

**Concepts of Thermodynamics**  
**Prof. Suman Chakraborty**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 51**  
**Entropy Transport for Flow Process: Examples**

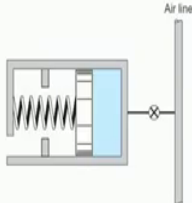
We have been solving problems related to entropy transport across the control volume. We will work out a few more problems in this lecture.

(Refer Slide Time: 00:25)

**Problem 7.10:** A frictionless piston/cylinder is loaded with a linear spring with a spring constant  $100 \text{ kN/m}$ , and the piston cross-sectional area is  $0.1 \text{ m}^2$ . The cylinder initial volume of  $20 \text{ L}$  contains air at  $200 \text{ kPa}$  and ambient temperature,  $10^\circ\text{C}$ . The cylinder has a set of stops that prevents its volume from exceeding  $50 \text{ L}$ . A valve connects to a line flowing air at  $800 \text{ kPa}$ ,  $50^\circ\text{C}$ , as shown in the figure. The valve is now opened, allowing air to flow in until the cylinder pressure reaches  $800 \text{ kPa}$ , at which point the temperature inside the cylinder is  $80^\circ\text{C}$ . The valve is then closed and the process ends.

- Is the piston at the stops at the final state?
- Taking the inside of the cylinder as a control volume, calculate the heat transfer during the process.
- Calculate the net entropy change for this process.

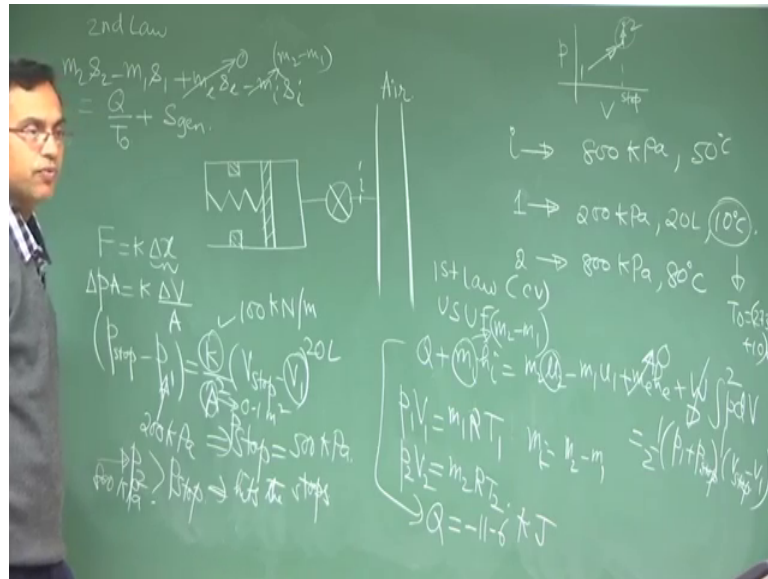
**Ans: (a) The piston hits the stops (b)  $Q_{cv} = -11.6 \text{ kJ}$  (c)  $\Delta S_{net} = 0.063 \text{ kJ/K}$**



So, the next problem is problem numbers 7.10. A frictionless piston cylinder is loaded with a linear spring with a spring constant of  $100 \text{ kilo Newton per meter}$  and the piston cross sectional area is  $0.1 \text{ meter square}$ . The cylinder initial volume is  $20 \text{ litre}$ . It contains air at  $200 \text{ kilo Pascal}$  and ambient temperature  $10 \text{ degree centigrade}$ .

The cylinder has a set of stops that prevents its volume from exceeding  $50 \text{ litre}$ . A valve connects to a line flowing air at  $800 \text{ kilo Pascal}$ ,  $50 \text{ degree centigrade}$  as shown in the figure. The valve is opened, allowing air to flow until the cylinder pressure reaches  $800 \text{ kilo Pascal}$ , at which point the temperature inside the cylinder is  $80 \text{ degree centigrade}$ . The valve is then closed and the process ends. Is the piston at the stops at the final state? Taking the inside of the cylinder as a control volume, calculate the heat transfer and net entropy change during the process.

(Refer Slide Time: 01:45)



So, let me draw a schematic of this problem. This is the air supply line. There are some stops. So, what is happening is air is flowing in this line and it is supplied so that this piston expands and it might hit the stop or not from the data we have to figure out whether it hits the stops or not.

So, when the piston is moving how the pressure is related to the change in volume? So, you have the force for a linear spring, force is proportional to the displacement. So,  $F$  is equal to  $k$  into  $\Delta x$  right;  $\Delta x$  is  $\Delta V$  by  $A$ ; where  $A$  is the cross sectional area of the piston and  $F$  is  $\Delta p$  into  $A$  right. So, the net force some  $\Delta p A$  right, some  $p$  minus  $p$  reference. So, this is just to let you know that the pressure versus volume curve will be a straight line with a slope of  $k$  by  $A$  square. So, you can write say  $p_{stop} - p_1$  is equal to  $k$  by  $A$  square into  $V_{stop} - V_1$  right.

So,  $k$  is given, the spring constant, this is 100 kilo Newton per meter, cross sectional area is 0.1 meter square ok,  $V_{stop} - V_1$ . So, initial volume is given 20 litre, you convert this to metre cube, initial pressure is 2 into 100 kilopascal; so,  $p_1$  is 200 kilopascal. So, from here you get what is  $p_{stop}$ . Area of cross section is given. What is that? 0.1 meter square. So, you will get what is  $p_{stop}$ ,  $p_{stop}$  is 500 kilo Pascal.

Now, from this data what is this  $p_{stop}$ ? That means, the pressure when this just reaches the stops. From these data can you tell whether it hits the stops or not? See the final pressure is told as 800 kilo Pascal which is more than this. That means, until and unless it

hits the stops pressure cannot be more than this pressure when it comes just before the stops. So, because  $p_2$  is greater than  $p_{stop}$ ,  $p_2$  is 800 kilo Pascal so that means, it hit the stops. See why thermodynamics problems are so interesting? I may plan for setting up exactly the same problem in your exam, but I change the final pressure from 800 kilo Pascal to 400 kilopascal then entire solution changes because then the piston no more hits the stops, it comes before the stops and the solution is different, although you know conceptual it is a similar problem.

So, this makes thermodynamics problems very interesting because the solution of the problem how it is evolving it depends on the data which comes as a part of the intermediate calculation. That does not make problems very stereotype where you have a unique approach of solving the entire problem.

So, then what is required is basically the entropy generation. So, let us write what are the states. The inlet state is 800 kilo Pascal, 50 degree centigrade. State 1: 200 kilo Pascal, 20 litre, 10 degree centigrade. State 2: 800 kilo Pascal, 80 degrees centigrade and  $T_0$  is this one, 10 degree  $273.15 + 10$  Kelvin. Ambient is 10 degree that centigrade that is given there is no exit right, only it is a tank filling with a variable area that is the problem.

So, if we apply the first law for the control volume which is the cylinder it is a uniform state uniform flow process. So,  $Q + m_i h_i$  is equal to  $m_2 u_2 - m_1 u_1 + m_e h_e + W$ ; so there is no exit. Therefore, this is 0. What is  $W$ ? Here there is a  $W$ . As I told you earlier and you should remember this very carefully, the work done equal to integral  $p dv$  does not depend on whether it is a control mass or a control volume. So, long as you have a moving system bounded in a quasi equilibrium process you can write  $p dv$ . So, this is integral of  $p dv$  from 1 to 2; so, half of  $p_1 + p_2$  into  $V_2 - V_1$ .

So, this is actually not  $p_2$ , this is  $p_{stop}$  into  $V_{stop} - V_1$  because after reaching the stop there is no work done, after this piston reaches the stop there is a constant volume process. So, there is no work done. So, this is the final pressure. So, far as this formula goes is the pressure when it just reaches the stops. So, if you consider the PV diagram for the process. So, 1 to 2, 1 to stop it is like this and from stop to 2 it is like this.

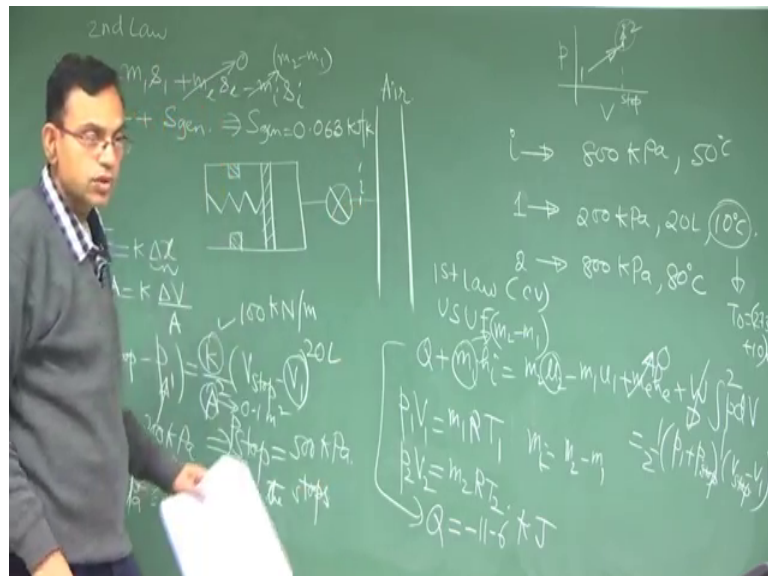
So, this part has no work. So, only this part has work. Then this tank is it this cylinder is it insulator? It is not insulator. So, you have to basically find out what is the heat transfer.

State 2 is known. So, this is air right, you can see look at the temperature range it is not that much 50 degree, 10 degree, 80 degree this range of temperature, you can assume constant  $c_p$   $c_v$ . So,  $m_1$  is  $m_2$  minus  $m$ . So, how do you know what is  $m_2$  and  $m_1$ ? So, you can use the ideal gas law right. So,  $p_1 V_1$  is equal to  $m_1 R T_1$  right and  $p_2 V_2$ ;  $p_2 V_{stop}$  or  $p_{stop} V_{stop}$ ,  $p_2 V_2$ ;  $V_2$  is same as  $V_{stop}$  is equal to  $m_2 R T_2$  ok.

So, from these two you can calculate what is  $m_2$  and  $m_1$ . So, I do not have the data for  $m_2$  and  $m_1$ , but you know you can calculate this; an  $m_i$  is  $m_2$  minus  $m_1$ . So, once you calculate these values and you can. So,  $m_i$  when you write  $m_2$  minus  $m_1$ , then you can split this into two terms; one is with  $m_2$  and another is with  $m_1$ ; then  $h$  you can write  $c_p T$  and  $u$  as  $c_v T$  ok. So, with constant  $c_p$   $c_v$  you can replace all these as functions of temperature and then you will get what is  $Q$ . So, that answer I have.  $Q$  is equal to so in this equation if you substitute everything. So,  $Q$  is minus 11.6 kilo Joule.

So why do you require  $Q$ ? Because for entropy generation you require  $Q$ . So, if you apply the second law, so, change in entropy  $m_2 s_2$  minus  $m_1 s_1$  plus  $m_e s_e$  minus  $m_i s_i$  is equal to  $Q$  by  $T_0$  plus entropy generation there is no exit; other values  $m_i$  is  $m_2$  minus  $m_1$  and you can use the change in entropy formula for ideal gas.

(Refer Slide Time: 14:44)



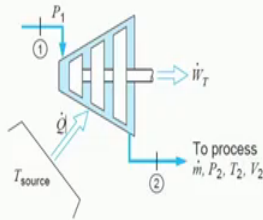
So, if you do all this you will get entropy generation as 0.063 kilo Joule per Kelvin ok. So, we will work out a couple of more problems, let me erase this one.

(Refer Slide Time: 15:04)

**Problem 7.11:** A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K, as shown in the figure. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent  $n = 1.20$ .

- What is the turbine inlet temperature?
- What are the power output and heat transfer rate for the turbine?
- Calculate the rate of net entropy increase if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

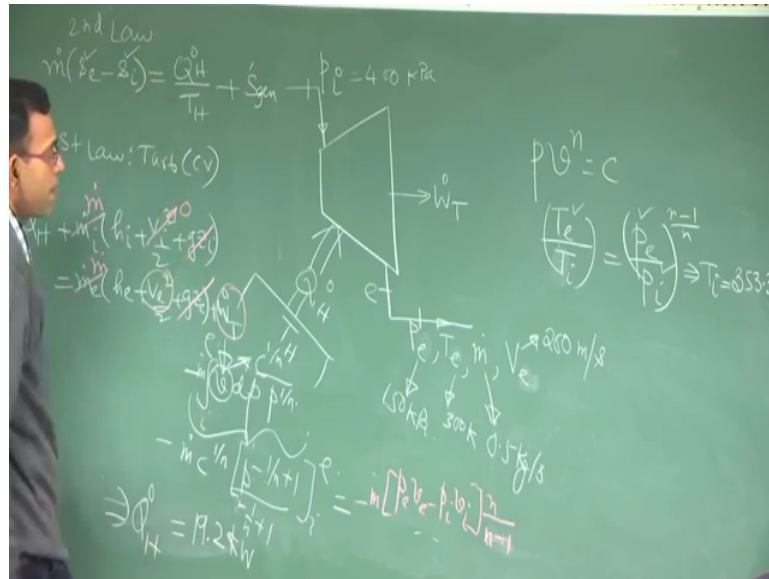
Ans: (a)  $T_1 = 353.3 \text{ K}$  (b)  $\dot{W}_T = 35.9 \text{ kW}$ ;  $\dot{Q}_T = 19.2 \text{ kW}$  (c)  $\dot{S}_{\text{net}} = 0.0163 \text{ kW/K}$



Problem number 7.11: A certain industrial process requires a supply of 0.5 kg per second of air at 200 meter per second at the condition of 150 kilo Pascal, 300 Kelvin. This air is to be exhaust from a specially designed turbine whose inlet pressure is 400kilo Pascal ok, the turbine process may be assumed to be reversible and polytropic with polytropic exponent as 1.2.

What is the turbine inlet temperature? What are the power output and heat transfer rate for the turbine? Calculate the net rate of entropy increase, this is essentially entropy generation if the heat transfer comes from a source at temperature 100 degree centigrade than the turbine inlet temperature. So, this is a turbine which produces some power output and it also produces steam at a given pressure, temperature, mass flow rate etcetera that is used for some chemical processing, there is a velocity also right.

(Refer Slide Time: 16:11)



So, we call this as i and we call this as e that is better, because for the turbine that is the steady state steady flow picture, we have one inlet and in this case one exit. So, i and e are our usual symbols ok. So, what is told?

Student: There is a heat transfer (Refer Time: 17:47).

That there is a heat transfer to the turbine; this takes place from a heat source at temperature T H ok.

So, let us note down what data is given; p i is 400 kilopascal, p e is 150 kilopascal, T e is 300 Kelvin, m dot is 0.5 kg per second and v e is 200 metre per second. The turbine process is reversible and polytropic with n is equal to 1.2. So that means, the process in the turbine can be described by pressure into specific volume to the power n equal to constant. If this n is c p by c v then its reversible adiabatic for air that c p by c v if you take constant c p c v it is 1.4. Here the value of n given is 1.2; that means, it is not reversible and adiabatic. It is reversible, but not adiabatic. This is the first message that we get from here, but if this is followed and also p v by T is equal to constant. So, from here for reversible polytropic process this much we can write. So, all this things are known except T i. So, T i is 353.3 Kelvin.

Now you need to find out what is the heat transfer because entropy generation depends on that. So, you apply the first law for the turbine as control volume ok. So, Q dot plus m

dot i, now here kinetic energy term is there. So, let us write the full thing and see what we neglect ok. So,  $V_i$  nothing is given, but we can neglect this one, you can neglect the difference between these two, you can neglect sorry you cannot neglect anything else this is given.

So, this you have to use the data;  $h_i$  minus  $h_e$  you can write,  $m \dot{i}$  and  $m \dot{e}$  are both  $m \dot{c}_p$  into  $T_i$  minus  $T_e$ ; what is this one? So, remember this is a reversible steady state steady flow process with single inlet and single exit. So, you can use the  $v dp$  formula right. So, this will be minus integral of  $m \dot{v} dp$  from  $i$  to  $e$  this is very important not  $p dv$  ok.

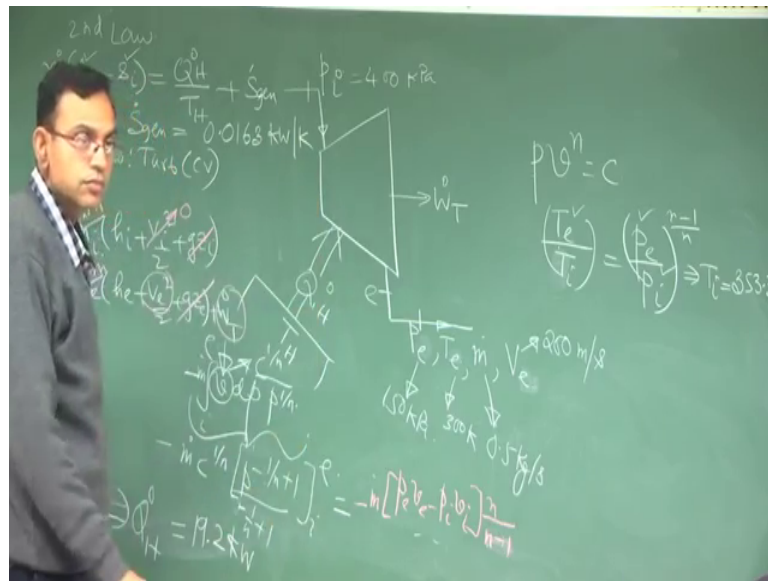
So, we showed that the limiting case of reversible isothermal process satisfies this formula, the limiting case for reversible adiabatic process satisfy this formula and any reversible process can be thought of as a succession of a large number of reversible adiabatic and reversible isothermal processes. Therefore, for this polytropic which is not adiabatic, but reversible you can use this formula. So, now, you can straight away  $v$  you can express  $v$  in terms of  $p$  that is  $c$  to the power  $1/n$  by  $p$  to the power  $1/n$  right.

So, then that integration becomes minus  $m \dot{v}$  let us write this term  $c$  to the power  $1/n$  then  $p$  to the power  $1/n$   $dp$ ; so,  $p$  to the power  $1/n$  plus  $1/n$  from  $i$  to  $e$ . So, this  $v dp$  integration this is minus actually. So,  $p$  to the power  $1/n$  into  $c$  to the power  $1/n$  is  $v$  right so, minus  $m \dot{p} v_e$  minus  $p_i v_i$  into  $1/n$  sorry  $n$  minus  $1$  right.

So, this is the work done, you can substitute all the values. The specific volume you can calculate by using  $p v$  is equal to  $r t$  then because this is air which you can approximate as ideal gas. So, from here you can calculate what is the rate of heat transfer. So, very interesting problem where you do not have reversible adiabatic any reversible process with heat transfer in the turbine; so, this is 19.2 kilowatt ok. And then we can apply the second law for the turbine.

So,  $m \dot{s}_e$  minus  $s_i$  is equal to  $Q \dot{H}$  by  $T_H$  plus rate of entropy generation right. So,  $s_e$  minus  $s_i$  again you can use the ideal gas formula  $Q \dot{H}$  you have calculated.

(Refer Slide Time: 25:57)



So, you will get what is entropy generation. So, entropy generation is 0.0163 kilowatt per Kelvin. So, you can attribute this entropy generation due to two things, one is heat transfer and the heat transfer the associated external irreversibility associated with heat transfer because within the turbine the process is reversible. So, that will not give rise to in entropy generation ok.

(Refer Slide Time: 26:50)

**Problem 7.11:** A certain industrial process requires a steady 0.5 kg/s of air at 200 m/s, at the condition of 150 kPa, 300 K, as shown in the figure. This air is to be the exhaust from a specially designed turbine whose inlet pressure is 400 kPa. The turbine process may be assumed to be reversible and polytropic, with polytropic exponent  $n = 1.20$ .

- What is the turbine inlet temperature?
- What are the power output and heat transfer rate for the turbine?
- Calculate the rate of net entropy increase if the heat transfer comes from a source at a temperature 100°C higher than the turbine inlet temperature.

**Ans:** (a)  $T_1 = 353.3 \text{ K}$  (b)  $\dot{W}_T = 35.9 \text{ kW}$ ;  $\dot{Q}_T = 19.2 \text{ kW}$  (c)  $\dot{s}_{net} = 0.0163 \text{ kW/K}$


So, let us work out another small problem, we have a little bit of time in this lecture. So, let us utilise that.



(Refer Slide Time: 27:00)

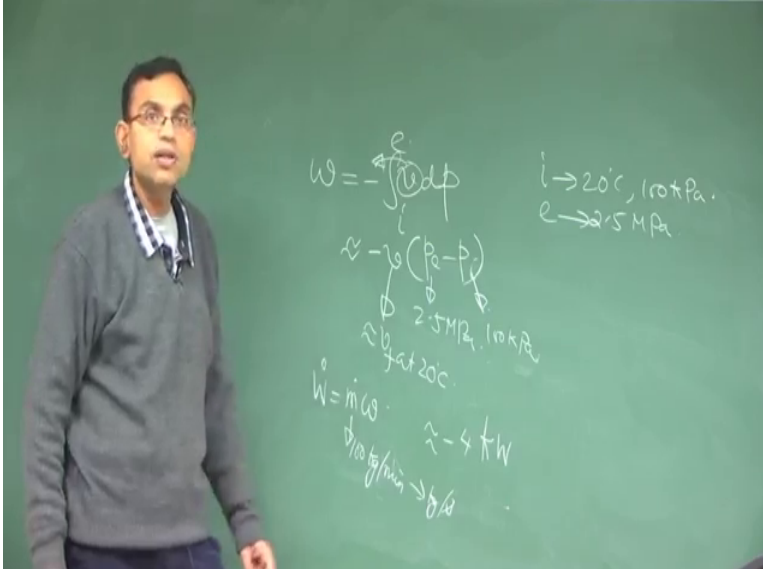
**Problem 7.12:** A small pump takes in water at 20°C, 100 kPa and pumps it to 2.5 MPa at a flow rate of 100 kg/min. Find the required pump power input.

**Ans:**  $\dot{W}_{\text{pump}} = 4 \text{ kW}$



Problem 7.12, this is a sort of review problem; a small pump takes in water at 20 degree centigrade, 100 kilo Pascal and pumps it to 2.5 MegaPascal at a flow rate of 100 kg per minute. Find the pump input power input. So, the key to this problem is that it is a small pump. So, heat transfer is negligible and if you assume the pump to be reversible because no other data is given, so the work done for the pump is given by the integral  $v \, d p$  formula.

(Refer Slide Time: 27:49)



$$\dot{W} = -\int_1^2 v \, dp$$
  
$$\approx -v (p_2 - p_1)$$
  
$$\dot{W} = \dot{m} v (p_2 - p_1)$$
  
$$\dot{W} = 100 \text{ kg/min} \cdot (2.5 \text{ MPa} - 100 \text{ kPa})$$
  
$$\dot{W} \approx 4 \text{ kW}$$

$i \rightarrow 20^\circ\text{C}, 100 \text{ kPa}$   
 $e \rightarrow 2.5 \text{ MPa}$

$\dot{m} = 100 \text{ kg/min} \rightarrow 1.67 \text{ kg/s}$

So, the work done minus integral  $v \, d p$  from pump inlet to pump exit.

So, the pump takes in water at 20 degree centigrade, 100 kilo Pascal and it pumps to 2.5 MegaPascal. The specific volume, see this is what as pump as an engineering device you must have an idea. Pump handles a fluid which is virtually incompressible; so; that means, this  $v$  is roughly constant. So, because this  $v$  is roughly constant you can take this  $v$  out of the integration. So, this approximately, not exactly constant, but approximately constant. So, this is minus  $v$  integral of  $dp$  is  $p_e$  minus  $p_i$ ;  $p_e$  is 2.5 MPa,  $p_i$  is 100 kilo Pascal and this  $v$  is specific volume, roughly  $v_f$  at 20 degree centigrade; which  $1/\rho_f$  will be roughly like you know 1000 kg per metre cube the density of water.

So, you have to find out what is the rate of work done. So,  $\dot{W}$  is  $\dot{m}$  into small  $w$  and  $\dot{m}$  is given as 100 kg per minute. So, you convert it into kg per second. So, this will be roughly minus 4 kilowatt that is the work input necessary for the pump. We have solved quite a few problems in the control volume analysis for entropy transport. We will continue with discussions on similar lines in the next lecture.

Thank you very much.