Concepts of Thermodynamics

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Lecture – 29
First Law for Unsteady Problems – Examples

In my previous lecture I was discussing about how to write the first law for a process taking place across the control volume when the control volume undertakes or undergoes an unsteady process. And eventually we came to special case of this you know unsteady state evolution of a control volume which is uniform state uniform flow process or US UF process. So, today we will work out a few problems to illustrate these US UF concept. So, the first is the transient, I mean the first is first in the transient problem list is problem 4.9 ok.

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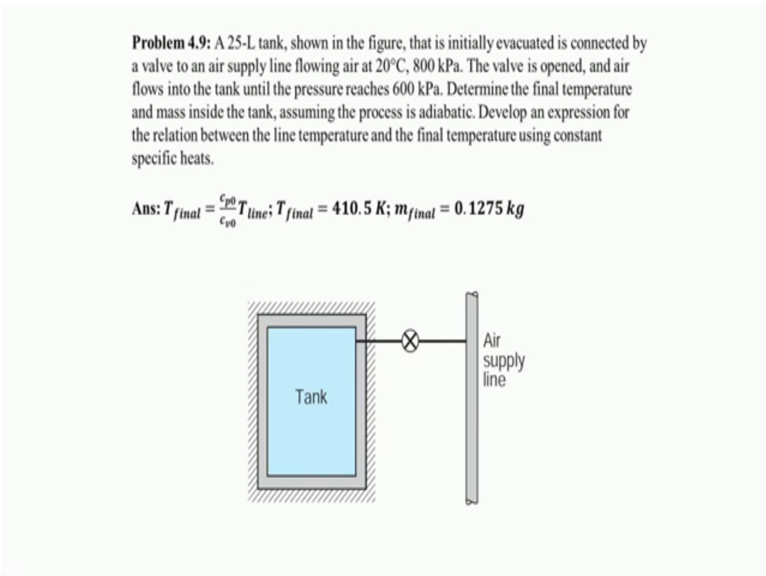

So, this problem 4.9 this is a very classical problem, this is called as a tank filling problem you know. So, what is the tank filling problem, there is a tank which is a 25 liter tank shown in the figure that is initially evacuated, it is connected by a valve to an air supply line flowing air at 20 degree centigrade 800 kilo Pascal, the valve is opened an air flows into the tank until the pressure reaches 600 kilo Pascal.

Determine the final temperature and mass inside the tank assuming the process is adiabatic. So, in the process develop an expression for the relationship between the line temperature and final temperature assuming that the specific heats are constant that is calorically perfect yes ok. So, we will work out the problem as we have done for all the problems in the previous chapters, we will draw a schematic of the problem and give you a outline of how to solve the problem in the board.

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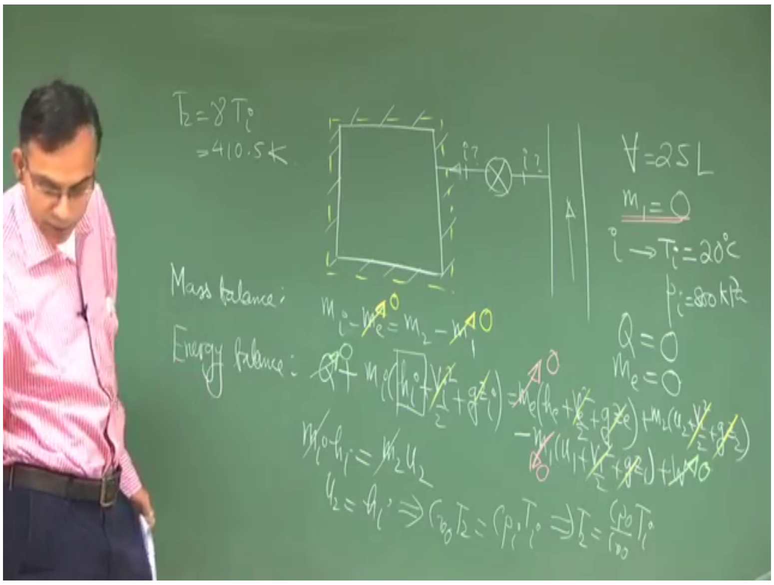

So, you have a tank this tank and there is a air supply line. So, this is a pipe which supplies air, air is moving like this when this. So, initially this tank is empty and this valve is closed, when this valve opens the air from the air supply line enters and fills the tank this is what as I called earlier as tank filling problem. So, this tank is insulated typically for insulated chambers this is how we draw symbols so this shows that it is insulated.

Let us write what is given so V is 25 liter which is the volume of the tank, it is initially evacuated; that means, m 1 is 0 mass within the tank which is the control volume. So, let us draw a control volume here ok, so this is there mass 1 is 0 then air supply line. So, this is the inlet I, state i the air supply line is T i is 20 degree centigrade and p i is 800 kilo Pascal, process is adiabatic so heat transferred is 0.

So, this is like normally I mean from my own student life I use to do this that whatever is given you know as a data somewhere you know write it. So, that it gives you a clarity of what is known and what needs to be found out, because based on that only you will apply the relevant equation. So, at I mean may be all these data may not be necessary for problem solving, but whatever is given you list it somewhere.

So, this is what is given and this tank has only 1 inlet and no exit. So, there is no m e right that also needs to be kept in mind. So, there are 4 states associated with this unsteady process i e 1 2. So, as per the symbols we have used for our derivations these the differences between these 4 should be absolutely cleared to you, i inlet e exit 1 initial state of the control volume and 2 final state of the control volume. So, we have to associate all these states. So, i is there, e is 01 is 0 and 2 you have to you know figure out that what is the state 2 that is the whole problem, so let us write the mass balance.

So, m 2 minus so m i minus m e is equal to m 2 minus m 1 right. So, here m 1 is 0, m e is 0, so m 2 is equal to m i this is so common sense that you know initially no mass was there, so final mass inside is same as whatever mass has come in. So, you really do not need to do with that elaborately, but I am trying to give you a structure to the problem solving. So, that how you use the fundamental equations to arrive at your final theme.

So, energy balance Q plus this, so if you have a feel of what you are writing you know these you do not actually have to memorize this is energy balance. So, you can write energy balance without you know memorizing the formula ok.

So, now let us you know simplify this equation based on what is given, so first of all m 1 is 1. So, because m 1 is 0 you know these term will be totally 0, then m e is 0 this term is totally 0. In this problem what is neglected is the change in kinetic energy and potential energy because, you know here the main problem associated is the transfer of thermal energy, not so much kinetic and potential energy.

So, kinetic energy anyway these term is 0, but for the other terms kinetic energy and potential energy even for any term you can write the changes are not important. On the top of that you know because it is insulated the heat transfer is 0 and because it is a rigid tank there is no system boundary displacement and there is no other form of work that is extracted from this device so this is 0. So you are left with a very simple equation mi hi is equal to m 2 u 2 right and from this equation you know already from mass balance that mi equal to m 2 so u 2 is equal to hi ok.

Now, further assumption that is given in the problem that you can treat this air as an ideal gas with constant C p C v. So, T 2 is equal to C p naught by C v naught C p naught C v naught as C p C v corresponding to a constant C p C v of ideal gas. So, T2 is nothing but T i into the ratio of specific heat C v and CV which is often called as gamma, so the final answer is T 2 is equal to gamma T i. So, very interesting you see try to understand the physics of this problem, then you will appreciate this problem more.

So, you have a tank there is a line which is filling up the tank, the temperature inside the tank is always greater than the temperature in the line because, C p is always greater than C v. Why? The reason is that to penetrate this fluid into the tank an additional energy beyond the internal energy had to be there because, it has to maintain the flow in presence of pressure and that is flow energy or flow work.

So, because in addition the internal energy the flow energy is also coming into the tank that makes the tank energized in a manner, so that it is temperature eventually is greater than the temperature here. Because when the fluid is inside tank the entire flow energy gets converted into internal energy, the internal energy here plus flow energy here, this total energy gets converted into internal energy within the tank and that makes the temperature you know shoot up from the temperature here. So, this is the physics of the problem.

The other important point that I intend to make is that you know when we think of state i, the inlet can we take it here or can we take it here is there any difference see for problem solving purpose there is no difference, why because eventually for the state I we require only the enthalpy right and enthalpy this is a throttle valve.

So, enthalpy before throttling is same as enthalpy after throttling right, with all those assumptions that we had earlier discussed. So, that makes this problem solving absolutely on light no matter whether you take i here or here, but when you take i here you incur those additional assumptions that make sure that enthalpy here is same as enthalpy here, because that is based on some approximation, but those assumptions are valid for most of the practical problems ok.

So, the answer to this problem is 410.5 Kelvin remember that when you write a relation like this the temperature has to be in Kelvin on both sides. So, let me erase the board and then we will move on to the next problem.

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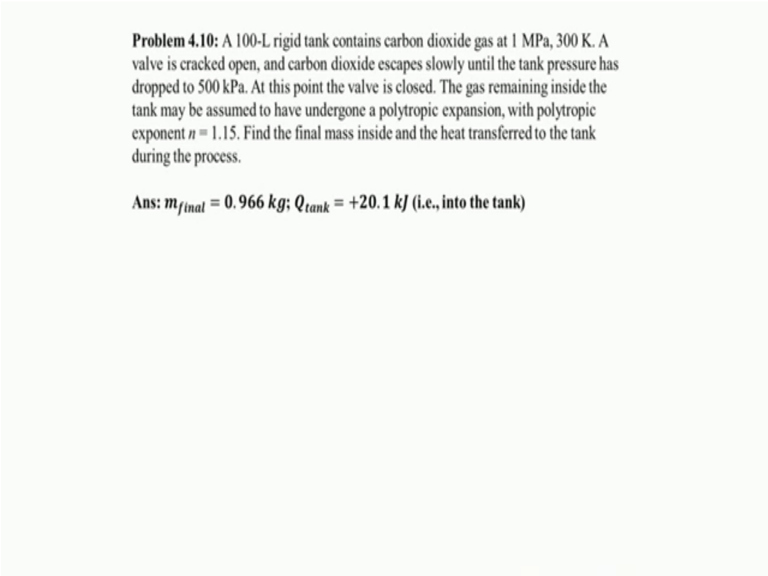

Problem 4.10: A 100-L rigid tank contains carbon dioxide gas at 1 MPa, 300 K. A valve is cracked open, and carbon dioxide escapes slowly until the tank pressure has dropped to 500 kPa. At this point the valve is closed. The gas remaining inside the tank may be assumed to have undergone a polytropic expansion, with polytropic exponent $n = 1.15$. Find the final mass inside and the heat transferred to the tank during the process.

Ans: $m_{final} = 0.966\ kg$; $Q_{tank} = +20.1\ kJ$ **(i.e., into the tank)**

Problem then problem number 4.10. A 100 liter rigid tank contains carbon di oxide at 1 MPa, 300 Kelvin. A valve is kept crack open and carbon dioxide escape slowly until the tank pressure has dropped to 500 kPa. At this point the valve is closed the gas remaining inside the tank may be assume to have undergone a polytrophic process polytrophic expansion with the polytrophic exponent n is equal to 1.15. Find the final mass inside and heat transferred into the tank during the process.

So, to summarize this problem is a tank emptying problem instead of a tank filling problem. So, in the previous problem fluid was filling up the tank in this problem fluid is leaving that.

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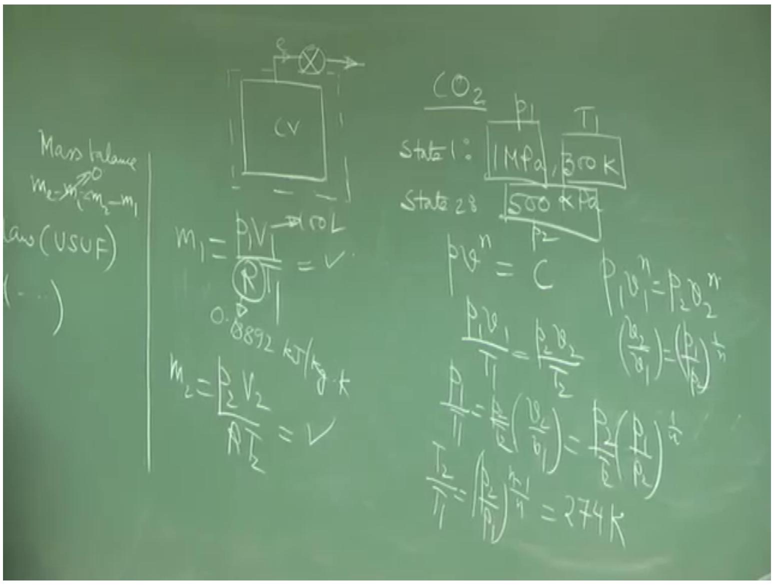

So, let us try to draw a schematic and as the previous problem try to identify the relevant properties. So, this is your control volume and you have a state e. So, the fluid is carbon di oxide state 1, 1 MPa 300 Kelvin; state 2, 500 kPa between state 1 and 2 you have a polytropic process which is defined as pv to the power n equal to constant remember polytrophic is not I mean polytropic is a generalization of all processes. Because you know if you put n equal to gamma which is the ratio of C p by C v then it is a special type of adiabatic process, which is called as reversible adiabatic process we will learn this later on.

When n is equal to 0 so that means p equal to constant so it is a isobaric process, similarly there will be cases when n is equal to one then p V equal to constant so that is called as reversible isothermal process and then you can also have a v equal to constant as a part of this process. So, you can write v equal to C into p to the power minus n ok. So, you can set what value of n you think about this to make sure that these constant so that is called as isochoric process. So, different types of processes can be modeled this is a model remember the value of n will dictate what is the physical process; so, in this case yes.

Student: sir, one (Refer Time: 18:58) I think it is p into specific volume to the power of n.

In this question yes you are correct, in this question it is p into specific volume to the power n, but you know always we write it specific volume just to recognize that it is for a given mass of gas. So, to make sure that the mass reference is same. So, why this is important? In this case state 1 and state 2 will have different mass to rationalize with respect to a given mass if you write the chain then it is specific volume instead of total volume. In case for our control mass system it does not make any difference instead in case of this particular case where the mass in the control volume is changing with time this is what is more appropriate.

So, now state 2 temperature you do not know, but what you know that you have p 1 v 1 by T 1 is equal to p 2 v 2 by T 2 this is also on a mass basis given mass right. So, p 1 by T 1 is equal to p 2 by T 2 into v 2 by v 1. So, you can write p 1 v 1 to the power n is equal to p 2 v 2 to the power n. So that means this into v 2 by v 1; so v 2 by v 1 is equal to p 1 by p 2 to the power 1 by n.

So, now, you can simplify this and write. So, T 2 by T 1 is equal to p 2 by T 1 to the power n minus 1 by n right. So, then this is T 1 this is p 1 this is p 2 so you can calculate what is T 2 from here, so T 2 is 274 Kelvin ok. So, m 1 now you use this one this is total volume because you want to get the mass, this R is not universal constant it is universal constant by molecular weight so universal constant divided by molecular weight of carbon di oxide.

So, this you can calculate because you have v 1 as 100 liter which is same as v 2 p 1 you know, then R for carbon di oxide you can find out and p 1 is given 300 Kelvin. So, you know what is the amount m 1 R of carbon di oxide just for your reference I am giving you this value 0.18892 kilo joule per kg Kelvin ok.

So, then you can also calculate what is m 2 ok. So, this you can find out. So, now you can use the first law, so first law so for mass balance me minus mi is equal to m 2 minus m 1 right. So, here the you have mi is equal to 0 there is no I, so me is m 2 minus m 1 m 2 m 1 you know. So, you know what is m then you come to first law for US UF.

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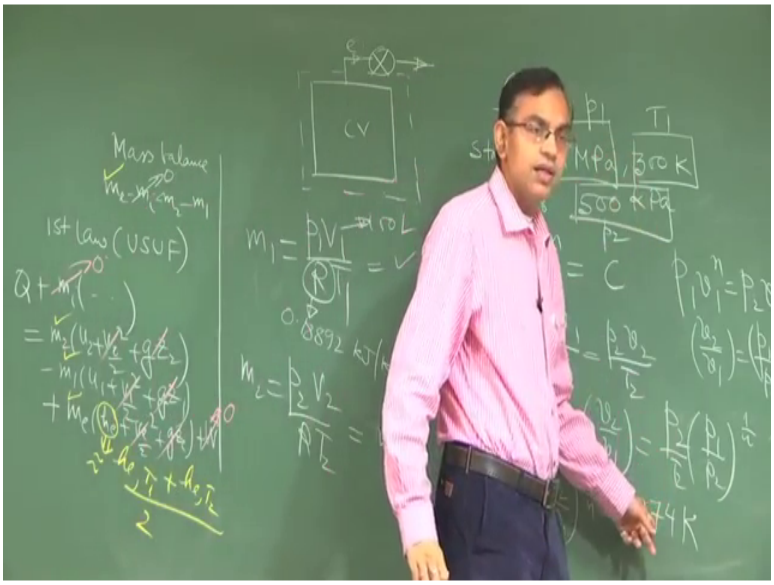

So, Q plus mi into whatever here mi is 0. So, I am not interested about that so m 2 into u 2 plus v 2 square by 2 plus gz 2 minus m 1 into u 1 plus v 1 square by 2 plus gz 1 plus m e into h e plus d square by 2 plus gz e plus W ok.

So, then again a rigid tank there is no W, but it is not an insulated tank anymore so there is heat transfer. In fact, that is the question find the heat transfer to the tank during the process. So, then this term is 0 we neglect changes in kinetic energy and potential energy.

So, you have m 2 m 1 all this known and then you have me also from this equation, you know you know u 2 and u 1 because state 2 and state one are already known assuming ideal gas they will be functions of temperature only. So, given the temperature 300 Kelvin and state to this 274 Kelvin you can find out. Their internal energies there is a critical part of this problem that what is he see the state e is the exit state, but you know this continuously changes as the gas is leaving, so how do you assess it.

So, essentially a good approximation to this remember this is what engineers commonly do, if we do not have an exact idea of the picture we give an approximation to that. So, what is the approximation? So, this is roughly like he at T 1 plus he at T 2 by 2 roughly, but T 1 is 300 Kelvin T 2 is 274 Kelvin the state e is continuously changing as that you know state in the tank changes from 300 Kelvin to 274 Kelvin. So, e is no more a fixed state which we assumed as a part of our derivation.

So, here there is a conflict between this the derivation of this equation and the physical situation, to resolve that conflict or paradox this is what we are assuming. So, this assumption is very very important you know otherwise you cannot solve this problem with a practical outlook and if you do that and then for you know u and h you can use constant C p C v.

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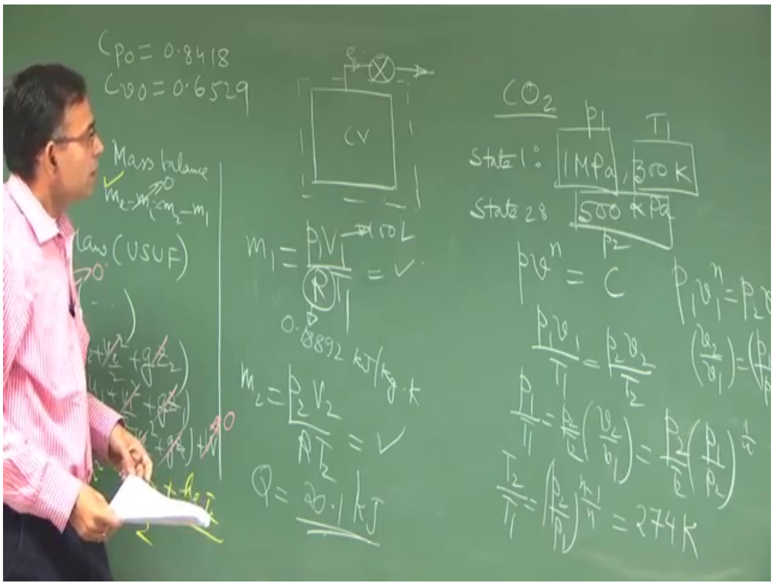

So, you can assume C p 0 is equal to 0.8418 this kilo joule per kg Kelvin and C v 0 is equal to 0 0.6529. So, if you consider these then you will get the heat transfer Q is equal to 20.1 kilo joule ok. So, why do you require a heat transfer to the tank in this problem? See whatever energy was in that tank the tank is trying to carry more energy out of it because the flow energy is also going out. So, to compensate for that additional loss of energy a heat has to be transferred ok.

So, let us stop here for the time being we will continue with more problem solving in a next lecture.