Robotics
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Lecture – 42
Biped Walking

Now, I am going to discuss on a new topic and that is on Biped Walking. Now before I start let me define what do we mean by this biped robot? Now, this biped robot is actually a simpler version of this particular the humanoid robot. So, humanoid robot is very much complicated and this biped robot is actually the; a simple version of that particular the humanoid robot.

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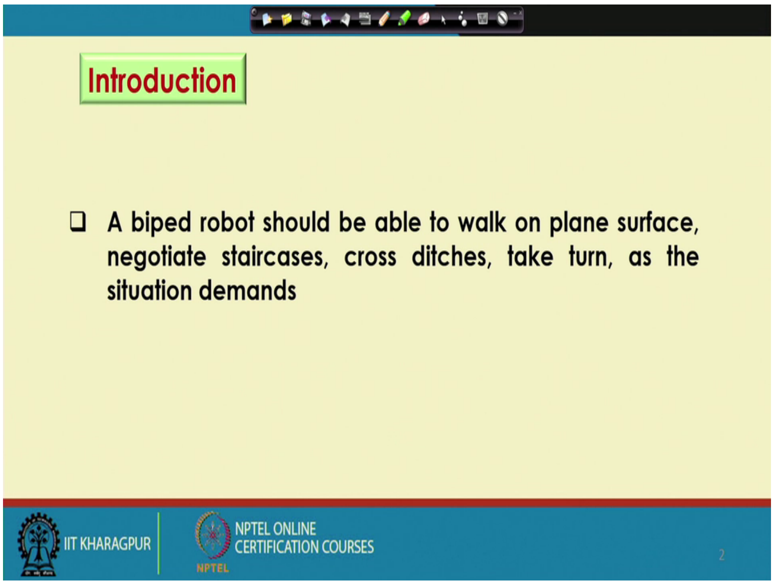

Now, a biped robots should be able to walk on the plain surface it should be able to negotiate the staircases, take turn, cross ditches as the situation demands. Now, while walking, so this particular biped robots should be able to maintain its balance and that balance is nothing, but the dynamic balance. Now I am just going to discuss in details like how can it maintain the dynamic balance.

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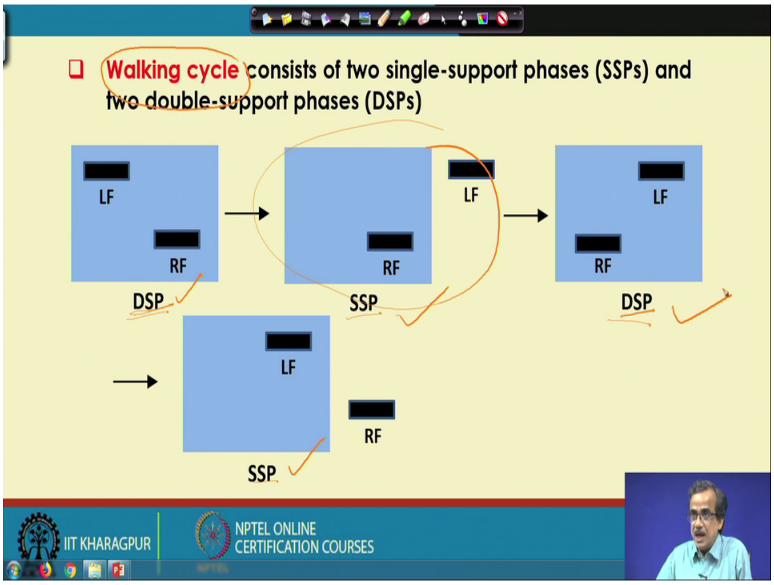

So, here I am just going to discuss the walking cycle of a biped robot; now if you see, if you concentrate on this particular that the this figure we can see that. So, this LF and RF are nothing, but the left foot and the right foot and these 2 foot are the ground foot. Now let me assume that; so, this particular the rectangular box indicates actually it is the; your the ground. So, both the feet are placed on the ground and this is nothing, but a double support phase.

Now, after this double support phase what happens? The right foot will remains at the same position and the left foot that will be taken away from the ground and now it is an air. So, here in this particular configuration for example, this particular configuration that is the single support phase configuration; the right foot is on the ground and the left foot is in air and this is a single support phase.

Now, after that; so there will be another double support phase now here; so, this particular right foot that is already there on the ground and the left foot that will be placed on the ground. So, here both the feet are on the ground and this is nothing, but the configuration of the double support phase. And after that; so this particular the left foot will be on the ground and the right foot will be put in air and this is once again a single support phase.

Now, starting from this particular double support phase then there will be one single support phase, then double support phase and single support phase that completes actually one walking cycle. So, one walking cycle consists of 2 single support phases and there will be 2 such double support phases. Now, while walking this particular biped robot should be able to maintain the dynamic balance during its single support phase as well as the double support phases.

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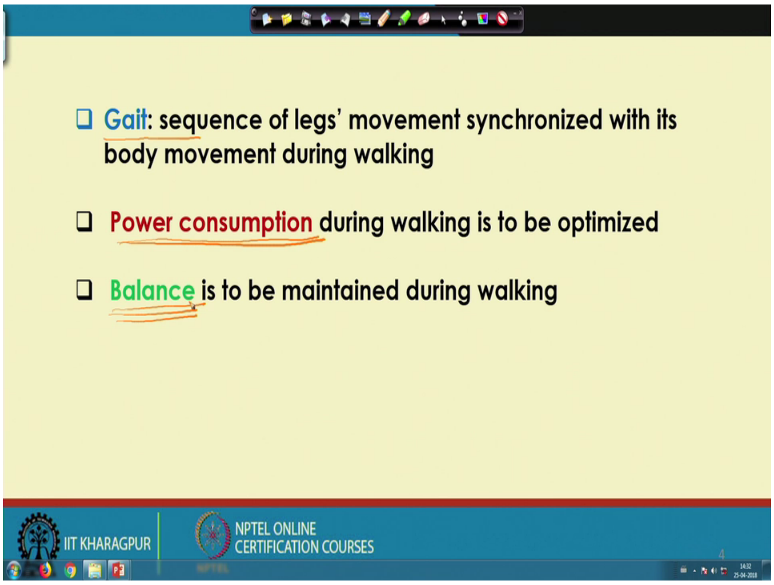

Now, here I am just going to discuss how to maintain that particular your the dynamic balance. Now, before that let me define like what do you mean by gait; the term gait is very frequently used in biped walking. Now, by gait we mean it is the sequence of legs movement in coordination of the body movement which is required for walking of that particular the biped robot.

Now, while walking; so this particular biped robot should be able to consume the minimum amount of power, but at the same time it should have the maximum dynamic balance margin. So, I am just going to discuss in brief how to determine; so, this particular the power conjunction during walking and how to maintain this particular the dynamic balance.

Now, here actually what I am going to do? I am just going to discuss in brief just to make it simple, but the exact derivation or the detailed derivation if you want to have a look. So, you will have to concentrate on the textbook that is the fundamentals of robotics written by me. So, there are all such things are dealt in much more detail, but here as I told for simplicity I am just going to discuss in brief like; how to determine the power rating for this particular biped robot and how to determine the balance margin.

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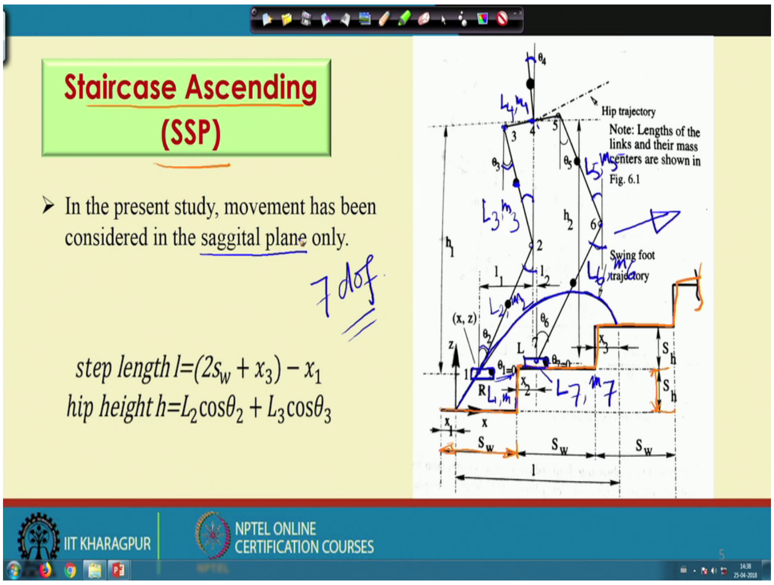

Now here I am just going to concentrate first on this single support phase; so, and here for simplicity; I am just going to concentrate on a particular task that is nothing, but the ascending of staircase. So, I am just going to discuss staircase ascending and that too for the single support phase. So, let us see what happens during the single support phase and whenever this particular biped robot is going to negotiate or going to ascend through the staircase. Now here on this particular figure the staircase; so that is denoted by these. So, these are nothing, but the steps of the staircases.

So, these are nothing, but the staircases now here. So, this S w that particular symbol indicates the width of the staircase and S h is nothing, but the height of this particular the staircase. And here, so this is the single support phase; so, only one foot will be on the ground and the other foot will be on the air.

Now, here out of this 2 foot; so this particular foot is on the ground and this particular foot is in air because this is a single support phase. And this indicates actually the trajectory of this particular the swing foot that is the foot which is in the air. So during this particular walking through the staircase; so, this is the locus of the swing foot; so this is nothing, but the swing foot trajectory.

Now, this particular swing foot trajectory I am just going to represent with the help of some mathematical expression and we will derive that particular the mathematical expression. Now before that let me tell you that this indicates say one foot, this is another foot, this is one link this is another link, another link, another link, another link and this is the foot.

So, let me write here; so the length of this particular the foot is denoted by say L 1 and its mass is denoted by m 1. Similarly for this particular link supposing that the length is your L 2 and the mass is m 2 and this particular mass is concentrated at this particular the point. Similarly, for this particular link the link length is L 3 and the mass is m 3 and may m 3 is concentrated here at this particular the point.

Similarly for this particular the length is L 4 and the mass is m 4. Now here the length is m 5 and mass is m 5 this here the length is L 6 and the mass is your m 6. And for this particular foot the length is L 7 and mass is nothing, but m 7.

So, here there are 7 links say L 1, L 2, L 3, L 4, L 5, L 6 and L 7 and here I am just going to consider for simplicity only 7 degrees of freedom. So, here I am going to consider a biped robot having 7 degrees of freedom and the joint angles. So, all the joints are actually the rotary joints and the joint angles are denoted by say theta 1. Here theta 1 is equals to 0 then comes here we have got theta 2; the second joint angle. So, this is also theta 2 then comes your theta 3; so, this is also theta 3 and the joint angle theta 4.

Similarly, the joint angle theta 5 then comes theta 6 and theta 7 and for simplicity we have assumed that theta 1 is equals to 0 and theta 7 is equals to 0. And this particular joint is actually the hip joint and here we consider that this joint that particular joint and this particular joint all 3 joints are coinciding. So, this is actually the ankle joint, this is the knee joint and this is the hip joint similarly on the other leg; so, this is nothing, but the knee joint and this is the ankle joint.

Now, here let us see how to determine the power consumption; like if this particular robot is planning to negotiate the staircase; in this particular the direction. And here for simplicity we are going to consider the movement only along the sagittal plane; that means, the sideways movement we are not going to consider for simplicity.

Now, let us see how to carry out this particular the analysis, but as I told that I am not going to discuss in details, the mathematical derivation which is available in the textbook of this particular the course. Now here the step length that is denoted by l is nothing, but 2 s w plus x 3 minus x 1. So here actually the step length; so, this is nothing, but; so the distance between this particular point and this particular point. So, this is actually the step length that is l and this l is nothing, but 2 s w minus x 1 minus x 1 plus x 3; so, this is your x 3.

So x 3 plus s w plus s w minus this particular x 1 that is from here to here; so, from here to here. So, this is nothing, but is your the step length that is nothing, but l. Similarly the height of this particular heap that is denoted by h is nothing, but L 2 cos theta 2 plus L 3 cos theta 3. Now, this is your L 2; the length of this particular the link; this angle is theta 2. So your L 2 cos theta 2 is nothing, but this; so from here; so this will be your L 2 cos theta 2.

Similarly, this is your L 3 and this particular angle is nothing, but theta 3. So, from here to here is nothing, but is your L 3 cos theta 3; so L 2 cos theta 2 plus L 3 cos theta t is nothing, but the height of this particular the hip. So, these 2 terms actually will have to define for the purpose of analysis and here another thing. So, this is the hip joint now during this particular that though the walking through the staircase; so this the hips should also follow a particular trajectory.

Now, here for simplicity we have assumed that the hip is going to follow a straight path. And the slope of this particular straight line is nothing, but the slope of this particular the staircase. So, if I just try to find out the slope of the staircase and the slope of this particular the hip; so, they are having the same slope. Now this is the way actually mathematically we are going to describe; so this particular the configuration for the purpose of analysis.

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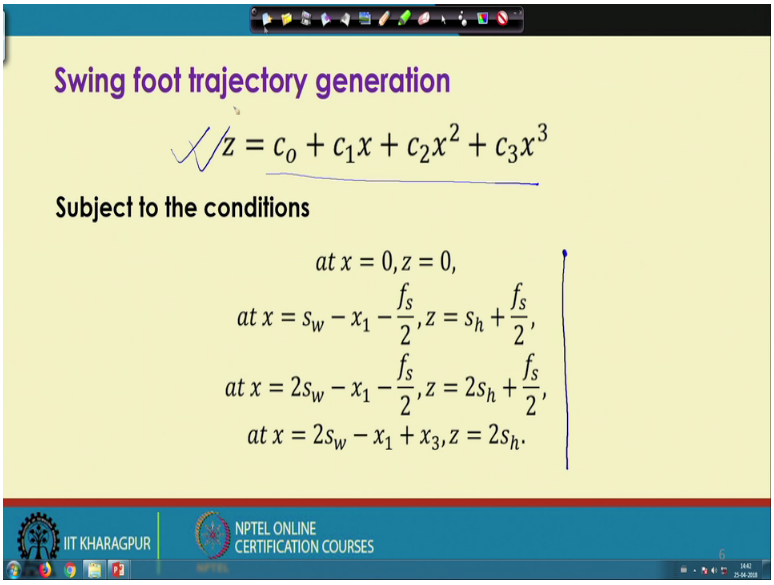

Now, here as I told that this particular swing food should have some trajectory now for simplicity we have considered that the swing foot is going to follow one cubic polynomial of this particular form; that is z is nothing, but c naught plus c 1 x plus c 2 x square plus c 3 x cube.

Now here actually for this cubic polynomial there are 4 unknowns like c naught, c 1, c 2 and c 3. Now if I want to solve it; I will have to take the help up for such known conditions and these are nothing, but the boundary conditions. Now, if I discuss that let me once again go back now this is nothing, but is actually your the hip trajectory so, which I am going to represent mathematically.

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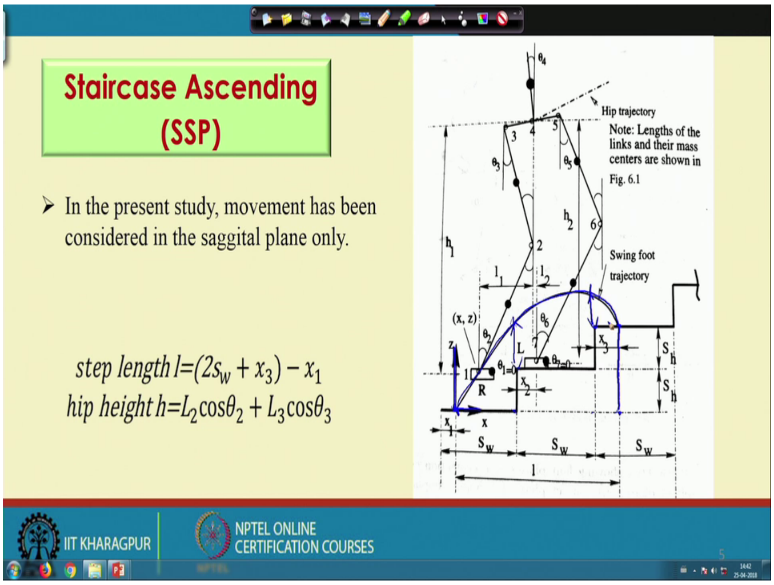

Now here, so this is the coordinate system this is your x and this is z. So, at this particular point the z height is actually is equal to 0; similarly when x is the is this at that particular situation. So, I can find out that this is nothing, but the height along this particular z and then we try to find out for a particular value of x.

So, when x is here; so, I can find out this much is actually your z when x is this much; that means, here. So, this is nothing, but the value of this particular the z. So, using these 4 conditions; so, I can derive this particular the cubic polynomial. So, I am just going to write down; so all such condition here; so, the conditions are written here.

So, these boundary conditions the 4 boundary conditions are written here for example, at x equals to 0; z is equals to 0 and so on. And if you use this boundary condition we can find out what should be the values for this c 1, c naught, c 1, c 2 and c 3. And once you have got the values of the coefficient; so, I can represent this swing foot trajectory.

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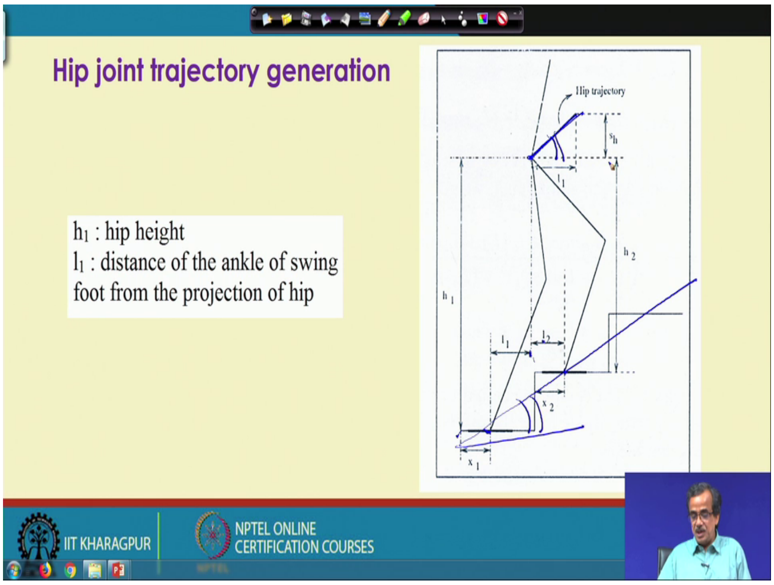

Now, this once you have got this particular the swing foot trajectory; next we try to find out the hip joint trajectory. And I have already mentioned that we have assumed that that this particular hip joint is going to follow a straight path and whose slope is nothing, but equal to the slope of this particular the staircase.

So, this particular angle and that particular angle so, they are the same. Now here actually there is a chance of optimization we can find out a suitable an optimal slope or optimal your the trajectory for this particular the hip joint. But here for simplicity we consider that the slope of this particular the trajectory is same as the slope of the staircase.

Now, if I concentrate on this particular hip joint; so, I can find out. So, if I take the projection of this particular hip joint; so I can find out the distance between this ankle joint and the projection of the hip joint. Similarly I can find out the distance between the projected point from the hip and the distance of this particular the ankle joint and that is denoted by l 2. And I can also find out what is h 1 that is the height of this particular the hip joint and I can also find out what is h 2 that is nothing, but the height of this particular the hip joint.

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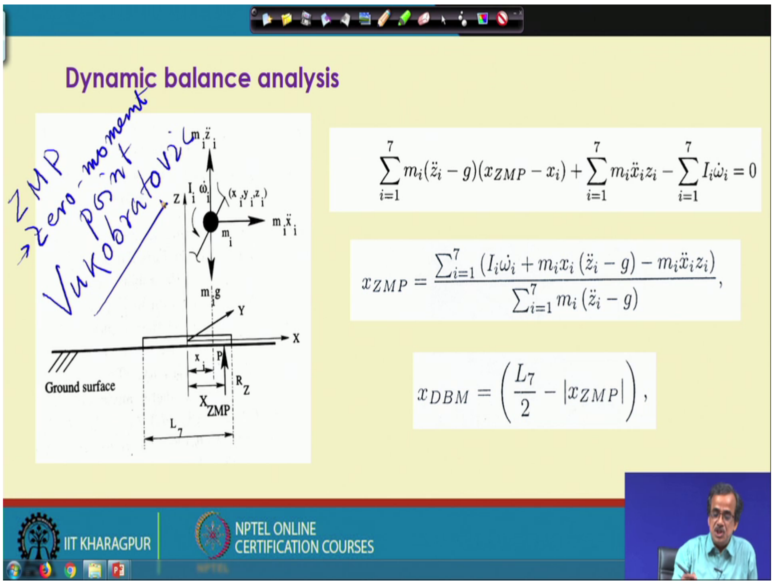

$$\sum_{i=1}^{7} m_i(\ddot{z}_i - g)(x_{ZMP} - x_i) + \sum_{i=1}^{7} m_i \ddot{x}_i z_i - \sum_{i=1}^{7} I_i \dot{\omega}_i = 0$$

$$x_{ZMP} = \frac{\sum_{i=1}^{7} \left(I_i \dot{\omega}_i + m_i x_i (\ddot{z}_i - g) - m_i \ddot{x}_i z_i\right)}{\sum_{i=1}^{7} m_i (\ddot{z}_i - g)},$$

$$x_{DBM} = \left(\frac{L_7}{2} - |x_{ZMP}|\right),$$

Now, knowing this particular so what you can do is we can carry out the analysis for this your the dynamic balance. And we can also find out the expression for the power consumption. Now, here I am just going to discuss little bit how to maintain the dynamic balance for this particular the biped robot.

Now, before I proceed further I just want to mention that we human being, we are not statically stable; we are dynamically stable. Even if we are standing at a particular location we are not statically stable, but we are dynamically stable. Now let us see how to maintain that this particular the dynamic balance. Now here I am just going to use the concept of the ZMP and that is known as the zero moment point. So, ZMP is the zero moment point that is 0 moment point and the concept of ZMP was introduced by Vukobratovic and this particular concept has become a very popular.

Now, let us try to understand how can you find out this particular ZMP or the zero moment point. Now to find out this particular zero moment point actually what I am going to do is; so, I am just going to consider say a particular the link for the robot. And for this particular biped robot say I am just going to consider say a particular the leg.

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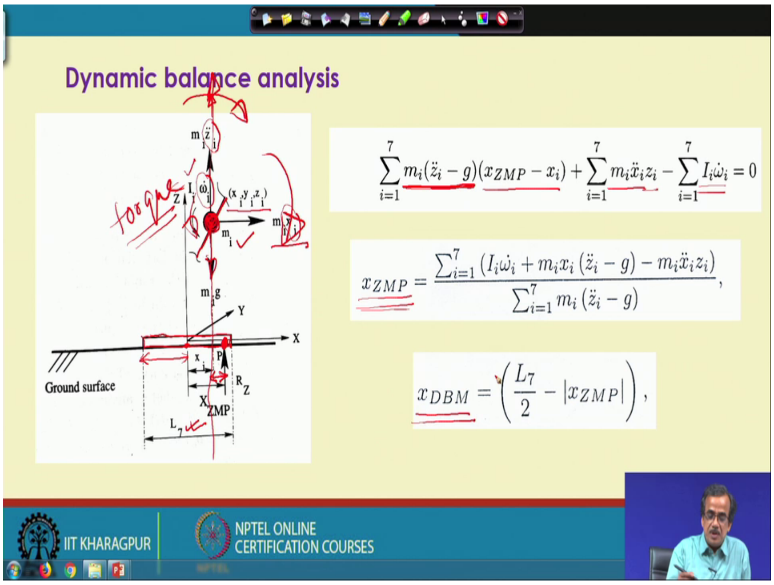

Supposing that so that particularly leg is denoted by this; so, this is nothing, but a link or a leg and this particular link or the leg is having one concentrated mass and this particular mass is denoted by this. And supposing that for this ith leg or the ith link; the mass is denoted by m i and this mass center is having the coordinate that is nothing, but x i, y i, zi. Now, let us see how to determine the your the ZMP that is the zero moment point. Now here; so this is nothing, but the foot which is in touch with the ground; so, this is nothing, but the foot.

And this foot is having the length and this length is denoted by L 7. And this is nothing, but the center of this particular the foot; that means, that is at the midpoint. So, this is this particular the length is nothing, but L 7 by 2; so, this is the midpoint.

Now, let us see how to derive and how to determine your this ZMP. So, before I define actually let me let me tell what do you mean by this particular the ZMP? Now ZMP is actually a zero moment point which is a hypothetical point and this is a point about which; the sum of all the moments becomes equal to 0. Now let me repeat; so ZMP is a [hypothec/hypothetical] hypothetical point now this is a point about which the sum of all the moments becomes equal to 0.

Now, here so this particular mass mi; so this is subjected to a few forces. For example, say; so g is the acceleration due to gravity. So, this particular m i g is acting vertically downward; so this is the direction along which this m i g is acting vertically downward.

Now here if I consider the movement of this particular mass along the x direction and the movement of this particular mass along the z direction. And if I say that along the x direction there is one acceleration that is nothing, but x i double dot and along this particular z direction there is one acceleration that is nothing, but z i double dot. Then we can say that there is a force acting along the x direction that is your m i x i double dot mass into acceleration is the force.

Similarly here along this particular z direction; so, m i, z i double dot. So, that particular force is acting and moreover; so here; so this is a rotary movement the link is rotating. So, here we will have to consider the moment of inertia and this i is nothing, but the moment of inertia of the ith link or the ith leg and this your omega I dot that is nothing, but the angular acceleration.

So your the moment of inertia multiplied by angular acceleration is nothing, but is actually a torque. For example, say force multiplied by linear acceleration sorry mass multiplied by linear acceleration in force. Similarly, the moment of inertia multiplied by the angular acceleration is nothing, but is actually the torque.

So, here; so this is subjected to the torque that is I i omega i dot and then it is subjected to the force like m i g; then comes a m i, x i double dot, then m i, z i double dot. Now, here I am on this particular the ground foot; so, I am just going to consider a hypothetical point. Supposing that the point is here; now corresponding to this particular point, let us try to find out what should be the moment and we just put the sum of those particular moments equal to 0.

Now, here the vertically downward force is nothing, but m i g and vertically upward is a m i, z i double dot. And truly speaking; so this m i, z i double dot is more larger compared to this particular m i g; because this is moving vertically upward direction. So, what is the difference between these 2 forces; the resultant force is nothing, but m i, z i double dot minus m i g. So, this is nothing, but the resultant force in this particular the direction.

Now, I will have to find out the moment; so, the force is acting, the resultant force is acting in this particular direction and how much is the moment? So, with respect to this particular moment; so, I will have to find out I will have to multiply it by this particular the distance. And what is this particular distance? That is nothing, but x ZMP minus x i.

So, ZMP minus x i; so I am getting the moment due to this particular the vertical force; Now, I am trying to find out the moment due to this horizontal force; now horizontal direction the force is m i x i double dot. So, m i x i double dot and your this particular height is nothing, but is your z i. So, m i; x i double dot multiplied by z i is the moment; now here, so this is going to create some sort of clockwise moment.

This is also create some sort of clockwise moment, but this is going to create some set of anticlockwise and that particular torque the summation of talk is nothing, but summation i equals to 1 to 7 because I have got 7 links. So, I i multiplied by omega i dot; so this is nothing, but the torque and summation of that and this is anticlockwise and other things are clockwise.

So, clockwise I have taken as positive and anticlockwise is negative and that is equals to 0 and if I solve if I simplify; so, I will be getting the expression for the x ZMP. So, I will be getting the coordinate of this particular point that is nothing, but the zero moment point. And once I have got this particular zero moment point now very easily I can find out what is this; your dynamic balance margin that is x ZMP. Now, x ZMP is nothing, but L 7 by 2; now L 7 by 2 if I consider now L 7 by 2 if I consider. So, I am here.

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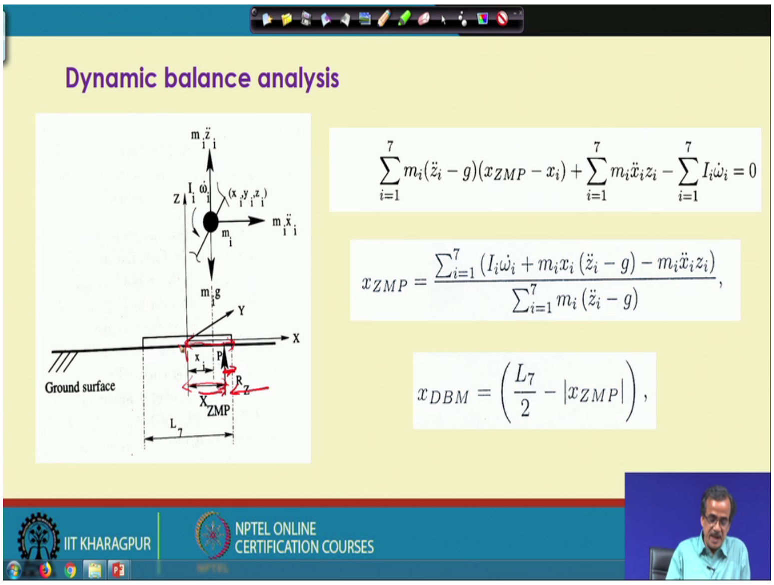

So, this is nothing, but L 7 by 2 minus your x ZMP. So, minus this x ZMP then what is the dynamic balance margin? So, dynamic balance margin is this much. So, this is nothing, but is your the dynamic balance margin. Now, if I consider x ZMP is here in that case I will have the maximum the dynamic balance margin. Now this is the way actually we calculate the dynamic balance margin for this particular during the biped walking.

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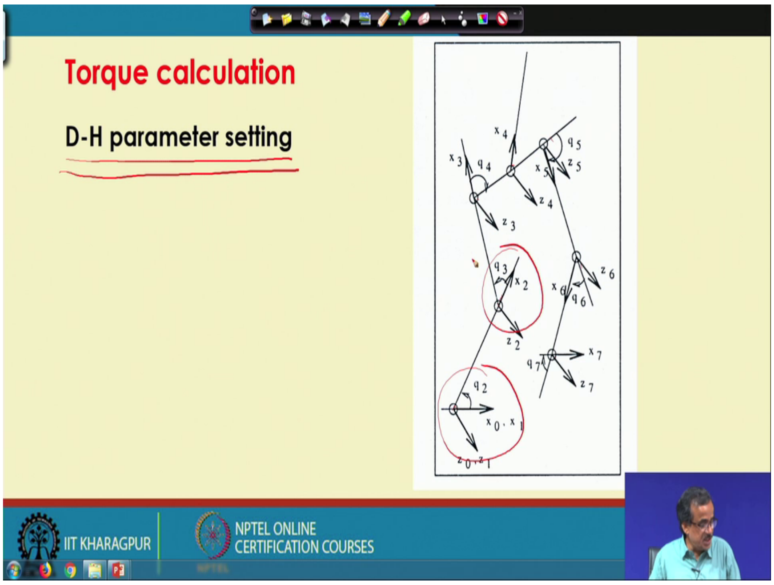

Now, in short let me discuss; let me tell you the procedure how to find out the joint talk and how to determine the joint talk I have discussed in much more details, while discussing the dynamics. Now, let me let me proceed little bit faster. So, this is actually how to how to assign the coordinate system at the different joints; according to this D-H parameter setting rule. Now this D-H parameter setting rule; so, I have discussed in details in the in the chapter of robot dynamics and those things I am not going to repeat.

Now using that particular principle of D-H parameter setting; so, at each of these particular robotic joint 1, 2, 3, 4, 5, 6, 7; So, I will have to assign this particular the coordinate system like x axis, y axis and z axis will have to assign.

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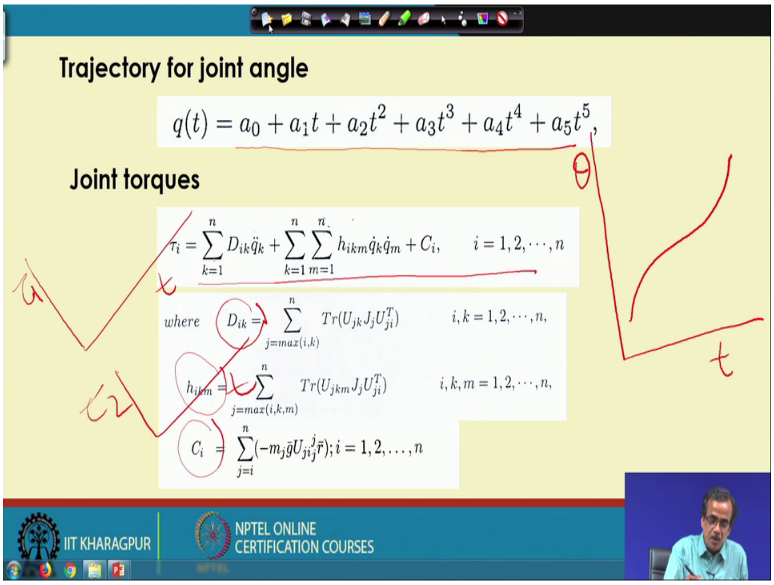

Now, once you have assign this particular coordinate system; now, if I want to find out the joint torque. So, what I will have to do is; so, I will have to find out what should be the variation of theta as a function of time. And we will have to assume actually the smooth variation of this particular; the joint angle.

Now while discussing the trajectory planning I have discussed in much more details; like how to fit. So this type of fifth under polynomial just to find out a smooth variation of theta; so, here q t is nothing, but theta t because this is nothing, but your the rotary joint. So, what I will have to do is; I will have to find out; so theta as a function of time some sort of a smooth curve I will have to fit.

And once you have got that particular thing; now I am in a position to find out what should be the variation of this particular joint torque that is tau 1 as a function of time, then tau 2 as a function of time and so on up to tau 7 and how to derive? So, those things I have discussed in much more details; so I am not going to repeat. Now, this is actually the final expression for the torque and this D i k is nothing, but the inertia terms which I have already discuss and derived; h i k m; correlation and centrifugal term and C i is nothing, but the gravity terms as we have discussed in much more details in robot dynamics. So, I am not going to spend much time on this.

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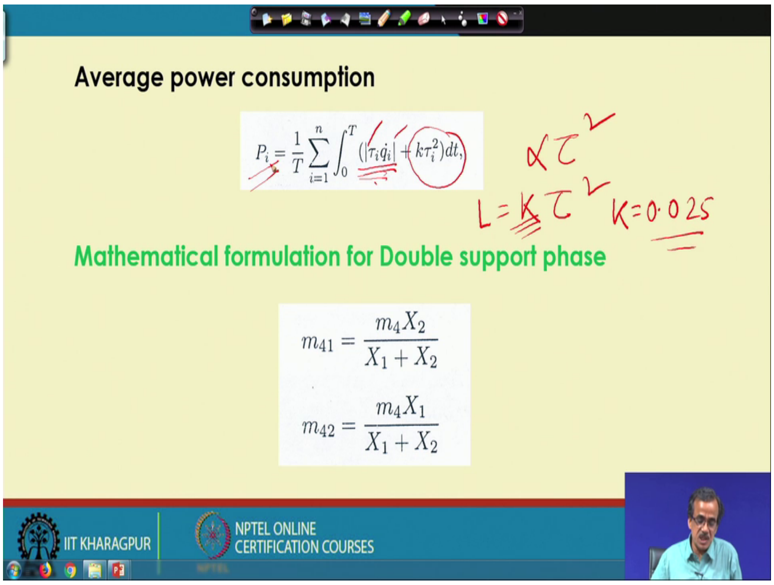

So, now I am in a position to think that for this particular biped robot; I am able to find out what should be the expression for the joint torque and how to determine actually your the dynamic balance margin. Now here actually what we can do is; so, I can discuss like how to determine this power consumption. Now, the expression for the power consumption; power we know that is nothing, but the rate of change of work done or a work done per unit time. So, here; so this particular tau i tau denotes actually the torque multiplied by qi dot that is nothing, but the angular velocity. So, torque multiplied with angular displacement is the work done per unit time; so, this is nothing, but the power plus here I have written k multiplied by tau i squared.

Now, I have already discussed that at each of the robotic joint we use some DC motor and whenever we are going to use DC motor, there will be some loss. And that particular loss is nothing, but; so loss in the in this DC motor is proportional to tau square; that means, here the loss L is nothing, but k multiplied by tau square and k is nothing, but is your constant of proportionality. And generally for the DC motor; so this particular k is taken to be equal to 0.025 or very close to that.

Now, if I know this particular; so I can find out how much is the loss due to this particular loss in this particular the DC motor. And this is the requirement of the torque and I can find out what should be the a power rating for the motor, which I am going to put at the different joints.

And using this particular principle actually we can find out what should be the power rating for this particular your that the motor connected at the joint. This is how to carry out the analysis for the single support phase.

Thank you.