Electronic Packaging and Manufacturing
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Lecture – 27
Thermal Management 6: Heat Transfer Correlation

Welcome back to the course on Electronic Packaging and Manufacturing, we will continue with our discussion on Thermal Management and cooling. And today as I had mentioned in the, at the end of the last lecture we will talk about Heat Transfer Correlations. Because, this is what is going to help us in determining what is the thermal resistance as a function of flow rate, as a function of temperature difference depending on the conditions or scenario under which the applet or of the implication that we of the application that we are talking about.

So, heat transfer correlations that is going to be the concepts that is covered that is going to be covered today. Now the thing is this again is probably a recap for people with the heat with the mechanical engineering background who have taken either undergraduate level or graduate postgraduate level heat transfer course. So, but for people who do not have who have not had these courses as part of their curriculum; I will try to explain some of the terms. So, apologies if it is coming at if it is going to come across as something repetitive to the students with mechanical engineering background alright. So, concept going to be covered today is heat transfer correlations.

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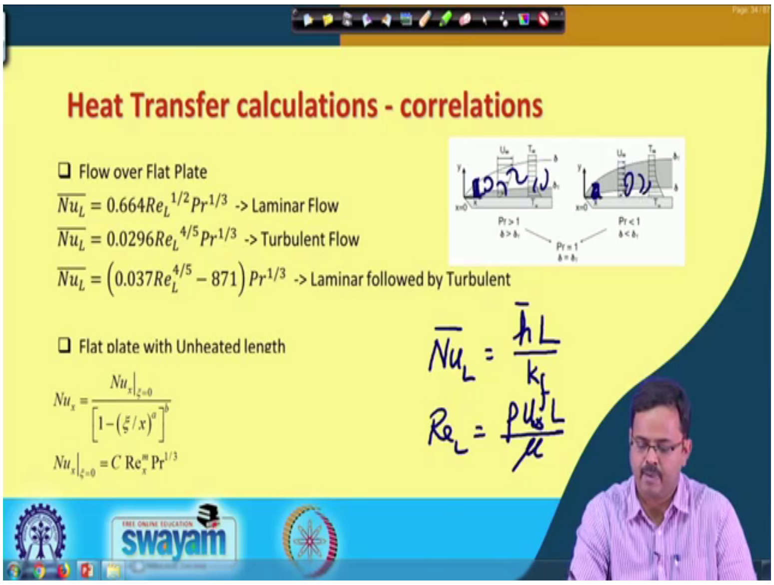

So, heat transfer correlation before that, what is it going or before going into some of these correlations let us first talk about what is the correlation and what is when I say correlation.

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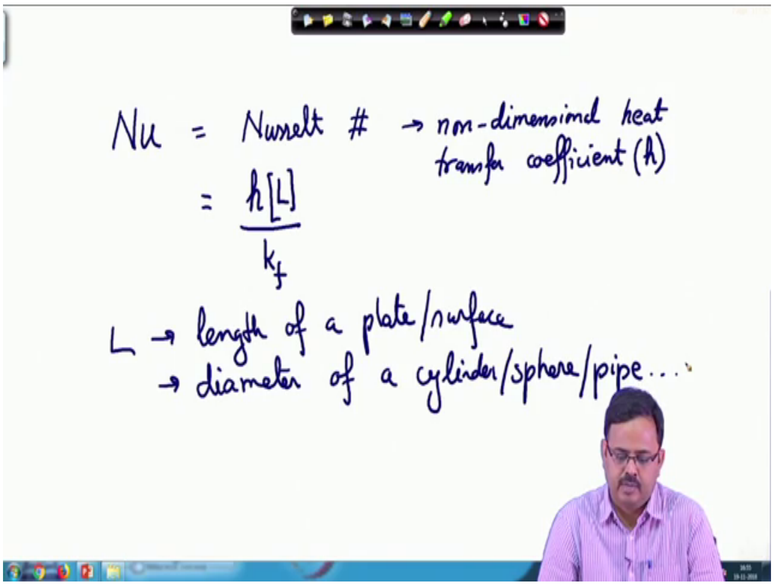

So, most of the correlations that we will see is going to be in terms of a parameter called Nu and Nu is something called a Nusselt number. Now, Nusselt number what is it? It is a non-dimensional I am going to try to keep this as simple as possible, it is a non-dimensional heat transfer coefficient. So, what is Nusselt number? It is non-dimensional heat transfer coefficient. So, what we do is we take h and then multiply it with some length scale which is L and divide it by the fluid thermal conductivity k f. You see if you just look at the units this is non-dimensional because, h is what per meter squared Kelvin and thermal conductivity is watts per meter Kelvin.

So therefore, h times a length divided by thermal conductivity is non-dimensional; now depending on what is L? L can be length of a surface of a plate surface, L can be diameter of a cylinder sphere pipe channel etcetera ok. So, Nusselt number is defined in this manner ok. So now, I am going to define three other terms non-dimensional terms.

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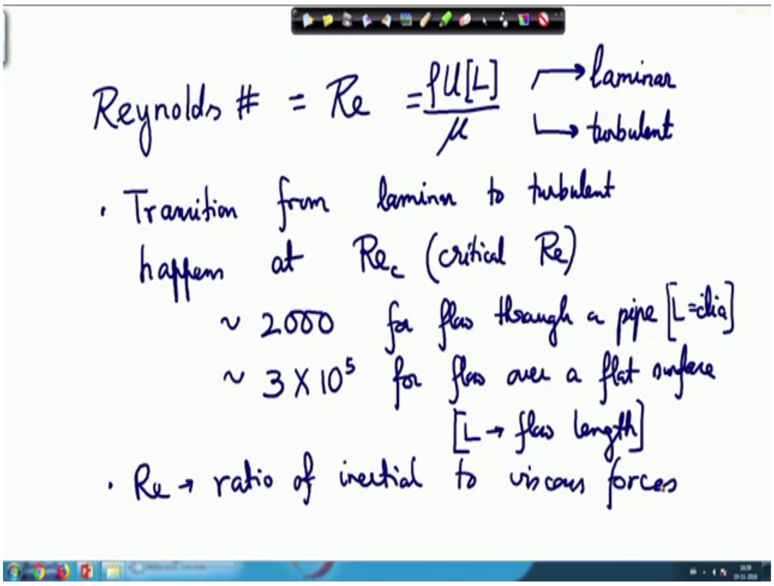

The next one I am going to talk about is called Reynolds number and denoted by Re. Reynolds number is a very interesting term it is a very one of the most fundamental quantities in fluid mechanics and that is given by u L again a length scale unless a rho u L over mu.

So, rho is the density of the fluid mu is the viscosity of the fluid u is the velocity and L is a length scale ok. So, again L like before can be length of a plate, it can the diameter of a pipe etcetera and Reynolds number determines if a flow is laminar or a flow is what is called turbulent. Laminar is when the flow happens very steadily you know you have one layer of fluid another layer of fluid like that turbulence is when you have all this mixing yeah ok.

So, the fluid is no longer moving uniformly in one direction, but there is a lot of churning and mixing and all that alright. But, also want to say that this transition from laminar to turbulent happens at the critical Reynolds number Re c which is typically around 2000 for flow through a pipe, in that case L is actually dia. It is actually some people say 3 into 10 to the power 5 some people say 5 in to (Refer Time: 06:42) flow over a flat surface where L is a flow length. So, Reynolds number is very important and actually Reynolds is ratio of inertial to viscous forces alright.

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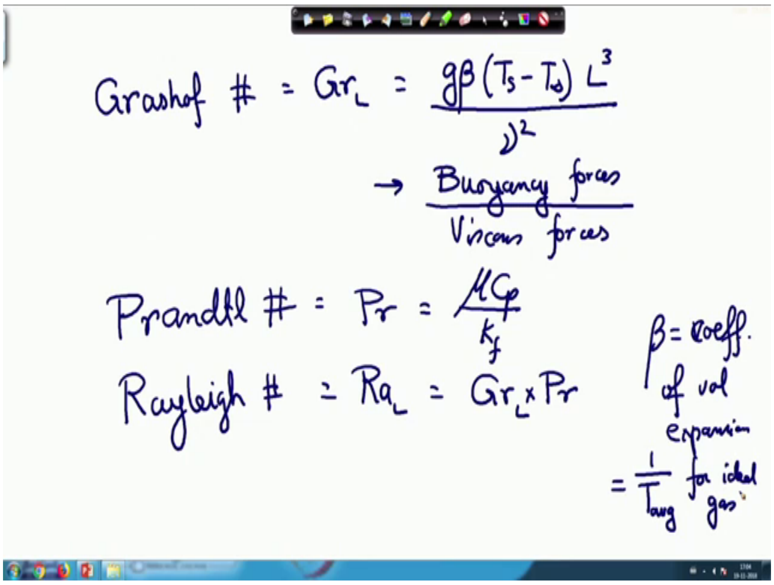

There is something else called especially when we come to natural convection there is something called Grashof number denoted by G r of L and that is given by g beta T surface minus T infinity L cubed over nu squared. And, this is a function of basically or this is a measure of buoyancy forces divided by viscous forces ok. Just like in naturally the first one is something that helps in natural convection heat transfer, it helps a liquid to go up or the other heated fluid to go up viscous forces tries to hold it back.

There is another thing which is of completely a fluid property which is known as Prandtl number Pr that is given by mu Cp over the fluid conductivity k f. And the last one is Rayleigh number Ra L and that is nothing, but Grashof times Prandtl. Now, Prandtl these all have physical significance I by the way Prandtl number actually is momentum diffusivity over thermal diffusivity. So, I am not going into those details right now I am just giving you that much details which will help you solve problems without going too much into the intricate physics of fluid mechanics and heat and mass transfer.

So, let us quickly recap what we say that these three Nusselt number one non dimensional term hL by k Reynolds number rho u L over mu and rush of rental and Rayleigh number ok.

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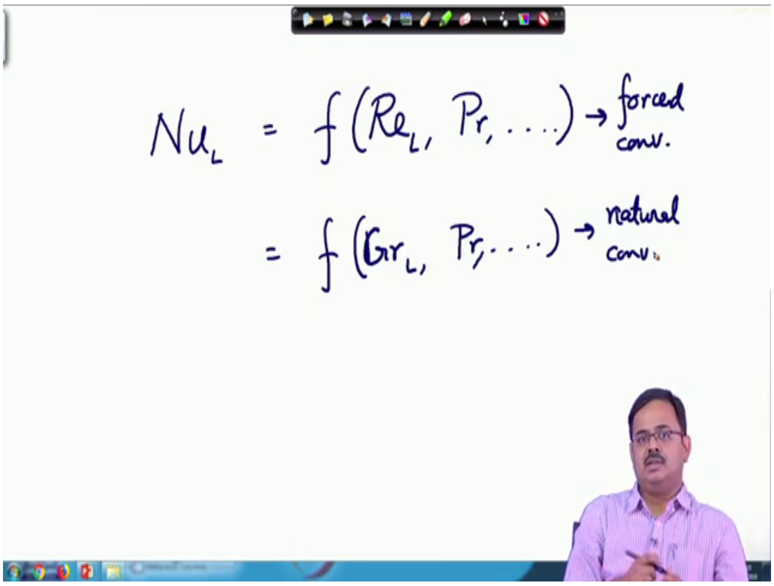

And then typically what we see is Nusselt is going to be a function of Reynolds Prandtl and maybe something else for forced convection and it is going to be a function of Grashof or la Prandtl and something else for natural convection. So, recall what we said many slides back when we are talking about force of convective heat transfer we said that the challenge in solving convection problems lies in the determination of the heat transfer coefficient h right.

How do you determine h? You determine h by calculating Nusselt number which is a non-dimensional form of the heat transfer coefficient and that Nusselt number depends on the variety of parameters depending on the flow scenario if its force convection it depends on Reynolds number and Prandtl number and maybe some of the geometry geometrical features. If it is natural convection it depends at a minimum on brush oven potential number and maybe something else.

And in turn Reynolds print Reynolds Grashof all depend on the temperature the velocity I mean Reynolds depend on the velocity the fluid properties the geometry Prandtl will depend on the fluid properties. Similarly, Grashof depends on the geometry the temperature difference and Prandtl of course, again in the fluid property right. One more thing I need to quickly mention is that this beta, beta is the coefficient of volumetric expansion and it is typically 1 over the absolute temperature of average for an average temperature for ideal gas.

So, if its air we many a times take it as 1 as an ideal gas and therefore, beta is 1 over the average temperature in the absolute scale. So, with this much background what we will do is we will move back to the slides and talk about correlations. So, this functional form what are those anyway by the way there is no need to memorize all this I also do not remember most of these, but it is important to know that you have a bunch of these correct correlations available to you.

So, the judgment as a thermal engineer depends or is really in identifying which correlation to use when to identify the flow scenario the situation and have the judgment of which of these correlations I can give you 50 different correlations which one to choose all right. So, flow over a flat plate for flow over a flat plate that average Nusselt number and what is going to be the average Nusselt number for flow over a flat plate is going to be h times average Nusselt number h bar L over kf.

So, that is given as a function of is given as 0.664 Reynolds to the power half Prandtl to the power one third if it is a laminar flow and what is Reynolds then rho u infinity L over mu. So, this is flow over a flat plate forget all this. So, that you have a fluid flowing over a flat plate and as a result of velocity as we know increases from 0 to the suit the mean means value as you move away from the plate the temperature if it is a heated plate it is highest at the plate and then reduces and slowly takes the takes the value of the ambient at which it came in the flow came in.

So, you will if its if you it is a hot plate if you bring your hand very close you feel the warmth if you move away you have really come on to this range and you do not feel it feel the warmth anymore. So, this thickness within which the temperature reduces from the hot surface temperature to the ambient temperature is known as the thermal boundary layer and it slowly increases in thickness as you move along the flow length. Similarly the thickness within which the velocity changes from 0 to the mean free velocity as we call it is known as a momentum boundary layer. And depending on the fluid for a fluid with higher Prandtl number Prandtl number greater than 1, the momentum boundary layer thickness is higher than thermal boundary layer thickness.

So, water is an example for Prandtl number less than 1 like for example, air the momentum thickness boundary layer thickness will be less than the thermal boundary layer thickness. So, this is a little more detail, but probably it does not harm to know all these and especially for mechanical engine students I think this is a good recap alright. So, for turbulent flow look at the correlation the dependence on Reynolds number is now four fifth.

So, few things Nusselt number is proportion in laminar flow is proportional to Reynolds the power half which means heat transfer coefficient is proportional to the square root of the velocity yeah. But, in case of turbulent flow where there is more churning it is really intuitive that the heat transfer is going to be higher. And therefore, the heat transfer coefficient is proportional to the velocity to the power 0.8 or four, fifth. Now as I said there is something called a transition Reynolds number and which will happen when the length of the plate is long enough. So, that the Reynolds number exceeds the critical value right.

So, till that length the flow is laminar followed by turbulent and in such a case this is the correlation that is used if it is laminar throughout correlation the top one. If it is lam turbulent throughout how will you do that well you can you can have some mixed air you can have some mixing some turbulator etcetera. We will come to that, you can actually you know introduce a disturbance over here and what is called trip these boundary layer. So, that now there is right after these there is mixing and there is flow churning ok.

So, that is thoroughly two fully turbulent, but otherwise if you just let the flow develop and finally, transition from laminar to turbulent it is going to do it after a certain length when the critical Reynolds number at that point is higher, when the Reynolds number is higher than the critical Reynolds number. And, the correlation to be used is a third one. Flat plate with unheated length what is that that is a case when sorry you have flat plate like this you have air coming in, but you start heating after a certain length sorry.

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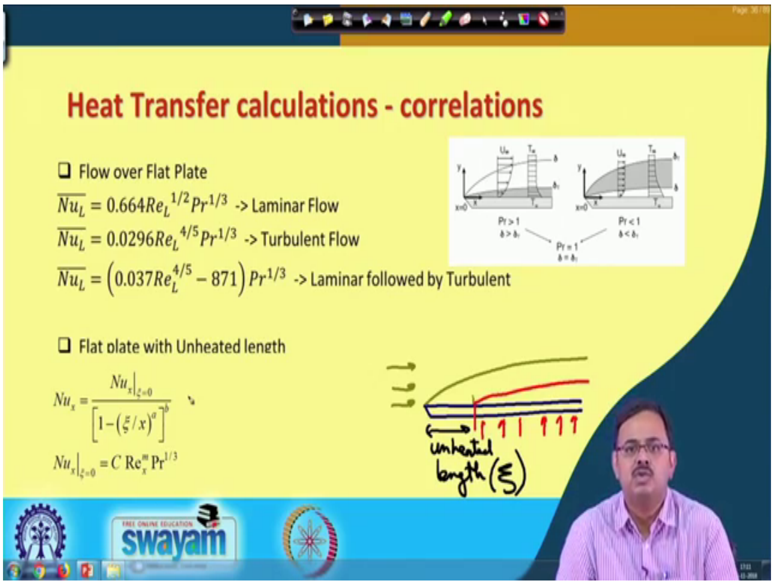

So, therefore, what happens the momentum boundary layer develops right from here, but the thermal boundary layer starts developing only from this point. So, there is an unheated length as you can see this is an unheated length, if that is the situation which is actually quite common if it is a circuit board. So, the air is moving for the circuit board and then you have a component which is heated it is not uncommon if that is the case you use this correlation this unheated length zeta ok.

This unheated length zeta is given by this correlation, you first get a correlation for Nusselt number from the top 3 assuming that zeta is equal to 0. And, then you basically use 1 minus zeta over x to the power a the whole to the power b and this a and b are standard values that are available ok.

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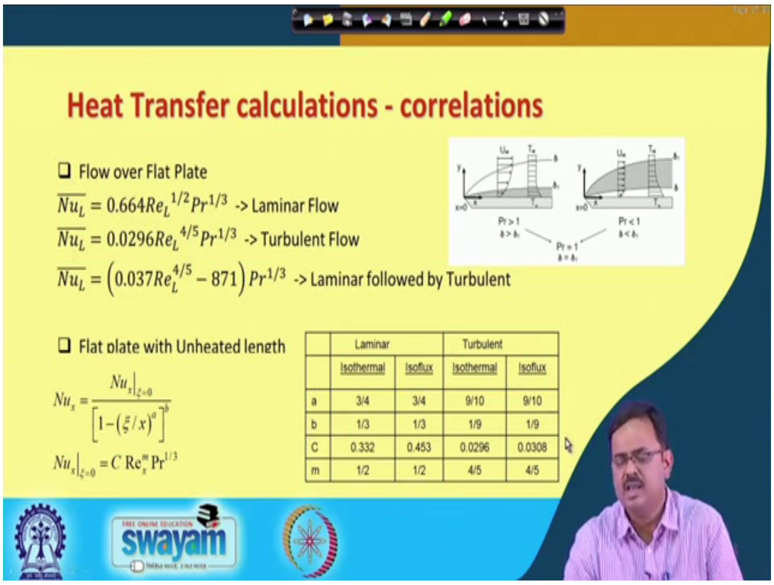

I have this table which gives you a b c and m values you can see here. So, c and m laminar isothermal 0.332 which is x actually the average is 0.664. So, these are all well documented.

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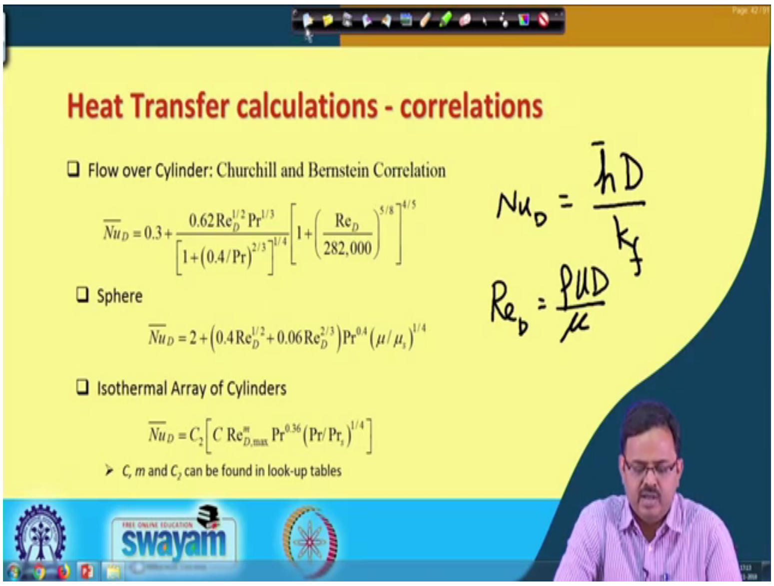

If it is flow over a cylinder the Nusselt number; however, now becomes a new D. So, Nusselt number of sorry I do not know where this came from if its flow over a cylinder or a sphere and similarly Reynolds number will be rho u D over mu. If it is an isothermal array of cylinders then it is a little complicated as is shown over here and c mc 2 all these correlation all these coefficients all these indices there are standard values and look up tables available for these.

So, you just need to know that these correlations are available and these values will be given to you again I repeat the challenge is to use the correct judgment or is in the judgment of using of choosing which correlation to use internal flow force convection internal flow.

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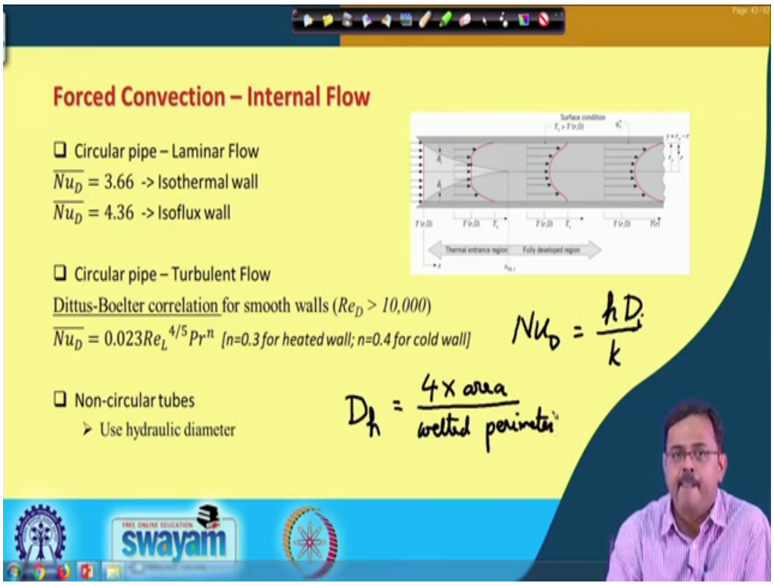

So, these were all external flows. So, flow is happening over at the body whether it is over a sphere, over a cylinder, over a plate, now if its flow through a channel or a pipe like this. So, if it is a circular pipe the Nusselt number which is again a Nu D sorry hD by k and D is the id internal diameter, that Nusselt number value comes out to be constant 3.66 and 4.36 whether its isothermal boundary condition which is constant temperature or iso flux boundary condition which is constant heat flux it is for laminar flow. If it is a turbulent flow then Nusselt number this is a very famous correlation called Dittus Boelter 0.023 Reynolds to the power four-fifth Prandtl to the power either 0.3 or 0.4 depending on if the wall is heated or if it is cooled.

So, in whether its heat being transferred from the heated wall to a colder fluid or whether it is a heat of all getting heated up because a hot fluid is flowing through it. Now here let us spend a little time on the flow physics here also there is a thermal boundary layer that happens or that that takes shape, but that if this is one of the cross section you see there is a thermal boundary layer that happens on this wall the top wall as well as the bottom wall and beyond a point they intersect. And as they intersect and the temperature profile therefore, at the beginning goes from the wall temperature to the fluid temperature at both walls and finally, it takes this parabolic paraboloid profile.

Now, the question is what happens if you use non circular tubes. So, for non circular tubes what we do is we use something called a hydraulic diameter D h which is given by 4 times area divided by wetted parameter.

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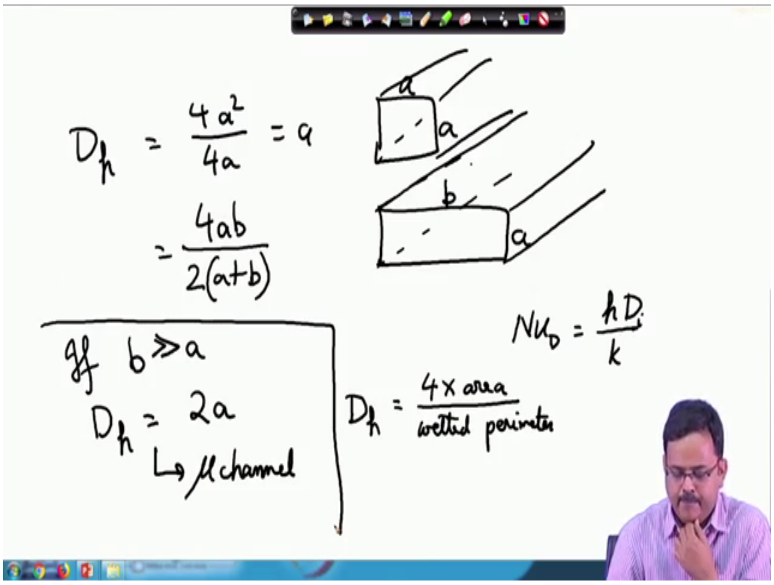

So, let us go over here what will be D h for a square it is 4 a squared 4 times area. So, the flow is this is the cross section of the pipe right instead of a circle, 4 a squared over 4 a equals 2 a. What happens if it is a rectangle? Think about it this is very similar to flow between 2 fins through a channel then what happens 4 a b divided by 2 into a plus b ok.

What happens if b is much much greater than a then D h becomes what? 2 a why? You divide everything by b. If you divide everything by b then what happens its -2 a divided by a by b plus 1 a bi be close to 0 because being is much much greater than a this is 2 a. So, this is a classic case of what is called a microchannel all right let us move back; so, for hydraulic for non-circular cross sections use hydraulic diameter ok.

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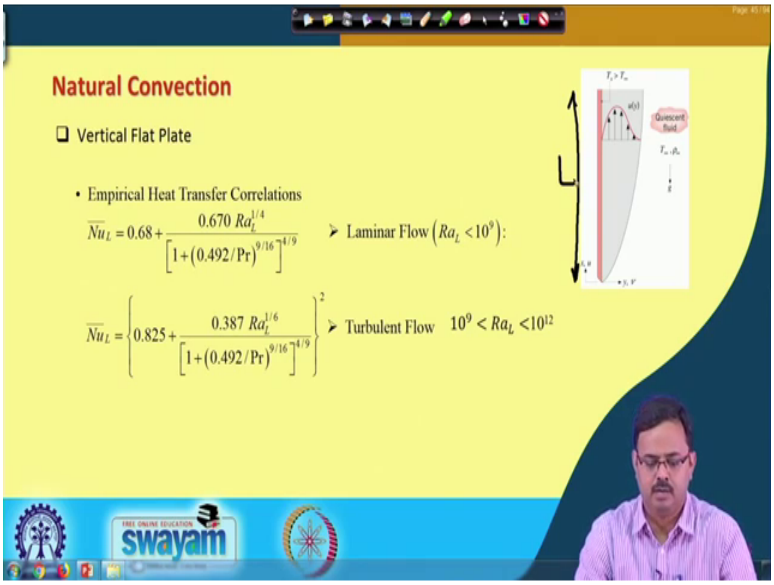

Let us quickly go to natural convection here also if it is a vertical flat plate with and here gravity is important. So, what happens the flow here becomes lighter and it moves up and as it moves up it carries heat away, this is how the temperature varies and if I draw this is how the velocity varies it is a little different from force convection. Because, here the maximum velocity occurs somewhere within the boundary layer there is no within this range and the thermal boundary layer of course, is again this is kind of the value ok.

Highest at the wall and then slowly becomes equal to the ambient temperature. So, here also we have correlations like this except now these are in terms of Rayleigh number Nusselt is a function of Rayleigh number and L is the height of the plate here because this is the flow direction correct. So, this is L vertical flat plate.

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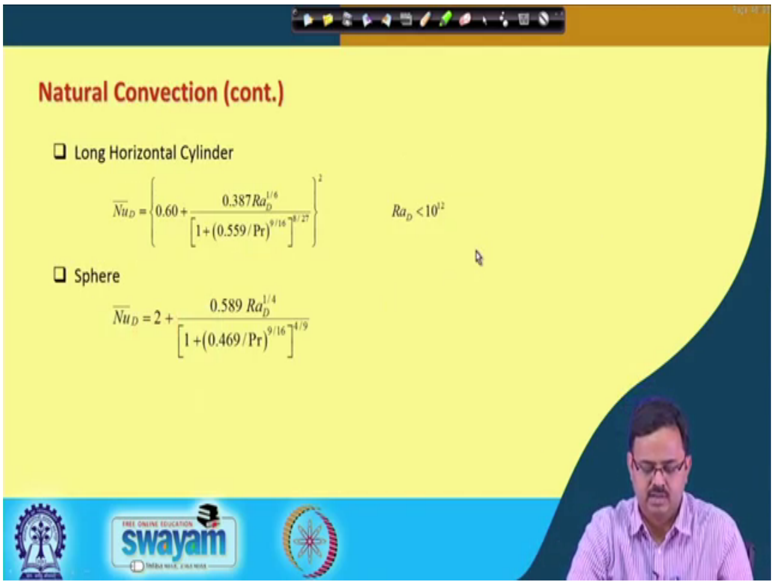

If its a horizontal flat plate then these are correlations, horizontal closed flat plate is the hot surface facing up then the flow will go up if the hot surface is facing down ok. And so, these are these are the same thing if it is a cold surface then this and this is the same, but if it is facing down what happens the flow cannot leave right.

So, therefore, this is a corrosion it is almost half as you can see because the hot fluid wants to rise, but it cannot because the plate is blocking it. So, it has to go around and go up. So, these are the correlations long horizontal cylinder again the Nusselt as a function of Rayleigh number sphere also Nusselt as a function of Rayleigh number.

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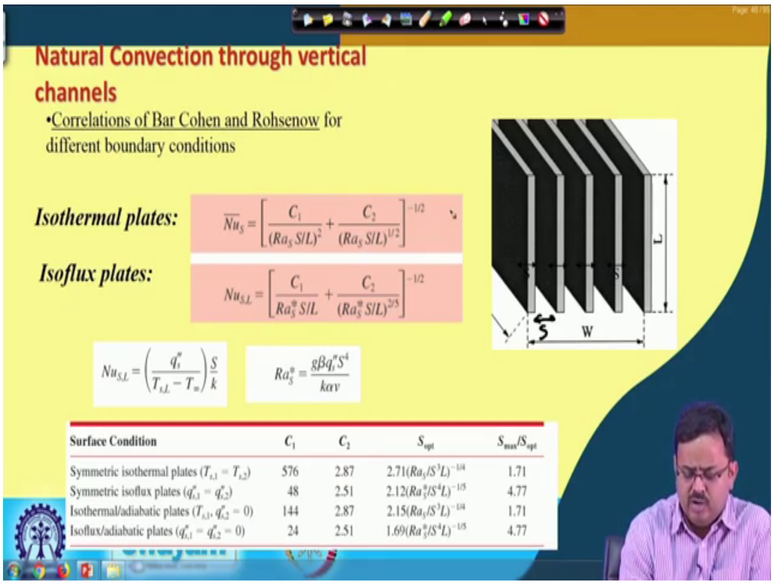

Natural convection through vertical channels very very important because for heat sinks its vertical channel then look at it is a complicated correlation. But do we know everything Nusselt as a function of spacing where S is the space between S is the spacing the correlation of bar Cohen Rohsen. Now, S is the spacing between S is this spacing L is the length and W is the total width, then what happens? For isothermal plates this is the condition for the temperature is the same C1, C2 needs to be found out and for iso flux plates instead of Rayleigh number because Rayleigh number you need delta T.

But you do not have a T s here it is nice of flux. So, this is a different definition Nu SL and Rayleigh star all right and then this is how you use it. See if the feed flux is given to you can calculate rally star and then you can find out the Nusselt number and from there you can find out the temperature at the position where, at the what is at the top of the plate which is where it is going to be highest.

So, these numbers what is C1 what is C2 they are all given here. Optimal spacing this is important what is optimal spacing look and this is where S max and S opt, these are important. S max is look when you have two surfaces next to each other if you move them far apart then the heat transfer from one plate is the highest because, it is not disturbed by the presence of another hot surface next to it. If you bring them very close then of course, this hot plate influences the heat transfer from the adjacent hot plate ok.

So, S max is when beyond the point these this plate does not even feel the presence of this plate and that is when these boundary layers that we saw two slides back I am sorry ok. The boundary layers that you saw two slides back there will be a boundary layer on this surface and if you have another one next to it there will be another boundary layer when this boundary layers do not overlap then that is optimal spacing where the heat transfer from this plate is going to be maximum right.

But then for a given W that is not good right because then you can only accommodate that many number of fins your heat transfer per plate is higher, but number of places less. On the other hand if you bring them closer heat transfer per plate is lower, but number of plates is higher or per surface is lower, but number of surfaces is higher every additional frame means two additional surfaces. So, then there is an optimal spacing which is going to be a function of the length because the boundary layer also increases with increasing length. So, that is what is given over here.

So, understand so, S max is the maximum heat transfer from one heated surface and that happens when the two surface to adjacent surfaces are so, far apart that the boundary layers do not intersect at all. So, it is just at that point when they do not intersect they are just probably touching at the top end that is it, beyond that if you move them apart does not matter the heat transfer per surface does not change because, right at that S max itself or hire one surface does not know that the other is present. But if you bring them closer heats of heat transfer per surface goes down, but number of surfaces go up for a given width right.

So, therefore, there is an optimal spacing that exists. So, S opt and S max is different and as you can see S max is greater than S opt and it is really for isoflux conditions that ratio is quite high almost 5 times yeah all right. So, that kind of wraps up our discussion on heat transfer correlations, we looked at various correlations and these are correlations that we are going to use.

So, if it is for example, of indeed sink under natural convection we are going to use the bar code and rows in our correlation which we saw in the last slide if its flow through a fin heat sink, I will take each of these the space between two adjacent fins as a channel and use the internal flow correlation for either whether its laminar or whether it is turbulent and from there I will get the Nusselt number, use hydraulic diameter by the way because that is a non-circular cross sectional geometry clear.

So, this is how we get the average heat transfer coefficient I know what is going to be my thermal resistance clear theta I say all right ok. Thank you very much, in the next class what we will do is we will quickly talk about how do we calculate also pressure drop there is a correlation for that as well and then move on to thermal you know some thermal technologies other than heat sink ok.

Thank you very much and see you in the next class bye.