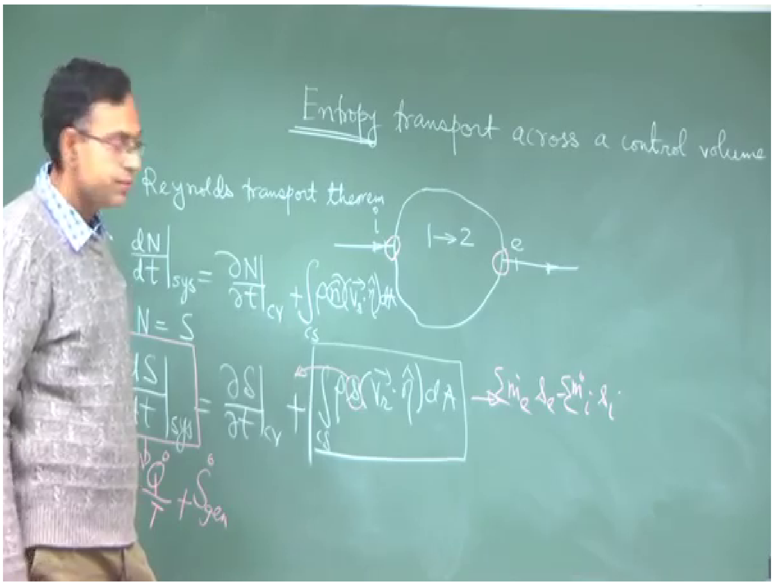
Concepts of Thermodynamics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 47
Entropy Transport for a Flow Process

In the previous lecture we were discussing about situations where the second law of thermodynamics is applied to control mass system. But there are many processes occurring in nature and engineering where the process concerns a of flow across the system boundary and then we require a control volume analysis. So, we should now devote our attention to understand control volume perspective of entropy change and entropy transport.

(Refer Slide Time: 01:20)



So, entropy transport across a control volume. Let us say that there is a control volume; across this control volume there are some inlets and exits, we symbolise inlet as i and exit as e. There can be a change in state within the control volume itself. So, if it is an unsteady state within the control volume it is 1 to 2 within the control volume; we want to calculate the net entropy change during this process. What we already know? We already know how to calculate the net change in entropy for a control mass. So, we have to take a call in expressing that known expression in terms of the change taking place within a control volume and that can be done by appealing to our Reynolds transport theorem.

So, the Reynolds transport theorem. So, the symbols are as usual, this is the net change of the extensive property n for a system this is the change within the control volume and this is the flow net transport of the property due to flow across the control surface. So, for this case, we are concerned about the transport of entropy. So, we consider N equal to S example. So, dS dt plus this is property per unit mass. So, capital S per unit mass becomes the lower case s; specific entropy. Now, this pertains to the change in entropy of the control system. Recall the formula delta S is equal to delta q by t plus the entropy generation.

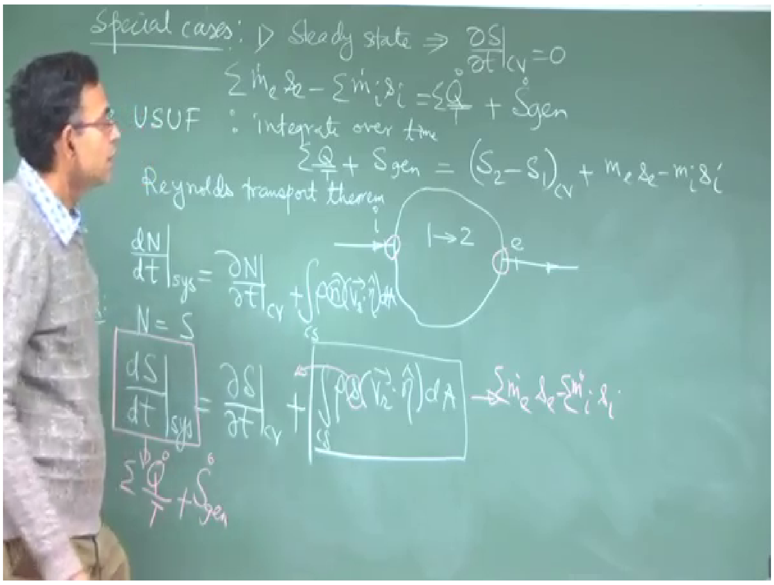
So, then if you divide that by time delta t and take the limit as delta t tends to 0 then this will become, the heat transfer will become rate of heat transfer by T. Remember that we are considering that the entire heat transfer is taking place at a given temperature T, but if the heat transferred is having different thermal reservoirs with which the heat exchange is done then you have to put a summation over Q naught by y t where every heat transfer is associated with the corresponding thermal reservoir.

So, in the control mass system expression the Q 1 2 by T that also can be generalized, it can be summation of heat transfer divided by the corresponding temperature of the system boundary across which the heat transfer is taking place.

So, here to simplify it I am not writing the summation, but it is if there are many thermal reservoirs, it is technically summation of this where this is the corresponding temperature of the system boundary, this is the corresponding heat transfer plus rate of entropy generation. Now we want to simplify this term.

So, what we make an assumption, as an assumption is that we assume that over the flow boundary the respective flow boundary the entropy is constant. That is this is one flow boundary this is another flow boundary over this the entropy is s i and over this the entropy s e. So, then if you bring that out the integral of rho Vr dot eta d A is nothing but the mass flow rate. So, this when will become m dot e s e minus m dot i s i again, you can have a summation over entrance and exit.

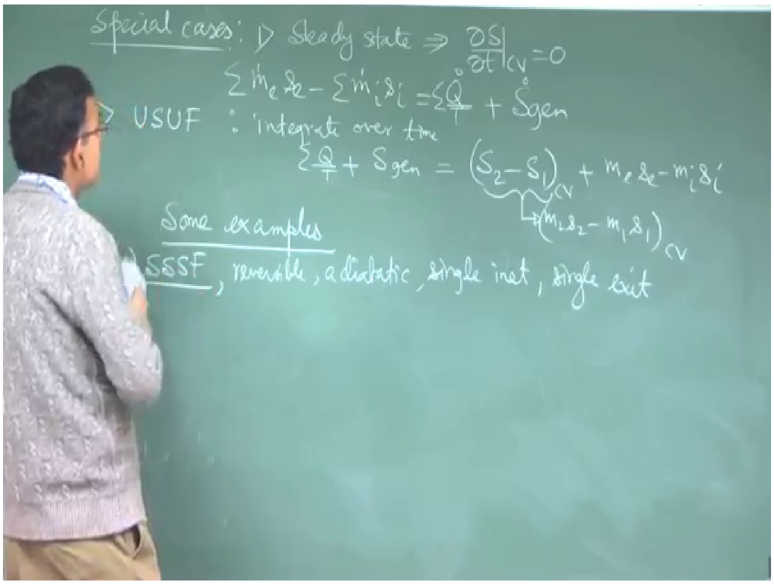
(Refer Slide Time: 08:12)



Now we will consider two special cases, steady state within the control volume there is no change of state overtime. But at a given time it is uniform so, state 1 is uniform state 2 is uniform in case of steady state there is no difference between states 1 and 2 so; that means, you can write summation of m dot e s e minus summation of m dot i s i is equal to summation of Q dot by T in the summation T is not outside, Q dot by T as a whole is the term plus rate of entropy generation so, this is case 1,.

Case 2 is uniform state uniform flow. So, in this case it can be un-study, but at a given time the property is uniform within the control volume. So, for this case what we do is we integrate this over time. So, if we integrate this over time; integrate overtime. So, summation of Q by T plus entropy generation all the time derivatives are gone because they are integrated over time ok. So, S 2 minus S 1 c v is m 2 s 2 minus m 1 s 1 ok. So, it is very much similar to entropy change is equal to heat transfer by T plus entropy generation, this is the broad structure that you have to keep in mind.

(Refer Slide Time: 11:26)



Special cases: ▷ Steady state $\Rightarrow \left.\frac{\partial S}{\partial t}\right|_{CV} = 0$

$$\sum \dot{m}_e s_e - \sum \dot{m}_i s_i = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

▷ USUF : integrate over time

$$\sum \frac{Q}{T} + S_{gen} = \underbrace{(S_2 - S_1)_{CV}}_{(m_2 s_2 - m_1 s_1)_{CV}} + m_e s_e - m_i s_i$$

Some examples

1) SSSF, reversible, adiabatic, single inlet, single exit

Now, based on this we will consider some examples, steady state steady flow reversible, adiabatic, single inlet, single exit. So, because the examples that we will consider in this lecture will be steady state steady flow, I am just erasing the uniform state uniform flow part of the theory to reduce your confusion. So, we will be using this equation steady state steady flow reversible, adiabatic single inlet, single exit, because it is single inlet and single m dot i equal to m dot e equal to m dot.

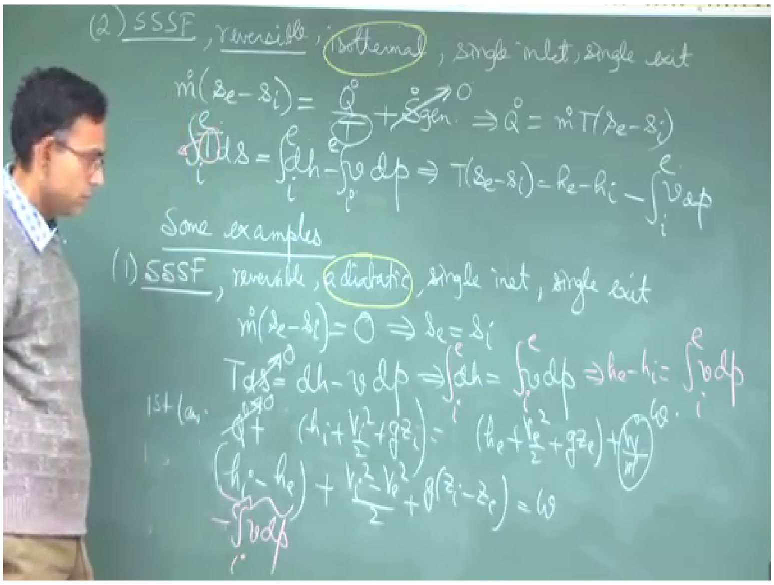
So, m dot into s e minus s i, adiabatic means heat transfer equal to 0 and reversible means entropy generation equal to 0. So, reversible plus adiabatic with single inlet and exit is this; that means, s e equal to s i. Now we can use the Tds formula; Tds is equal to dh minus vdp right this we can always use across any property. So, in this case because ds is 0 between states i and e, any state in between will have ds equal to 0 because the process in equilibrium is changing from state i to state e without creating any net change in entropy between any successive state.

So, ds equal to 0, integral of d s is 0, but ds individually between steps is also 0 so; that means, you can write dh is equal to vdp. We will use this in a first law for the steady state steady flow q dot plus m dot i h i plus p i square by 2 plus gz i is equal to m dot e h e plus ve square by 2 plus gz e plus w dot c v ok. So, here it is adiabatic.

So, let us just write down w dot without writing c v. So, q dot is 0, m dot i and m dot e are m dot. So, if you divide both the sides by m dot then this w dot by m dot let us call this as specific work small w. So, h i minus h e plus V i square minus v e square by 2 plus g into z i minus z e is equal w. To calculate this h i minus h e, we can integrate this from i to e or e to i whatever this d h is equal to vdp right. So, h i minus h e or in this case is h e minus h i is equal to integral vdp from i to e.

So, h i minus h e is nothing but it equal to minus integral vdp from i to e ok. So, we get an expression for the work done which is minus integral vdp plus V i square minus V e square plus 2 z i minus z e.

(Refer Slide Time: 17:08)



(2) SSSF, reversible, isothermal, single inlet, single exit

$$\dot{m}(s_e - s_i) = \frac{\dot{Q}}{T} + \cancel{\dot{S}_{gen}}^{\,0} \Rightarrow \dot{Q} = \dot{m}T(s_e - s_i)$$

$$\int_i^e T\,ds = \int_i^e dh - \int_i^e v\,dp \Rightarrow T(s_e - s_i) = h_e - h_i - \int_i^e v\,dp$$

Some examples

(1) SSSF, reversible, adiabatic, single inlet, single exit

$$\dot{m}(s_e - s_i) = 0 \Rightarrow s_e = s_i$$

$$T\cancel{ds}^{\,0} = dh - v\,dp \Rightarrow \int_i^e dh = \int_i^e v\,dp \Rightarrow h_e - h_i = \int_i^e v\,dp$$

1st law

$$\cancel{\dot{q}}^{\,0} + \left(h_i + \frac{V_i^2}{2} + gz_i\right) = \left(h_e + \frac{V_e^2}{2} + gz_e\right) + \frac{\dot{W}}{\dot{m}}$$

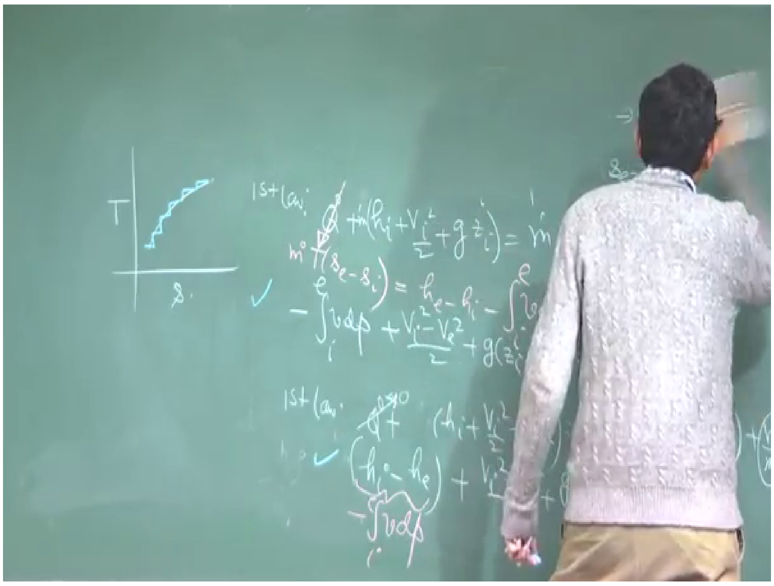
$$\underbrace{(h_i - h_e)}_{-\int_i^e v\,dp} + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e) = w$$

Next example that we will consider is another limiting process, steady state steady flow, reversible, instead of adiabatic it is isothermal, single inlet, single exit. So, only difference is this one, I am highlighting the different here it was adiabatic, here it is isothermal ok.

So, let me write the entropy transport equation m dot into s e minus s i is equal to q dot by p, there is only one temperature here because it is isothermal. So, no summation is required plus entropy generation because it is reversible entropy generation is 0. So, Q dot and this T is constant. So, Q dot is equal to m dot T into s e minus s i. Now you use this Tds exactly the same steps we will follow as the previous case, Tds is equal to dh minus vdp ok.

So now, you integrate it from i to e ok. So, if you integrate it from i to e and here T is a constant because it is a isothermal process. So, you will have T into s e minus s i is equal to h e minus h i minus integral vdp from i to e right and then we will apply the first law.

(Refer Slide Time: 20:11)

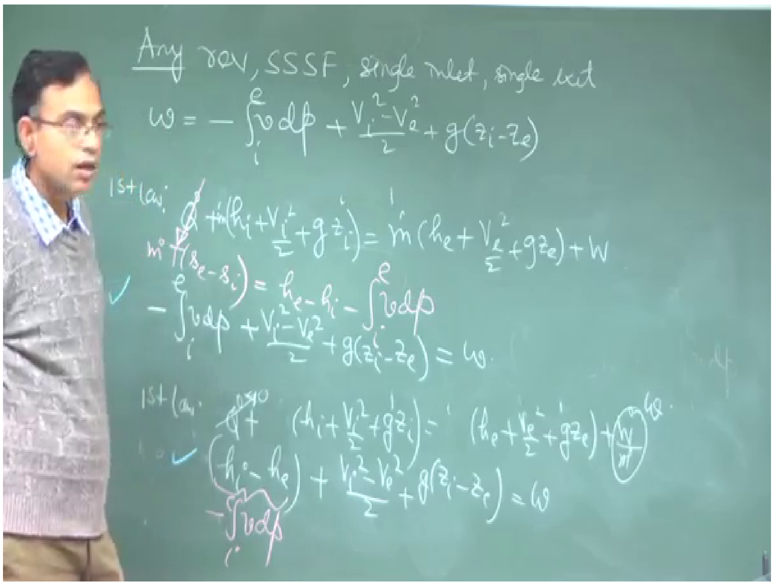


So, first law q dot plus h i; sorry, m dot into h i plus V i square by 2 plus gz i is equal to m dot into h e plus V e square by 2 plus gz e plus w dot. In place of q dot, this is reversible process. So, q dot is m dot into integral Tds. So, m dot T into s e minus s i right, you can use that Tds formula if it is reversible process. This is h e minus h i minus integral vdp from i to e. So, if you substitute that you will see h e and h i terms get cancelled and you are left with minus integral vdp from i to e plus V i square minus V e square by 2 plus g into z i minus z e is equal to w that is w dot y m dot.

See very interestingly, no matter whether it is a adiabatic process or an isothermal process you get the same expression back at the end. Now, any reversible process can be conceptualized as a succession of a number of a reversible adiabatic and reversible isothermal processes because these two are very limiting processes. And using these two limits, you can construct any other reversible process which may not be adiabatic or may not be isothermal, but it can be thought of as a succession of a number of adiabatic and isothermal processes ok. So, I will tell you why, see consider the T s diagram temperature entropy diagram. Let us say this is an arbitrary process. So, you can think it as a collection of steps like this, small steps, the horizontal line is constant temperature and vertical line is constant entropy which is reversible adiabatic.

So, any process can be thought of as a succession of reversible adiabatic and reversible isothermal process this concept you have to develop. That means, we can say that now for any reversible process this equation is true, provided it is reversible steady state steady flow, single inlet and single exit.

(Refer Slide Time: 23:41)



Any rev, SSSF, single inlet, single exit

$$w = -\int_i^e v\,dp + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e)$$

1st law:

$$\cancel{Q} + \dot{m}\left(h_i + \frac{V_i^2}{2} + g z_i\right) = \dot{m}\left(h_e + \frac{V_e^2}{2} + g z_e\right) + W$$

$$\dot{m}\,\cancel{T}(s_e - s_i) = h_e - h_i - \int_i^e v\,dp$$

$$-\int_i^e v\,dp + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e) = w$$

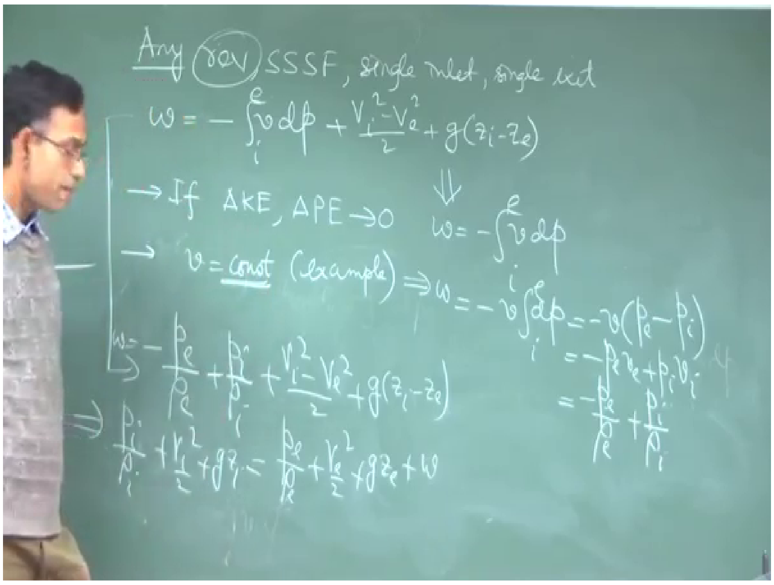
1st law:

$$\overset{=0}{\cancel{q}} + \left(h_i + \frac{V_i^2}{2} + g z_i\right) = \left(h_e + \frac{V_e^2}{2} + g z_e\right) + \boxed{\frac{W}{\dot{m}}}^{w}$$

$$\underbrace{(h_i - h_e)}_{-\int_i^e v\,dp} + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e) = w$$

So, the generalisation of our theory which is very interesting is that for any reversible process; so, any reversible, steady state steady flow, single inlet and single exit process we can write w is equal to minus integral vdp from i to e plus V i square minus V e square by 2 plus g into z i minus z e ok.

(Refer Slide Time: 24:40)



Any rev SSSF, single inlet, single exit

$$w = -\int_i^e v\,dp + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e)$$

→ If ΔKE, ΔPE → 0 ⇓ $w = -\int_i^e v\,dp$

→ $v = \underline{\text{const}}$ (example) $\Rightarrow w = -v\int_i^e dp = -v(p_e - p_i)$

$= -p_e v_e + p_i v_i$

$= -\frac{p_e}{\rho_e} + \frac{p_i}{\rho_i}$

$$w = -\frac{p_e}{\rho_e} + \frac{p_i}{\rho_i} + \frac{V_i^2 - V_e^2}{2} + g(z_i - z_e)$$

$$\Rightarrow \frac{p_i}{\rho_i} + \frac{V_i^2}{2} + gz_i = \frac{p_e}{\rho_e} + \frac{V_e^2}{2} + gz_e + w$$

If kinetic energy and potential energy changes are negligible then this will lead to; see the formula for the work done is no more pdv, but minus vdp. And the reason is that here the work associated with this is not due to movement of the system boundary, but it is also has a; it has primarily something to do with the flow energy or flow work. The work done to maintain the flow in presence of pressure, that has nothing to do with the moving boundary type of work. This is a work done for a control volume process and that particular work done is primarily associated with if there has no other form of work, then it is primarily associated with flow energy or flow work, work done to maintain the flow in presence of pressure.

So, for such a case you have to use the formula minus vdp and not pdv. I am telling this repeatedly because this is a place where students make mistakes very commonly. So, this is one case, another case is v equal to constant example that is a specific volume is constant. So, if v is constant then w is equal to minus vdp from i to e. So, minus v into p e minus p i. So, minus p e v e plus p i v i because specific volume is 1 by density. So, this is equivalently this one. So, now if you substitute it here minus p e by rho e plus pi by rho i. So, w equal to sorry, plus V i square minus V e square by 2 plus g into z i minus z e.

So you can write, p i by rho i, that rho i and rho e are both equal to rho plus V i square by 2 g z i is equal to p e by rho e plus V e square by 2 plus gz e plus w, it is a sort of mechanical energy conservation equation. Looks like Bernoulli's equation, if you set w equal to 0 is becomes it looks like Bernoulli's equation very deceptive this is not Bernoulli's equation. Bernoulli's equation is applied between 2.1 and 2 maybe along a streamline or maybe two different points in the flow field depending on whether the flow is irrotational or rotational. Here i and e are not two points, two they are two sections.

So, it represents a gross mechanical energy balance between them two sections, if it is reversible then there is no dissipation. Reversible process means there is no friction there is no dissipation, in terms of fluid mechanics it mean when there is no viscous effect.

So, if it is reversible, steady state steady flow with single inlet and single exit plus constant density. Then the mechanical energy at section one, this is flow energy, kinetic energy, potential energy together mechanical energy. Mechanical energy at section i is same as mechanical energy at section e plus some work that could be possibly expected. If the work extracted is equal to 0 then the mechanical energy at section i is exactly same as mechanical energy at section e.

So, it is; so, you can see the combination of first law and second law for a steady state steady flow process leads to the consideration of mechanical energy conservation which for a special case gives rise to an equation which looks like the Bernoulli's equation. So, today what we have discussed? We have discussed, the second law of thermodynamics expressed in terms of entropy or quantified in terms of entropy transport across a control volume both for steady and unsteady problems and how that can be applied for steady state steady flow processes with single inlet and exit.

Thank you very much we will start solving some problems on the second law on the entropy transport across control volume from the next lecture onwards.