Concepts of Thermodynamics
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Lecture – 19
Control Volume Conservation Reynolds Transport Theorem

So, far we have discussed about first law of thermodynamics, in the context of situations, where we are considering control mass system. That means, a system of fixed mass and fixed identity. Now, we will try to consider situations, where we will apply the first law of thermodynamics, for not a control mass system, but for a flow process. So, when there is a fluid flow taking place, the situation may no more be addressed by a control mass system and the reason is as follows.

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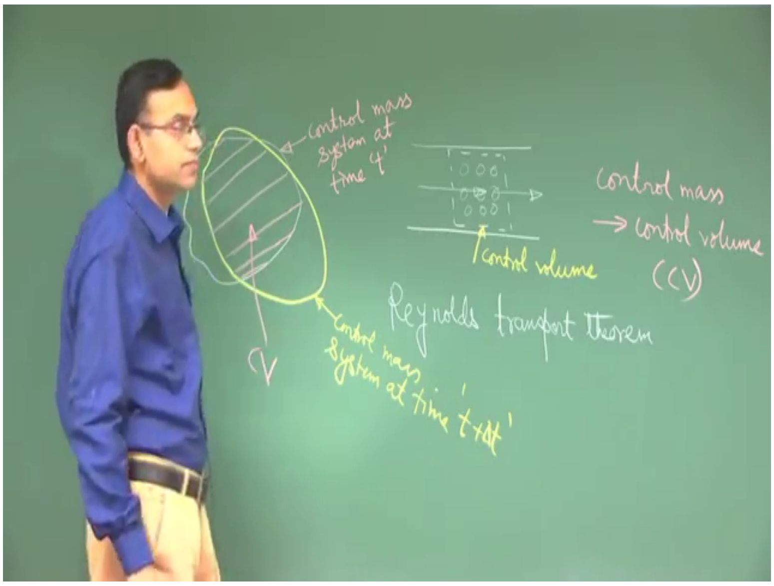

So, if you have a pipe like this and say some fluid is flowing, so this fluid can no more be considered as a control mass system, because within the pipe not the same fluid is flowing at all the times. So, if you have by same fluid I mean same fluid molecules. So, if you have a specified region in the pipe, then in this specified region what is happening is; some fluid is entering, some fluid is leaving. At a given instant of time maybe the mass within this, may not change, because if it is a steady flow of the same rate the mass enters, that is the same rate at which the mass leaves, but the identity of the mass has changed.

So, remember the definition of a control mass system is something of fixed mass and fixed identity. So, the molecules here, the total mass maybe fixed at a given instant of time may or may not be. It is fixed, if it is a steady flow, but different molecules are occupying, this at different instance of time and therefore, it is not a control mass system anymore. So, this is better represented by something called as control volume. So, what is this control volume?

So, this control volume is a specified region in space, across which there is flow of there is transport of mass, there is transport of momentum, there maybe transport of energy. So, there is some kind of transport across this region in space. So, what is identified here is not this particles of molecules, but a region across which the fluid flow is taking place.

So, physically it is as if you sitting with a camera and focusing your camera on this specified region. So, some mass is entering, some mass is leaving, some momentum is entering, some momentum is leaving, some energy is entering, some energy is leaving. So, this is alright so, we have a shift of paradigm from control mass approach to control volume approach.

In short sometimes for control volume, we call it CV. So, from control mass to control volume approach, when we make this shift of paradigm, then the question is that are our equations representing conservation of mass, conservation of momentum, conservation of energy. Are these equations remaining the same or are these equations getting altered? The principles remain the of course the same, but their mathematical expressions tend to get altered. Why their mathematical expressions tend to get altered? The reason is straight forward, our classical ways of expressing the conservation laws are all based on control mass approach like, Newton’s second law of motion.

It is a traditionally expressed, using a control mass approach. Now, when you have to change the approach from control mass to control volume, you have to make some adjustments in your mathematical formulation and we will learn, what kind of adjustments, you will require in your mathematical formulation, which is realized through a theorem called as Reynolds transport theorem.

So, Reynolds transport theorem is a theorem that commits a control volume based conservation principle or conservation law to the same conservation law applied to a with reference to a control mass system. So, to understand the this theorem let us say that; this is the control mass system at time t. So, within this there are some molecules or particles whatever you call it, macroscopically particles. Now, after some time let us say that this occupies this configuration just arbitrary.

This is control mass system at time t plus delta t. If delta t is very small, then these two configurations are almost overlapping. So, this two different configurations are drawn just for clarity in distinguishing their positions, but imagine that when delta t is very small, they are almost overlapping with each other.

So, when they are almost overlapping with each other, their common zone can easily be identified and this common intersection region is the identified region in space across which we are studying the transport so, this is our control volume. So, see so nicely the control volume picture is emerging out of a control mass system picture. So, we have a control mass system at time t, you have the same control mass system over a time change delta t, so that the system has only infinitely changed its configuration. So, the common intersection between these two is maybe your region of interest, which you call as control volume.

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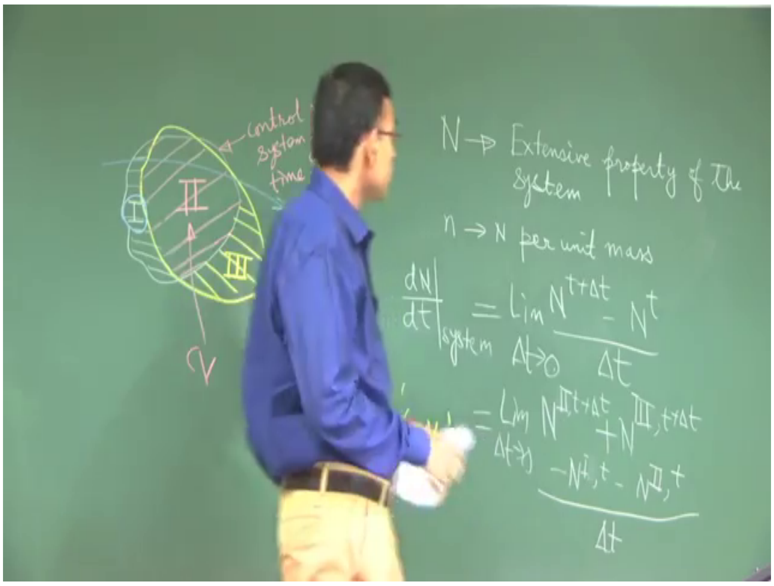

So, then let capital N be an extensive property. So, extensive property you have already understood the definition. Extensive property of the system means a property of the system, which depends on the total extent or total mass.

For example; total mass, total momentum, total energy like that and small n is capital N per unit mass. Now, let us divide this entire domain, which is there into three parts; the 1st part is this one, which we call as I, the 2nd part is this one, which we call as II and the 3rd part is this one, which we call as III. So, when the system, when the control mass system is sort of sweeping in this direction, then this I region is like the region across which the fluid is entering the control volume and III is the region across which fluid is leaving the control volume.

So, this is like inflow and this is like out flow. So, now we are interested in calculating dN dt of the system, that is the rate of change of N, capital N with respect to time for the system.

So, when we do that, remember that we are interested for the expression for rate of change, when expressed in terms of the control mass system ok. So, what is our objective? Our objective is to express this in terms of a corresponding rate of change of N, but with respect to the control volume. So, system based analysis to control based analysis and vice versa. So, by definition this is as good as limit as delta t tends to 0, N at t plus delta minus N at t divided by delta t.

Now, when you say limit as delta tends to 0 N at t plus delta t minus N at t by delta t this N comprises N at I and N at II, because at time t the region occupied is I plus II. So, this is limit as delta t tends to 0 and when you write the sorry, this is N t plus delta means II and III right. So, at t plus delta t it is II plus III and at t, it is I plus II.

So, N II t plus delta t plus N III plus delta t II, plus III these two regions make it at time t plus delta t minus . So, 2 plus 3 is a delta t t plus delta t I plus II at t.

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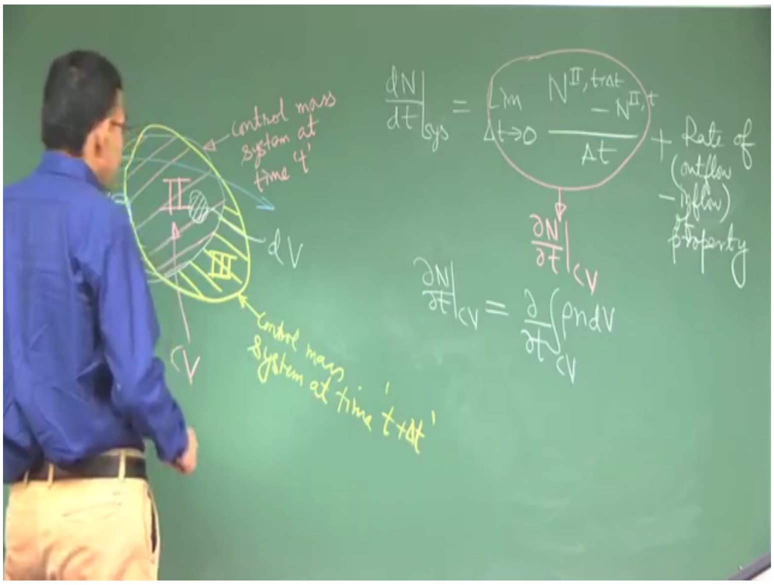

So, now you can isolate these II terms with a commonality. What is the commonality? Both of these represent the region II, which the control volume. So, dN dt of the system plus. When you write plus you have this term and this term divided by delta t. So, what is this term divided by delta t? This term divided by delta t is the rate of transport of the physical quantity across this three and that essentially means outflow, if it is sweeping in this direction and this similarly means inflow, so this plus rate of outflow minus inflow of property.

So, finally, what is this? This is the rate of change of N with respect to time as seen from the control volume. See very interestingly we write it as a partial derivative with respect to time, because we are freezing up position and then finding rate of change with respect to time. So, rate of change with respect to time for a given position, whatever position we are fixing up.

So, it means that you have dN dt of the system, this is the control mass of the system base rate of change, this is the dN dt with respect to control volume, this is the control volume based rate of change of N and this is rate of outflow minus inflow of property. Now what is this del N del t with respect to control volume? So, you can take a small volume dV, so then this is rate of change of the property within the control volume. So, within dV the property is, within dV the mass is rho into dV small n is property per unit mass. Similarly we have to write an expression for the rate of outflow minus inflow, which is the final task remaining before we are able to complete the mathematical description of these entire term, because it is till now qualitative lead, we have to write a quantitative expression for that.

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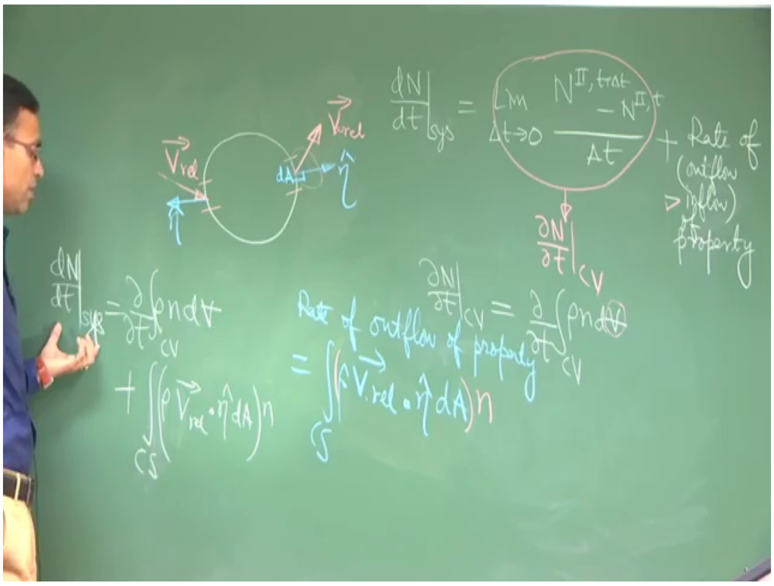

So, imagine that there is a control volume. So, let me erase this figure so, that I can draw a fresh figure, where we will have only the control volume. So, you have a control volume like this, where this is an outflow boundary; that means, fluid is flowing out. See this, the essence of transport phenomenon is like when fluid is flowing in or out, it is carrying with it some transport quantity; like it is carrying with it mass, momentum, energy.

So, that is why, that is how fluid flow is connected with say for example, heat transfer and work transfer in thermodynamics. So, fluid flow is carrying away some energy and that energy transfer may lead to some heat transfer work done, mass transfer etcetera and that is how the fluid flow and energy transfer is related. So, here let us say that the velocity vector is u. This velocity is essentially velocity of the fluid relative to the control volume, because that is what which essentially matters for the transport of mass momentum, energy whatever. Let eta be the unit normal vector perpendicular to this surface, we have taken a small area dA, so that you know you can have a fixed eta otherwise eta will change across the surface.

The normal direction will change and this is a very important vector, because the direction of this vector essentially describes the orientation of this area ok. So, what we can write is that the rate of outflow. What is that? Rate of outflow of property integral. So, what is the rate of flow here? So, this is the rate of flow and rate of flow of property is this times the property n, which is property per unit mass.

So, this is integrated over the surface of the control volume which is called control surface. Now so far it is fine for describing the outflow, but what about inflow. So, for inflow you have a surface like this with fluid entering the surface. The difference between inflow and outflow boundary is, for the outflow boundary the outward normal is the unit normal outward to the surface is oriented in a similar direction of that as the flow. Whereas, in this case the outward normal is not in the similar direction.

Therefore, this dot product will give positive, but this dot product will give negative algebraic expression. So, because this negative term is here, then a negative value of this will automatically imply that it is inflow. Because if you take this algebraically outflow minus inflow, we do not care about whether it is this expression is they are separately for outflow and inflow. If it comes out to be plus it will be automatically outflow, if it comes out to be minus it will be automatically inflow.

So, we can write eventually. Now before concluding this lecture I will make a change of the symbols, because some of the symbols in fluid mechanics conflict with some of the symbols in thermodynamics. For example, u in thermodynamics we have earlier preserved for internal energy right. So, it is better not to use u for velocity here. We will better use V for velocity instead of u.

So, we will switch from u to V, but when you switch from u to V there was another term which contained V as a symbol and this is volume. So, what we will do is, we will just put a different symbol to indicate volume here. So, the change in symbol is accounted for here and it becomes dN dt of the system is equal to partial derivative with respect to time rho and dV, this V is for volume and then rho V relative dot eta dA into n.

So, what we have successfully done here, is we have converted an expression of the rate of change of N with respect to a control mass system to a rate of change with respect to a control volume based system. So, this is what is the essence of the Reynold’s Transport Theorem.

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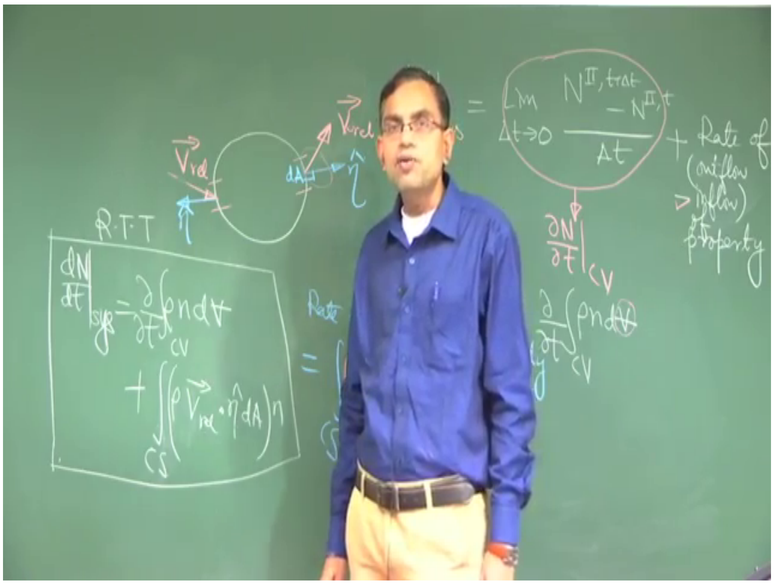

In the next class what we will do is, we will try to use this Reynolds Transport Theorem for conservation of mass and conservation of energy, which are essentially the hallmarks of applying the 1st law of thermodynamics for a flow process.

Thank you.