Concepts of Thermodynamics
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Lecture – 13
Heat and Work : Representative Problems (Contd.)

We will continue with solving problems related to Work done and Heat transfer.

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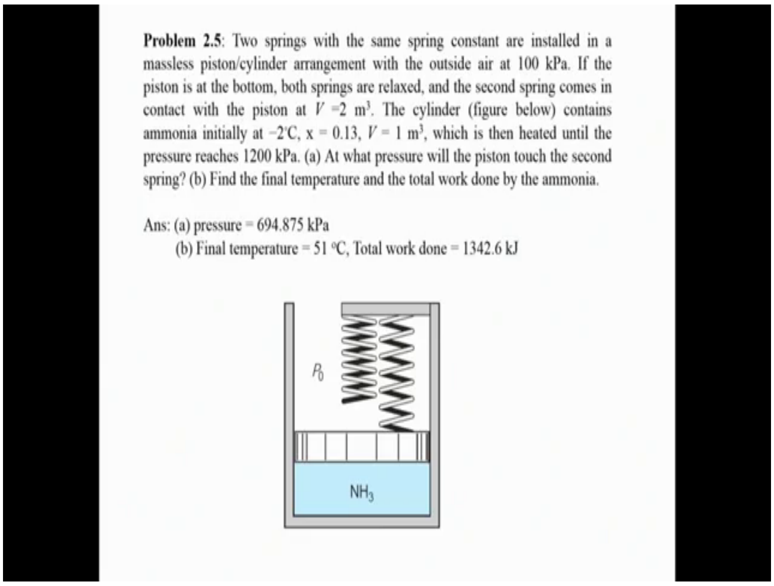

And we will start with the problem which is displayed in the screen. So, this problem is little bit advanced as compared to some of the previous problems, where we had only one spring. Now, here you have two springs; two springs with the same spring constant are installed in a massless piston cylinder arrangement. So, there is the piston as negligible mass and the outside air is 100 kilo Pascal. If the piston is at the bottom both springs are relaxed and the second spring comes in contact with the piston at 2 meter cube volume.

The cylinder contains ammonia initially at minus 2 degree centigrade quality equal to 0.13 and volume is equal to 1 meter cube which is then heated until the pressure reaches 1200 kilo Pascal. At what pressure will the piston touch the second spring, find the final temperature and the total work done.

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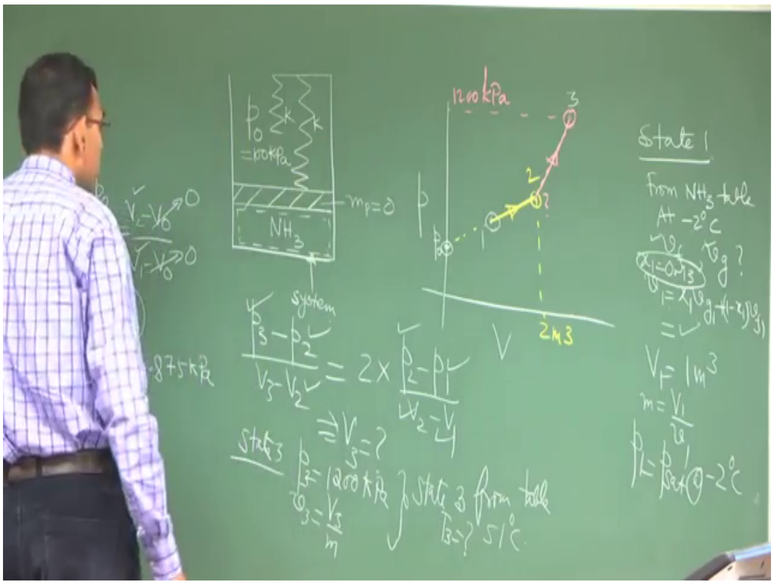

So, let us go through this problem carefully. So, both the springs have spring constant k mass of the piston is 0, p 0 is 100 kilo Pascal. So, whatever data is given what we will try to do is that based on the given data, we will try to plot PV diagram for the process. So, let us so, the first thing is given that if the piston is at the bottom; this is the hypothetical, if the piston is at the bottom not really that it is actually at the bottom. If the piston is at the bottom means if the volume is 0, then both the springs are relaxed; that means, there is no spring force. And therefore, pressure will be equal to p 0 so, this is again a hypothetical point.

The hypothetical point so, you may argue that when the volume is 0 then how do you have a pressure; see you have no fluid inside. So, here you have to consider a limiting volume V which is tending to 0 because, the piston is just at the bottom some limiting volume is there. And for that limiting volume there is an inside pressure which is balanced by p 0, because both the springs are relaxed and the weight of the piston is also 0. So, what balances at that relaxed condition is only the atmospheric pressure.

So, it helps you only to plot the data from an hypothetical given condition not that it is a real physical state. So, but this is not the real you know situation. Now here the process takes place through heat transfer and the piston goes up, when the piston goes up there is a particular point when the piston is in contact with both the springs. So, the first point is the state point 1 at which it starts so state point 1. At state point 1 the system is ammonia so, from ammonia table just like water property table your ammonia property table, at minus 2 degree centigrade which is the initial state, you get what is v f and what is v g; this is from saturated table.

So, then v 1 is equal to x 1 v g 1 plus 1 minus x 1 v f 1. This we learnt in properties of pure substances. So, from this v f v g and what is x 1; x 1 is 0.13 this is given. So, you can calculate what is v 1 ok. So, you can get so, total volume at state 1 it is given that is 1 meter cube so, small v 1 we will give you what is the mass. So, state 1 what is p 1 because, it is in the 2 phase region p 1 is the saturation pressure corresponding to p sat at minus 2 degree centigrade. So, state point 1 you know p 1 v 1. So, once you know the state point 1 then you know actually the locus which will be this straight line, which will continue up to the point when this piston touches both the springs, because when it touches both the springs the restoring force will double.

Earlier the restoring force was related to this spring only, but when it touches this spring also the restoring force will double. And because the restoring force will double the slope of the PV diagram will also double. So, this point 2 we do not know, but wherever this point 2 is from here the slope of the PV diagram will double and state 3 the pressure is 1200 kilo Pascal. So, the actual PV diagram is something like this, but this point 2 is not known. So, part of this problem is that what is the pressure at this point 2. What is known is: what is the volume at point 2, because it is given that at this point the volume is 2 meter cube at which the other spring also touches the piston ok.

So, now because both the springs have same spring constant. So, we can write this is state point 3 p 3 minus p 2 by V 3 minus V 2 is equal to 2 into p 2 minus p 1 by V 2 minus V 1 right. So, now from this hypothetical data point you can calculate this, how you can calculate this; the point 0 1 2 were parts of the same straight line. So, p 2 minus p 0 by p 1 minus p 0 is equal to V 2 minus V 0 by V 1 minus V 0. Just like y minus y 1 by y 2 minus y 1 is equal to x minus x 1 by x 2 minus x 1 the equation of a straight line that is what we are doing here.

So, V 0 is 0 V 1 and V 2 are given so, from here you can calculate what is p 2; that p 2 you substitute here, you will get what is p 3 right sorry V 3. This is p 3 is given so, from here you can calculate what is V 3 so p 3 is given p 2 you have got from here. So, this is known V 2 is known, V 2 known, V 1 known p 1 known so, from here you will get what is V 3. So, then so, let me give you the answer to the first part this p 2 is 694.875 kilo Pascal ok. So, then the question is what is the final temperature and the total work done. So, final temperature is what? Final temperature you have to identify state 3 from the table.

So, one data point is given as p 3 is equal to 1200 kilo Pascal. The other data point is V 3 is equal to this one. So, this combination will give you state 3 from the table so, state 3 means what is T 3 from these two data. So, that T 3 is 51 degree centigrade. And what is the total work done?

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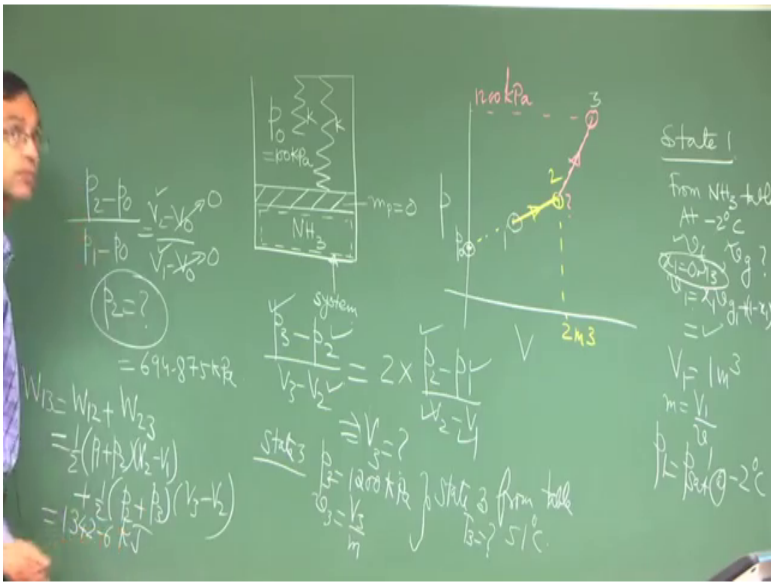

So, W 1 2 this is again a trapezium area under this. So, half into p 1 plus p 2 into V 2 minus V 1 plus half into p 2 plus p 3 into V 3 minus V 2. So, if you substitute all the values this will be 1342.6 kilo joule ok.

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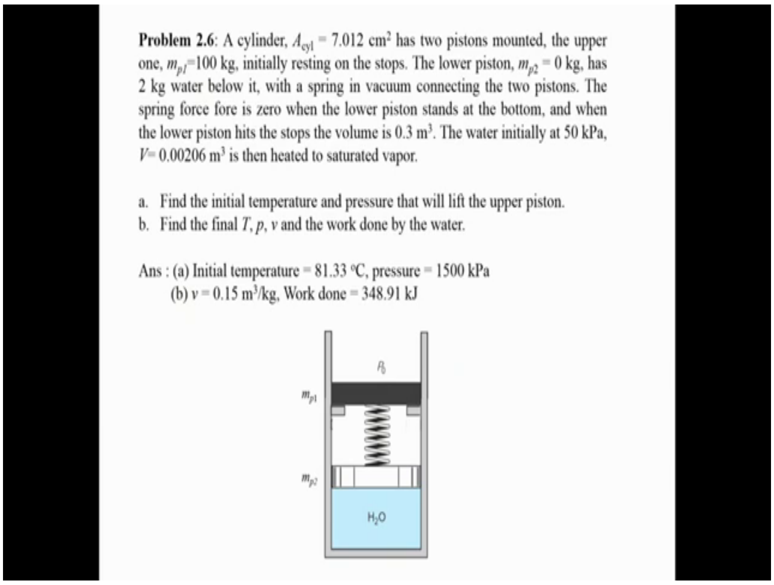

So, we will work out another problem from this chapter heat and work; let me erase the board before we come to that problem. So, that problem is problem 2.6 which is displayed in a screen. So, you have a cylinder with area of cross section as 7.012 centimetre square has two pistons, the upper piston which is given as p 1 in the diagram has a mass of 100 kg, it initially rest on the stops. The lower piston has a mass of 0 kg; that means, it is effectively mass less has 2 kg of water below it, with a spring in vacuum.

So, the medium in which the spring is there is in vacuum, there is no pressure from that medium. The spring force is 0 when the lower piston stands at the bottom; again it is a hypothetical state. And when the lower piston hits the stops, the volume is 0.3 meter cube. The water initially at 50 kilo Pascal and 0.00206 metre cube is heated to saturated vapour. Find the initial temperature and pressure that will lift the upper piston and final temperature pressure specific volume and work done. So, it is a very comprehensive problem, we will look into this problem in some details.

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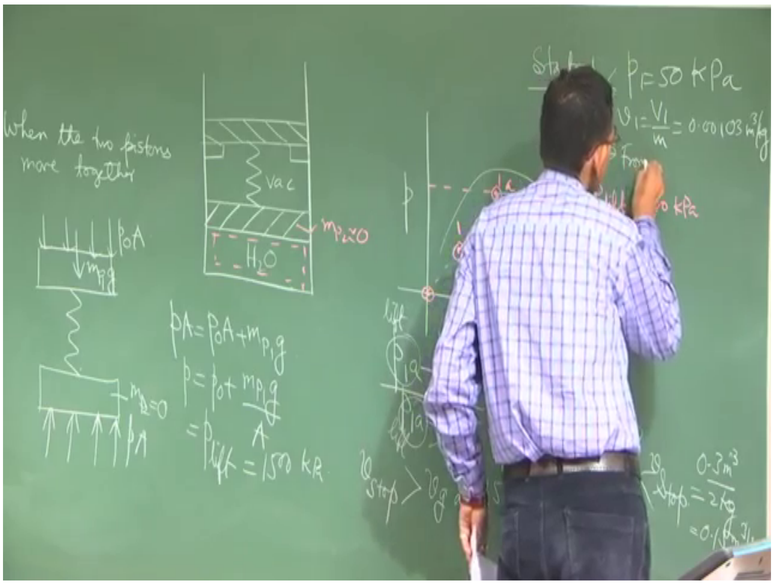

So, here is water, then there is a spring, then there is another piston; two pistons and the upper piston is initially resting on the stops. We will try to understand the process by drawing PV diagram. So, I am just preparing it to you know plot the pressure versus volume. Now, first of all so here there is vacuum. First of all you tell me one thing, what is the system here? So, the system here is nothing, but this water and we are drawing the PV diagram for this system. So, let us see what is there, what is given. So, what is given is that the spring force is 0 when the lower piston is at the bottom ok.

So, what is the resistance force due to pressure here which is 0 because, it is vacuum due to spring force plus due to the weight of the piston. This piston has this is, this piston has weight negligible. So, only force that it is encountering when this is at the hypothetical bottom state is the spring force, which is 0 that is given. Then so, this is one end of the spectrum, the other end of the spectrum is that when heat is transferred to it this piston alone starts moving, before it encounters the situation when this piston is also lifted and they move together.

So, let us start let us identify that condition when these two pistons move together. So, when the two pistons move together. So, the free body diagram will look something like this, this is a very interesting situation, because they are moving together see this spring force is like internal action reaction force; it contributes to nothing. So, as if these two pistons together is moving like a single unit.

So, you have this p 0 into A p A and m p 1 g right m p 2 is 0. So, p into A is equal to p 0 into A plus m p 1 g. So, p is equal to p 0 plus m p 1 g. This p we say equal to p lift, where by p lift what we mean; by p lift we do not mean that when this is lifted. By p lift we mean that when both of these pistons are lifted together. So, this p lift if you calculate it is 1500 kilo Pascal.

So, now let us try to draw the PV diagram. So, this is a two phase region the p lift, this is 1500 kilo Pascal say so, then we have to figure out that what is the state; what is the volume here, which will lie on this line. What is the volume at which both the pistons move together and then they will follow this line ok. So, up to this it will be a linear variation, because the spring is a linear spring and there is a state point 1 which is lying on that ok. So, state point 1 it is given state 1 p 1 is 50 kilo Pascal and v 1 is 0.00103 metre cube per kg. So, let us calculate, let us consider, let us say that this is state 1 a.

So, up to this there will be a straight line and we can write the equation of the straight line as follows: p 1 a minus p 1 by p 1 a minus p 0 is equal to V 1 a minus V 1 by V 1 a minus V 0 right. So, p 0 is 0 this is equation of the straight line, V 1 is sorry V 0 is 0 p 1 a is p lift p 1 is given V 1 a we do not know V 1, we know then p 1 a is p lift. So, from here you can calculate what is V 1 a.

So, if you calculate it so, you can write also it in terms of the specific volume v 1 a; you will find that after calculation this is less than v stop; v stop means that is 0.3 meter cube by 2 kg that is the v stop. So, this is less than v stop. So, where is the v stop? Let us see what is v g at here. So, v stop which is 0.15 meter cube per kg, if you look into this 1500 kilo Pascal data then you will see that that this v stop, this is less than v g at 1500 sorry, this is greater than v g at 1500 kilo Pascal, it is not visible ok. So, I will write it at a separate place. So, v stop whatever is calculated it is greater than v g at 1500 kilo Pascal.

So, v stop is physically here whatever small v or capital V depending on these case you are drawing a small v that. So, what it says is that it is heated to saturated vapour; that means, these two cylinders will go together, but the lower cylinder will not heat that stops. Because, at the lower cylinder hit the stops it has to go beyond this, but the process will stop hitting becomes saturated vapour that is what is given.

So, from 1 a to 2 it will continue but the lower cylinder will not sorry, yes the lower piston will not hit the stops. So, the PV diagram is from 1 to 1 a and then from 1 a to 2.

So, based on this the initial all the problems, all the and parameters for the problem can be obtained. So, first is initial temperature. So, from p 1 v 1 from table you will get what is T 1; table means water table.

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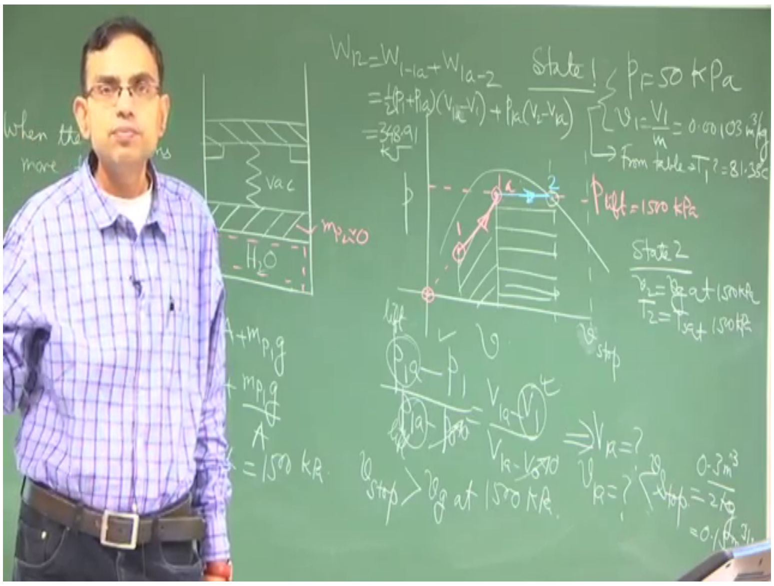

So, T 1 is 81.33 degree centigrade ok, then the final temperature pressure and volume. So, state 2 you have v 2 as v g at 1500 kilo Pascal, T 2 s T sat at 1500 kilo Pascal and pressure is already 1500 kilo Pascal. What is the work done? So, this first part of the work done is the area under this trapezium, the second part is the area under this rectangle. So, this is the area under the trapezium, this is the area under the rectangle all these are known; so, per unit so, the total work done will be 348.91 kilo joule.

So, again the problems which we have worked today we wanted to illustrate through this that by visualizing the problem through a proper PV diagram, it is very convenient to calculate the work done. So, instead of just hypothetically trying based on the initial state and the final state, you draw the processes and then integrate p d v to get the work done.

Thank you very much, we will continue in the next lecture.