

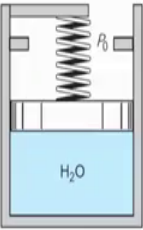
Concepts of Thermodynamics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 12
Heat and Work: Representative Problems (Contd.)

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Problem 2.3: Two kilograms of water is contained in a piston/cylinder (Figure below) with a massless piston loaded with a linear spring and the outside atmosphere. Initially the spring force is zero and $P_1 = P_0 = 100$ kPa with a volume of 0.2 m³. If the piston just hits the upper stops, the volume is 0.8 m³ and $T = 600^\circ\text{C}$. Heat is now added until the pressure reaches 1.2 MPa. Find the final temperature, show the P - V diagram, and find the work done during the process.

Ans: (a) Final temperature = 770°C
(b) Work done = 330 kJ



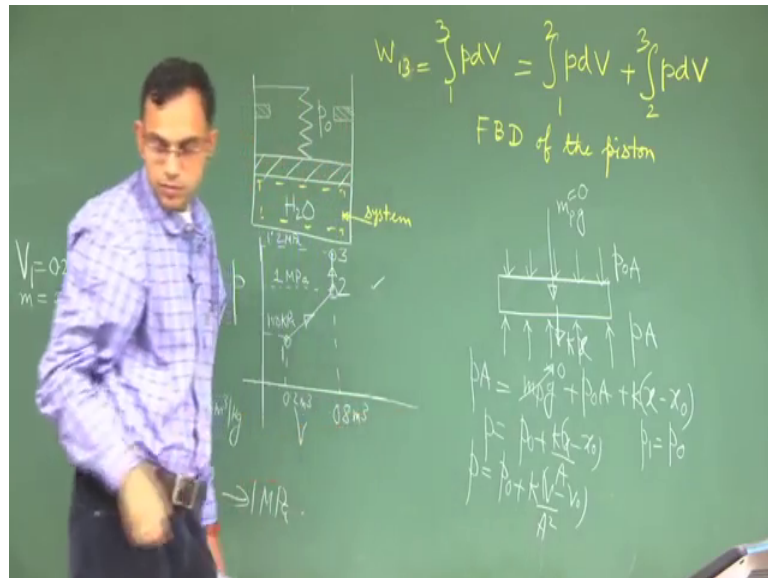
We will continue with the problems and the next problem that we will consider concerning heat and work is problem 2.3. Let us read out the problem as it is there displayed, 2 kilograms of water is contained in a piston cylinder as shown in the figure.

This kind of problem we have encountered previously, but the difference here is that this has a linear spring along with the piston. So, you can see that there is a linear spring with a piston and the outside atmosphere. So, the piston is balanced with the aid of a linear spring and outside atmosphere with atmospheric pressure P_1 is equal to P_0 is equal to 100 kilo Pascal.

Initially the spring force is 0 so, that the external loading is P_1 is equal to P_0 is equal to 100 kilo Pascal with a volume of 0.2 meter cube. If the piston just hit the stops the volume is 0.8 meter cube and T equal to 600 degree centigrade. So, this is again a hypothetical state it is not the case that it is given that the piston hits the stops, what is given is that had it hit the stops, then the volume would have been 0.8 meter cube and temperature is 600 degree centigrade.

Heat is now added until the pressure reaches 1.2 mega Pascal, this is the actual state 2, you have to find out the final temperature show the P V diagram and final work done during the process. So, let us draw a schematic of this problem.

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So, to understand how the pressure within the water is varying; let us draw the free body diagram of the piston considering that this is our system which is the water let us draw the free body diagram of the piston. So, you have the atmospheric pressure, the piston must be having its own weight, now it is given its a mass less piston for this particular problem let us draw the free body diagram with $m g$, but here this is.

Mass less means, see practically nothing can be mass less, in this case mass less means that the contribution of these weight of the piston in terms of the total forces is negligible as compared to the contribution due to the pressure that essentially boils down to mass less. So, then how it is moving? Heat is added, if heat is added the piston will have a tendency to go up.

So, if the piston has a tendency to go up the spring will try to restore it downwards. So, you have a spring force which is $k x$ and this force is $k x$ because it is a linear spring had it been a non-linear spring it would have been related to x in a non-linear manner. And then you have four switches p into a where p is the pressure from the water side.

So, this piston is moving in a quasi equilibrium process that means, all forces are balanced. So, you have $p \text{ into } A$ is equal to $m g$ which is 0 in this case plus $p_0 A$ plus $k x$. So, p is p_0 plus $k x$ by A . So, at state 1 x is 0; that means, the spring is completely relaxed and that is why at state 1 you have p_1 equal to p_0 which is given. So, this is possible only if the spring is just relaxed at state 1.

So, let us try to draw the PV diagram for this process state 1. So, p_1 is 100 kilo Pascal and V_1 is 0.1 metre cube per kg. So, this is what is given. So, how do you know that V_1 is 0.1 meter cube per k g actually your given V_1 total volume is 0.2 metre cube and mass is 2 kg this is what is given.

So, combining these two these two things are there, remember we are doing with water. So, state point has to be identified from the water property table and not from ideal gas relationship. So, state 1 is identified, then if the piston just hits the stops the hypothetical state then the volume is 0.8 metre cube and T is 600 degree centigrade. So, state 2 and T_2 is 600 degree centigrade. So, as we have told many times, but I have no problems in iterating it because it is a simple compressible pure substance you require two independent intensive properties to specify the state.

So, using these two properties you can identify what is p_2 from water table. So, you have state 1, but this is state 2 hypothetical right, remember this is not the actual state 2 this had it hit the stops. So, this is hypothetical state 2. So, p_2 hype hypothetical. So, 2 hypothetical you get p_2 and you get V_2 so, 2 hypothetical 2 hype is somewhere here.

So, why this is given? It is given because pressure versus volume is. So, how is pressure related to volume p is equal to p_0 plus $k \text{ into } V$ by A square because x is V by A .

Student: (Refer Time: 10:27).

Yes, so, this x is also x minus x_{naught} right that x_{naught} is 0. So, this is ΔV you can say the change in volume right so, your reference volume is 0.

So, you can of course, write it as x minus x_{naught} and this as V minus V_{naught} I means it depends on what reference you choose the reference is arbitrary. So, the important thing is pressure is linearly related to the volume ok. So, because pressure is linearly related to the volume your PV diagram will correspond to the straight line that

connects these two points right. So, these hypothetical data point actually helps you to construct the PV diagram without knowing what is the spring constant.

In this problem because the spring constant is not given, you are not able to get a pressure versus volume until and unless two data points are given and because two data points are given a unique straight line can be drawn using this, but remember this is the hypothetical state point. Your actual state point is when pressure is 1.2 M P a. So, pressure is 1.2 M P a.

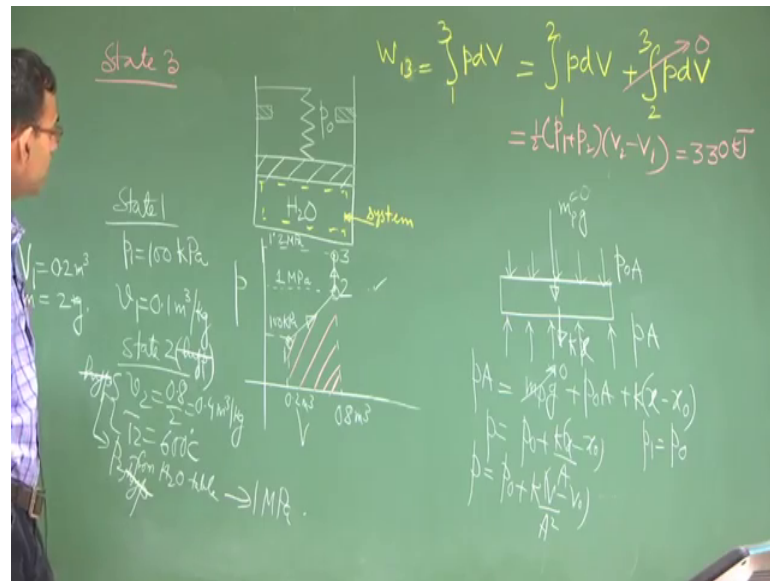
Now heat is added until the pressure is 1.2 M P a now whether this hypothetical point becomes a real point what is the this pressure let me see this is 1 M P a. So, this pressure is 1 M P a from the table. Now you tell is it hypothetical or real? The state 2, see it started with 100 kilo Pascal if it is moving the pressure is also increasing, if the final pressure is given 1.2 mega Pascal and it is not greater than 1.2 mega Pascal it is less than 1.2 mega Pascal.

So, this hypothetical becomes real had this been greater than 1.2 mega Pascal then that would have been hypothetical, but it is less than 1.2 mega Pascal which is the final pressure given. So, this hypothetical we drop now, this hype this will cut it is a real straight point and then. So, at state 2 it hits the stops physically after hitting the stops physically what is happening at constant volume its pressure is increasing.

So, at state 3 the pressure is 1.2 mega Pascal. So, let us just write the pressures, this is 1.2 MP a this is 1 M P a and this is 100 k P a. So, I mean before I complete the solution I mean here just to make it consistent with this x minus x 0 here also we write x minus x 0 ok.

So, just for completeness anyway let us come back to this diagram. So, what is this V 2 this is 0.8 meter cube V 1 is 0.2 meter cube. So, you can see all the data points you have on this. So, what is the work done? W 1 3 right from 1 to 3.

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So, from 1 to 2 what is the work done? That is from this diagram is area under this trapezium. So, this is half into p_1 plus p_2 into V_2 minus V_1 and this part because the volume is constant the work done is 0. So, you can see that by plotting the data and drawing a P V diagram how easy it becomes to calculate the work done. So, in this case the work done is 330 kilo joule, also there is another part of the problem that what is the temperature at state 3.

So, how do you find out the temperature at state 3? So, state 3 you have the following properties, what are the properties? P_3 is 1.2 M P a and V_3 is equal to V_2 is equal to 0.4 meter cube per k g. So, using these two independent data you can from table what are property table, you can find out what is T_3 . The answer to this problem is T_3 is 770 degree centigrade. So, we will work out another problem, let us erase these.

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Problem 2.4: A piston/cylinder (figure below) with $A_{cyl}=0.01\text{ m}^2$ and $m_p=101\text{ kg}$ contains 1 kg of water at 20°C with a volume of 0.1 m^3 . Initially the piston rests on some stops with the top surface open to the atmosphere, P_0 . (a) To what temperature should the water be heated to lift the piston? (b) If it is heated to saturated vapor find the final temperature, volume and work, ${}_1W_2$?

Ans : (a) Temperature = 120.2°C
 (b) temperature = 120.2°C , Volume = 0.8857 m^3 , ${}_1W_2 = 157.45\text{ kJ}$

So, the next problem is displayed in the screen a piston cylinder with the figure below is having the area of cross section of the cylinder as 0.1 m^2 and mass of the piston 101 kg , it contains 1 kg of water at 20°C with a volume of 0.1 m^3 .

Initially the piston rests on some stops with the top surface opened to atmosphere with P_0 . To what temperature should the water be heated to leave the piston. If it is heated to saturated vapour find the final temperature volume and work done.

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F-B-D of piston

$P_0 A_{cyl}$ (up)
 $P_0 A_{cyl}$ (down)
 $m_p g$ (down)
 $P_{cyl} A_{cyl}$ (up)

$P_0 A_{cyl} = P_0 A_{cyl} + m_p g$
 $P_{cyl} A_{cyl} = P_0 A_{cyl} + m_p g$
 $P_{cyl} = P_0 + \frac{m_p g}{A_{cyl}} = 200\text{ kPa}$

System: H_2O

state 1
 $v_1 = 0.1\text{ m}^3/\text{kg}$
 $T_1 = 20^\circ\text{C}$

state 2
 $x_2 = 1$
 $P_2 = P_{cyl}$ if it is moving
 $(P_2, v_2) \rightarrow \text{state 2}$

So, let us try to solve this problem. So, here it is given let us write the given data area of cross section of the cylinder as 0.01 meter square. Then mass of the piston is 101 k g and then water at state 1 what is given, this is the system. So, state 1 is given by v_1 is 0.1 meter cube per kg and T_1 is 20 degree centigrade, state 2 is saturated vapour; that means, quality equal to 1.

And then add state 2 if it is moving, p_2 is equal to the equilibrium pressure if it is moving it means the piston. So, we have to figure out that whether it is actually moving at state 2 or not, if it is moving its pressure is same as the equilibrium pressure. So, how do you calculate the equilibrium pressure? You draw the free body diagram of the piston, this is $p_{\text{equilibrium}}$ into A cylinder $m g$.

So, $p_{\text{equilibrium}}$ into A cylinder plus right. So, $p_{\text{equilibrium}}$ is equal to p_0 plus $m g$ by A, this p_0 is 1 atmosphere take at take 101 kilo Pascal. For example $m p$ is given g 9.81 and A cylinder is 0.01 metre square.

So, if you calculate this $p_{\text{equilibrium}}$ we will come out to be 200.4 kilo Pascal. Now question is what was p_1 ? So, that you can get from the table. So, from table you have p_1 so, I do not have the data with me, but you will get p_1 from water table. Once you get these data for this particular problem you will find that p_1 is less than $p_{\text{equilibrium}}$.

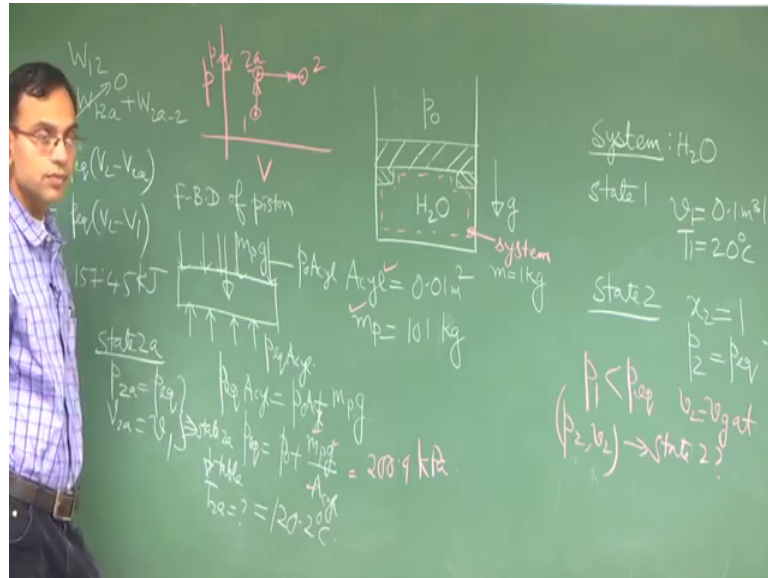
So, that you will get once you put this numerical value you can check it will be like this. So, if p_1 is less than $p_{\text{equilibrium}}$ then what is happening, what is happening is that see the inside pressure is less than the equilibrium pressure. So, outside pressure plus weight it is dominating and it is trying to bring the piston down, it cannot bring it down because the stops are preventing it from doing that.

So, then the next is it is heated. So, once it is heated the pressure will go up and pressure will first reach the equilibrium pressure. So, if you draw the P V diagram from state 1 it will come to 2 a where this will be equilibrium pressure, then only this will start moving. So, the piston will not start moving until and unless the pressure here is equal to equilibrium pressure initially pressure here is less than equilibrium pressure.

So, heat is supplied to pressurize this water until and unless heats pressure becomes the equilibrium pressure it does not start moving. So, its pressure becomes equilibrium pressure and then it starts moving. So, it comes to state 2. So, what is v_2 for example,

state 2 quality is 1; that means, v_2 is v_g at p equilibrium. So, you can identify the state 0.2 from v_2 and p_2 . So, p_2 comma v_2 will give you what is state 2 and then the work done is same as the previous example.

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So, W_{1-2} so, 1 to 2 the work done is $\int_{V_1}^{V_2} p \, dV$ at p equilibrium into V_2 minus V_1 . So, which is same as p equilibrium into V_2 minus V_1 .

So, if you substitute these values, these values will give you 157.45 kilo joule. In this case the problem data is very comfortable because the mass is given as 1 kg. So, the mass of water is 1 kg. So, specific volume and total volume numerically they are the same, so.

Student: (Refer Time: 28:01) other aspect to the 1 more question. Like to what temperature the water should be heated to (Refer Time: 28:07) leave the piston.

To what temperature the.

Student: Water should be heated.

The water should be heated to leave the piston. So, that will be the temperature corresponding to state 2 a. So, this is another part of aspect of this question that what should be the temperature at state 2 a. So, state 2 a; how do I identify state 2 a? Two properties you have to search 1 is p_{2a} is equal to p equilibrium and v_{2a} is same as v_1 .

So, this two will give you the state 0.2 a and from table you can, from water table you will get what is T 2 a. So, the answer is 120.2 degree centigrade.

Student: Basically the answer for both (Refer Time: 29:08) because that line 2 a to 2 is inside the saturation (Refer Time: 29:14). So, that is the time of constant (Refer Time: 29:15).

Yes, from 2 a to 2 this straight line has to be in the saturation region because this was in the saturation region, this is in the saturation region and this is quality equal to 1.

So, this cannot cross the saturation region and go to superheated region it is this is in the saturation region or 2 phase region. So, from 2 phase region it is going towards superheated, but it does not cross super heated because at state 2 it is quality equal to 1. So, because it is this is the same saturation line. So, the temperature here and temperature here both are actually saturation temperature corresponding to p equilibrium which is two 120.2 degree centigrade.

Let us stop here today we will continue with the other problems in the next lecture.