipation per board ok. So, remember what happens here if I if I draw the boundary layers recall there is a boundary layer formed over here there is another boundary layer formed over here, they are symmetric by the way I did not draw it well. And so, the maximum temperature of the board this is uniform flux right. So, the maximum temperature happens at the tip this is where will be the $T_b \text{ max}$ that we have to know all right. So, rest is easy let us go back to the last lecture to this curve.

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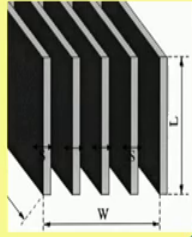
Natural Convection through vertical channels

Isothermal plates:
$$\overline{Nu}_s = \left[\frac{C_1}{(Ra_s SL)^2} + \frac{C_2}{(Ra_s SL)^{1/2}} \right]^{-1/2}$$

Isoflux plates:
$$\overline{Nu}_{s,L} = \left[\frac{C_1}{Ra_s^2 SL} + \frac{C_2}{(Ra_s^2 SL)^{1/5}} \right]^{-1/2}$$

$$\overline{Nu}_{s,L} = \left(\frac{q''_s}{T_{s,L} - T_w} \right) \frac{S}{k} \quad Ra_s^2 = \frac{g\beta q''_s S^4}{\alpha \nu^2}$$

Surface Condition	C_1	C_2	S_{opt}	S_{max}/S_{opt}
Symmetric isothermal plates ($T_{s,1} = T_{s,2}$)	576	2.87	$2.71(Ra_s^2 S^3 L)^{-1/4}$	1.71
Symmetric isoflux plates ($q''_{s,1} = q''_{s,2}$)	48	2.51	$2.12(Ra_s^2 S^4 L)^{-1/5}$	4.77
Isothermal/adiabatic plates ($T_{s,1}, q''_{s,2} = 0$)	144	2.87	$2.15(Ra_s^2 S^3 L)^{-1/4}$	1.71
Isoflux/adiabatic plates ($q''_{s,1} = q''_{s,2} = 0$)	24	2.51	$1.69(Ra_s^2 S^4 L)^{-1/5}$	4.77



So, what we have to do is this is an uniform flux that is what they say. So, we have to take this iso flux correlation and C 1 C 2 is given to you clear 48 2.51 can I do I know everything do i know S yes 25 mm it was I think given the spacing was given the length was also given 0.4 meters or. So, 40 centimeters right S L is known. So, this is going to give me this Nusselt at L. So, which is defined in this form I do not know q double prime S which is what i have to find out I have been asked to find out what is the surface heat flux I do not even know I know TS L because that cannot exceed 77 degrees ok. So, I know everything at the Nusselt number do I know everything in Reynolds number in saline Rayleigh number, here also the only unknown is this surface heat flux ok.

So, you will see that there is only one this is an equation with only one unknown which is q double prime S. So, if you go through it you can solve and find out what is the surface equal axis directly put in the formula the only thing that we need to keep in mind in this problem is the maximum temperature occurs at the top most at the tip at the topmost point all right. Thank you very much, I think these are I just wanted to give you the field 2 problems force convection and natural convection and the rest of it, and once you if you have understood these problems and the basic philosophy and the approach then you should be able to solve most of the other problems ok.

And these are also problems where you have to find out what is the heat transfer coefficient or Nusselt number alright. Except in the second one it was not exactly Nusselt

number it was heat flux directly, but you understand what you what how to solve this. In the second problem, if we had given you instead of uniform heat flux I said uniform temperature then also you could have solved it using the appropriate correlation. So, I think thank you very much that is all for this lecture and we will come back and try to wrap up our discussions on thermal technologies in the next lecture.

Thank you.