Concepts of Thermodynamics
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Lecture - 37
Carnot Theorem and Absolute Temperature Scale

In the previous lecture, we were discussing about what is a reversible process and we got into an example of a reversible cycle that comprises all processes as reversible ones. And one such cycle, we put forward as an example that was the Carnot cycle. So, now, Carnot cycle as I have mentioned earlier is a very ideal cycle, which may not be realized in practice, but it can bench mark certain aspects of reversible cycles.

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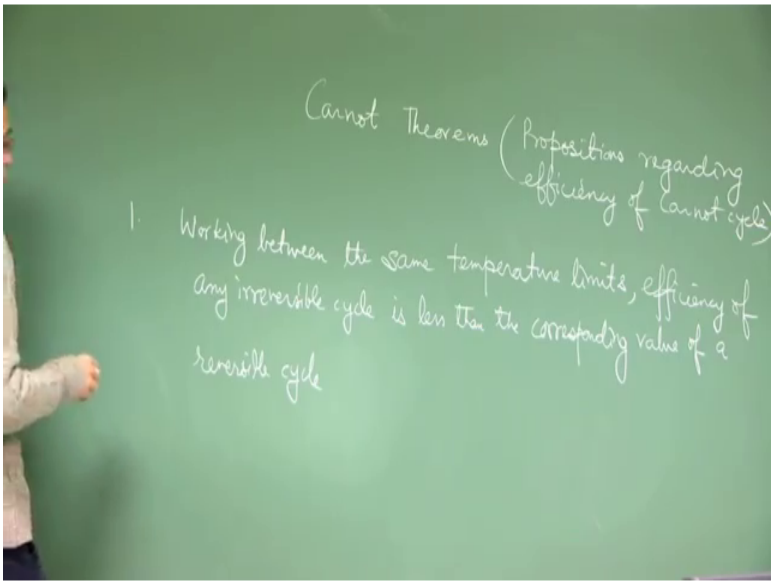

So, to understand that we will refer to certain statements, which are popularly known as Carnot theorem, which are essentially propositions regarding efficiencies of the Carnot cycle. So, these are propositions. So, the first of these propositions is that working between the same temperature limits, efficiency of any irrever cycle irreversible cycle is less than the corresponding value of a reversible cycle ok. So, that means, that the reversible cycle will have the highest efficiency out of all possible cycle working between the same two temperature limits.

How do we prove this? So, let us say we want to prove this by contradiction that means, we assume that the reverse is true that an irreversible cycle may have an efficiency greater than that of a reversible cycle working between the same two temperature limits.

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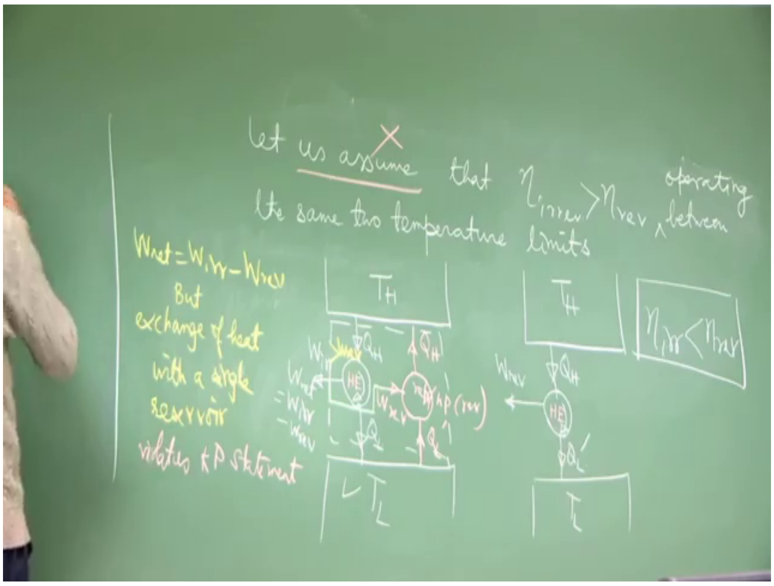

So, let us assume that the efficiency of irreversible cycle is greater than efficiency of reversible cycle between the same operating between the same two temperature limits operating between the same two temperature limits.

So, now the schematic of the system is something like this, you have a high temperature body T H, you have a low temperature body T L. There is a reversible cycle and there is an irreversible cycle. So, let me draw these two different T.

Let us say this is the irreversible cycle. And let us say this is the same corresponding to a reversible cycle. This Q L, so the whole idea is that we are trying to compare two cycles, one reversible, another irreversible that take the same heat Q H, but convert different fraction of that Q H to work. If we assume that you know it eta irreversible is greater than eta reversible, then we are implicitly assuming that for the same Q H W irreversible greater than W reversible right for the same Q H.

Now, we make an interesting arrangement, because this is reversible we can reverse it without changing any net effect. And then this work output will just be the work input, it can be reversed to make a refrigerator or a heat pump. And let us put that refrigerator or a heat pump here ok. So, this is refrigerator or heat pump reversible, this is heat engine, and this is also heat engine.

Now, because W irreversible is greater than W reversible, actually one may make an arrangement like this that from this W irreversible a part of the work is W reversible that is used to run this heat pump. So, far it may appear to be a very feasible arrangement, but as I have already mentioned, we have to check whether this violates any basic statement of the second law of thermodynamics.

So, now what we will do is we will consider these two devices as a whole. Effectively, it does a net work which is what, the net work that comes out of it W net is W irreversible minus W reversible right, because W reversible is fed to run this heat pump.

So, there is a net work output, but exchange of heat only with this reservoir, because with the heat source the net exchange of heat is 0, it is Q H drawn from here and Q H rejected here. So, here what is happening is there is a W net, which is W irreversible minus W reversible, but exchange of heat with a single reservoir ok. So, this violates the Kelvin Planck statement.

So, the very fact that we assumed the efficiency of irreversible cycle greater than efficiency of reversible cycle between same T H and T L that is not a correct assumption. So, let us assume this assumption is incorrect. So, by contradiction, we show that you must have eta irreversible less than eta reversible that must be the case.

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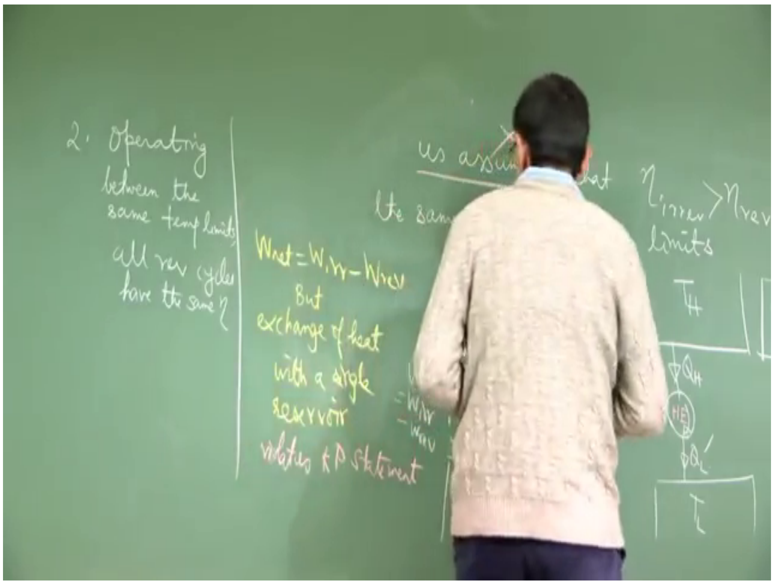

So, following this the next theorem is a corollary of this one, which is that operating between the same two temperature limits or same temperature limits, all reversible cycles have the same efficiency. So, the proof is very simple, instead of a reversible with another irreversible, you say this is reversible 1, this is reversible 2 right. And the contradiction can be resolved only if in the limiting case it has net zero work output that means, W net equal to 0. So, if W net equal to 0, the work output here is same as the work input here. And for same Q H that means, that efficiency of this one can be in the limiting case efficiency of this one provided both are reversible.

So, we have therefore learned two very important propositions. One is that between the same two temperature limits, the efficiency of irreversible cycle is always less than efficiency of reversible cycle, but you know there could be several reversible cycles operating between the same two temperature limits and their efficiencies will all be the same.

So, from this the major conclusion is that the efficiency of a reversible cycle is a function of only the temperature limits, because operating between the same temperature limits. If the efficiencies of all reversible cycles are equal that means, these reversible cycles have their efficiencies which are dependent only on the temperature limits.

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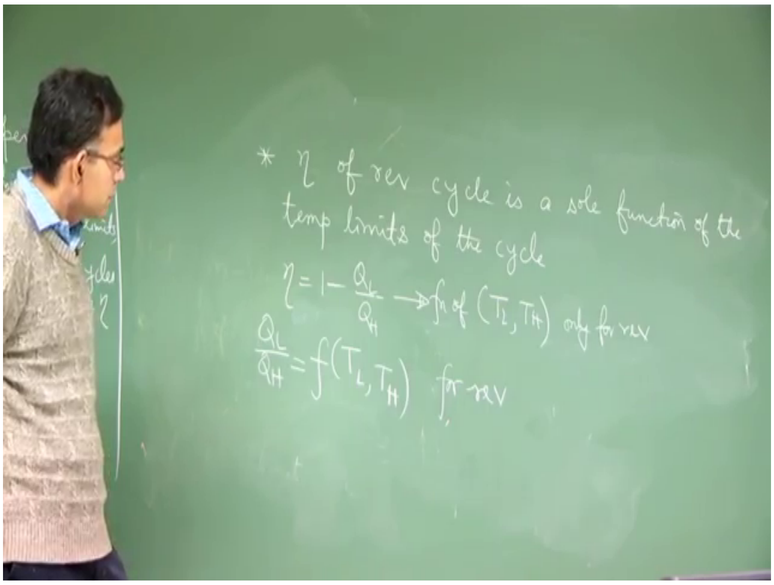

So, the very important conclusion is that efficiency of reversible cycle is a sole function of the temperature limits of the cycle. So, in other words, efficiency is a function of efficiency is equal to 1 minus Q L by Q H. So, this should be a function of only T L and T H right that means, Q L by Q H we can express this as a function of only T L and T H, question is what type of function ok, what is this typical functional form?

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This is eta reversible right. So, eta is so eta is equal to 1 minus Q L by Q H, this is true for all cycles. But, it is a function of T L by T H only for reversible, so that means Q L by Q H is equal to a function of T L by T H for reversible that is what I wanted to mean. So, the only point that remains to be addressed is what kind of functional form.

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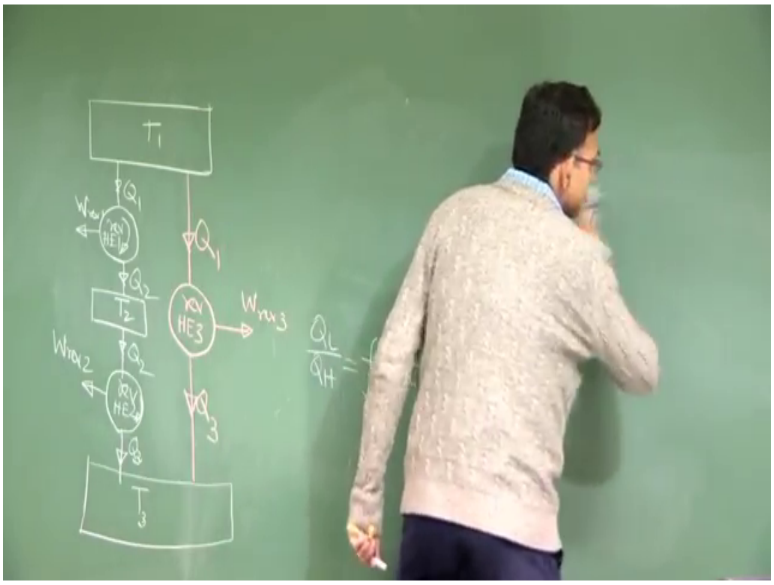

So, to understand that we form a hypothetical setup. So, this is a heat source at temperature T 1, heat Q 1 is taken from this there is a reversible heat engine 1 and then it rejects heat Q 2. There is a thermal reservoir at temperature T 2.

Again the same heat Q 2 is drawn from here. There is a heat engine reversible heat engine 2. So, this heat engine is doing some work W ref 1, this is W ref 2 and then this is rejecting heat Q 3 to a thermal reservoir at temperature T 3; this is one arrangement. Another arrangement is that instead of being mediated by this thermal reservoir at T 2, you have Q 1 taken from thermal reservoir 1. There is a reversible heat engine 3, the q 3 is rejected to this one directly T 3 directly.

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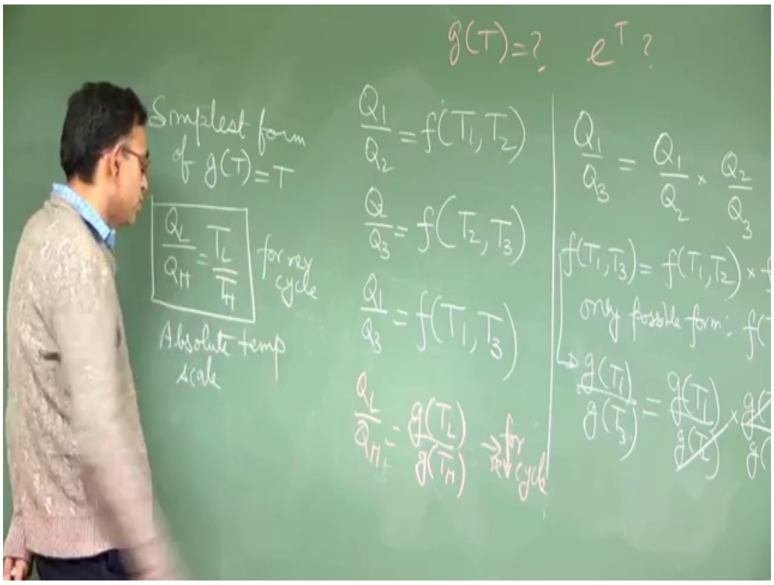

Now, all these are reversible, therefore we can write Q 1 by Q 2 is a function of T 1, T 2 right, because this is reversible heat engine right. Remember that this is true only if it is reversible, although we have not written anywhere irreversible, but in the diagram we have mentioned that this is reversible, so that should be kept in mind. Again I am repeating Q 1 by Q 2 is a function of T 1 comma T 2 only, if it is reversible, otherwise not.

Then similarly, Q 2 by Q 3 is the same function very important, because operating between the same two temperature limits, the efficiency of all reversible cycles are equal that means, the functional dependence of the efficiency with the temperature limits is the same for all reversible cycles. So, the same function f, not another function g, so, this f T 2, T 3 and Q 1 by Q 3 is f T 1, T 3.

Now, we can write Q 1 by Q 3 is same as Q 1 by Q 2 into Q 2 by Q 3 right simple you know nothing to discuss about this. So, f T 1, T 3 is equal to f T 1, T 2 into f T 2, T 3 have a look into this. So, you have f T 1, T 3 is equal to f T 1, T 2 into f T 2, T 3. In this side there is no T 2 in the left hand side. In the right hand side, you have T 2 that means, somehow T 2 got canceled in the right hand side to get T 2 cancelled the only possible form f T 1, T 2 is a function g T 1 by g T 2 right.

Because, then this becomes g T 1 by g T 2 well there could be even other forms which could satisfy this, but you cannot generalize. It in only some special forms, we will satisfy these. To generalize this you can write right so to generalize, we can write f so Q l by Q H is g T L by g T H for reversible cycle right, but what could be this function g in principle, g T could be anything.

So, for example somebody might take e to the power T, mathematically this is not ruled out. But, mathematically you know you are interested about this function not in an effort to do some mathematics, but in an effort to describe the ratio of heat transfer in terms of some ratios of temperature. So, we should use the simplest form of the function g T, which is T itself.

So, the simplest form is a linear form. And then you come up with this relationship for reversible cycle, this defines the temperatures defined in this way are called as absolute temperatures. So, this is the construction of a temperature scale called as absolute temperature scale.

Remember that while beginning with the description or discussion on second law, we promised that we would use the second law to come up with a temperature scale, which is independent of the properties of the thermometric sub-substance. So, here you see that the temperature ratios are functions of heat transfer ratios, these are not dependent on which is the thermometric fluid that is being used.

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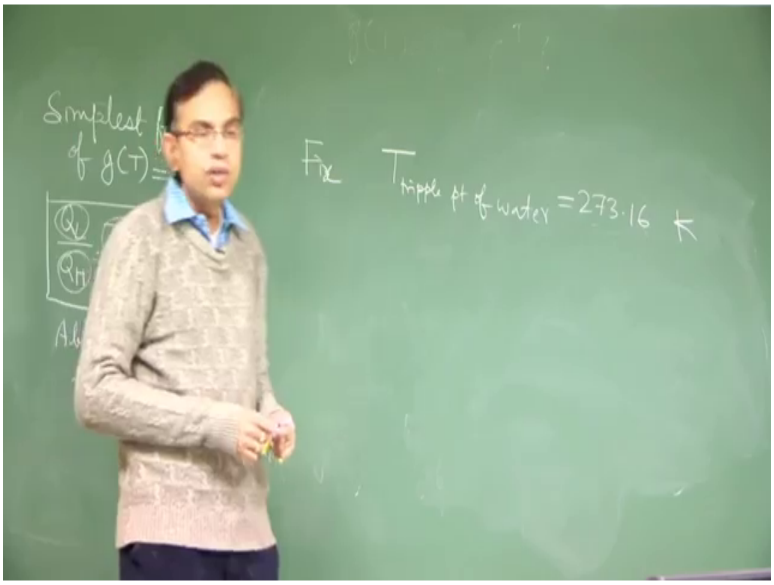

But, still the T L and T H their absolute values are not fixed through this, so this remained to be you know a temperature scale that is theoretically valid, but practically not usable until in 1954 in an international conference on units and measures. It was discussed that a particular value of this one that fix T of triple point of water as 273.16, this was fixed. Triple point of water is a point state point where solid liquid vapour can coexist. And in terms of centigrade, it is 0.01 degree centigrade.

So, in this unit to honor lord Kelvin, who contributed a lot towards developing the absolute temperature scale; this was fixed as 273.16 Kelvin. Remember normally with Kelvin, we do not write degree Kelvin, it is just K Kelvin that is the SI way of you know expressing Kelvin. So, then 1 Kelvin is 1 by 273.6 of this value, so that is how one Kelvin was fixed.

And the other important constraint that was put is that the difference between temperatures for in the unit of Kelvin is same as the difference in temperatures in the unit of centigrade. So, then you know for example, if you could fix up T L as the triple point of water and find out somehow hypothetically Q L, then T H can be obtained, if you can somehow hypothetically obtain Q H.

So, the establishment of this temperature scale as a function of heat transfer, although it is hypothetical because this is for a reversible cycle. So, this cannot be actually done by measurement, but in terms of concept it is for the first time that a temperature unit is established without referring to the volumetric expansion of a thermometric substance, and that is why this is called as absolute temperature scale.

So, to summarize we have discussed about a motivation of learning second law and various statements of second law of thermodynamics, they are equivalences. We have studied what is a reversible process and what are the factors that can make a process irreversible. And then we have discussed about the Carnot cycle as a model, reversible cycle. And eventually, we have come up with a description of an absolute temperature scale, which is independent of the properties of the thermometric substance.

So, with this let us conclude this lecture. And next time when we meet, we will be working out some problems related to whatever concepts we have learned by this time.

Thank you.