Robotics
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Lecture – 39
Robot Motion Planning (Contd.)

Now, we are going to discuss how to determine the collision free path for the robot in the presence of some moving obstacle.

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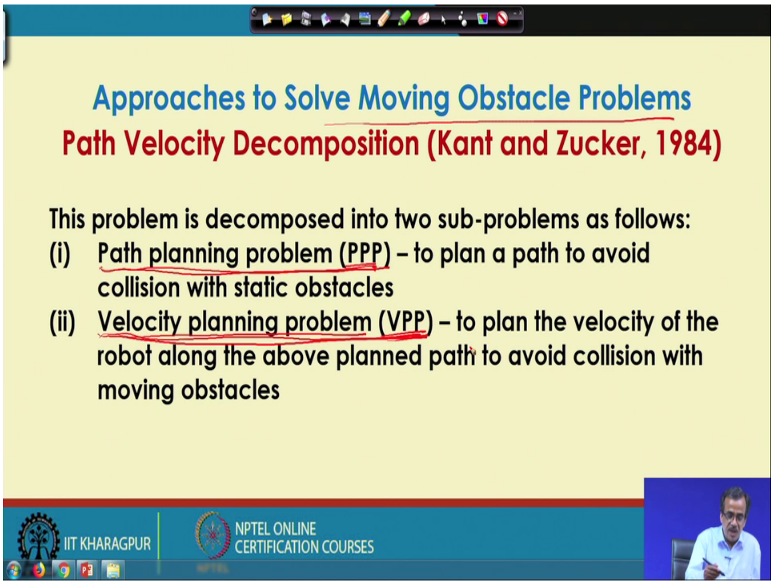

Now here the obstacles are moving, the robot is moving the obstacles are also moving; how to find out, how to ensure the collision free path? Now, if you see the literature the first approach which was proposed in the year 1984 by Kant and Zucker is the most popular approach which is known as the path velocity decomposition.

Now, let us see how to use the concept of this path velocity decomposition; to solve the problem the navigation problem of a mobile robot in the presence of some moving obstacle. Now this particular problem is popularly known as the dynamic motion planning problem.

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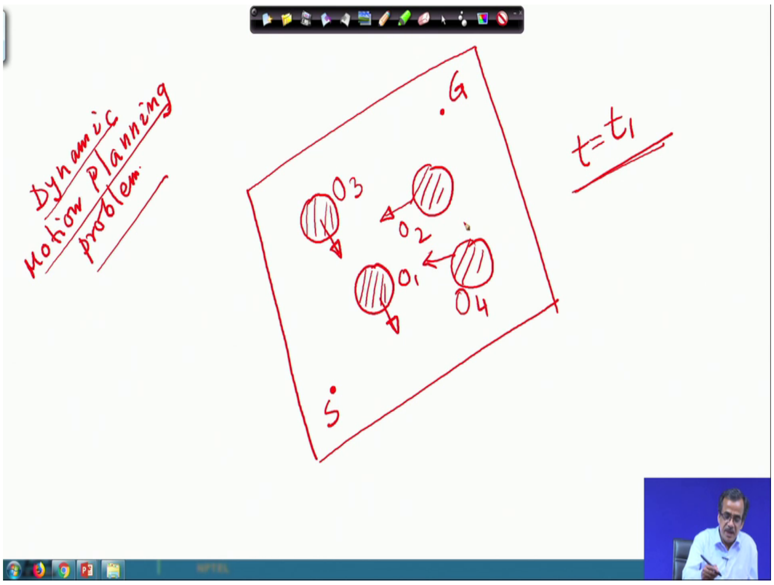

So, this is known as the dynamic motion planning problem; dynamic motion planning problem. Now, the problem is as follows supposing that I have got one field say this is the field, and I have got a point robot at position S. So, this is the starting position for the robot and the goal could be here that is denoted by G now here. So, the robot we will have to start from S and it will have to reach G and it will have to find out some set of optimal path.

Now if there is no obstacle then starting from S, directly it will reach that particular the point G. But supposing that there are some moving obstacle; so, this is obstacle 1, O 1 and this is moving in this particular direction with some speed. I have got another obstacle here say this is O 2 and this is moving in this particular direction with some speed. I have got say another obstacle; so, this is the direction of movement with some speed say O 3.

Then how to ensure or how to find out the collision free path and the time optimal path for this particular the robot? So, this is actually the problem and let me consider one more obstacle here and suppose this is moving in this particular direction with some speed. So, this type of problem is known as the dynamic motion planning problem.

Now, this is a dynamic motion planning problem; that means, it is varying with time ok? So, it is varying with time; now this particular dynamic motion planning problem can be converted to a find path problem at time t equals to t 1. So, at a particular instant at time t equals to t 1; so, this will become a find path problem. So, I know at time t equals to t 1; I know the predicted position of this particular obstacle and there is a possibility that I will be getting. So, this type of problem which is nothing but so, this is the field.

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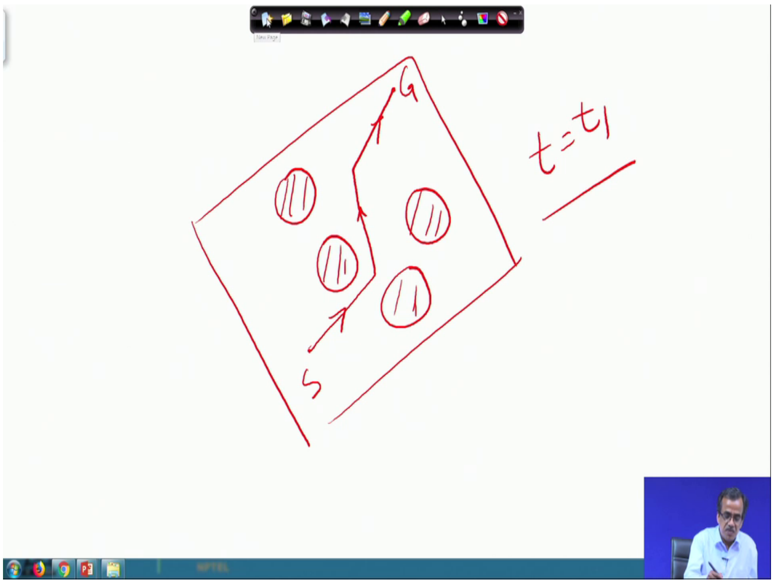

And I have got the starting point here, the goal is here and this is one obstacle, this is another obstacle, this is another obstacle, this is a another obstacle. So, these are all obstacles; so, at time t equals to t 1; so this becomes a find path problem. So, for simplicity; so this particular dynamic motion planning problem is converted into a find path problem at time t equals to t 1 and actually they solved; so this particular the find path problem. Now supposing that for this particular problem say supposing that it has got a collision free path something like this. So, this could be one of the possible the collision free path for this find path proble, ok.

Now, once it has got this particular find path this particular feasible path; now it will have to do something to ensure the collision free movement because truly speaking. So, these particular obstacles are moving ok; so, here actually in this particular method. So, what we do is so we try to solve this dynamic motion planning problem using two sub problems. So, this dynamic motion planning problem is actually considered as a combination of two sub problem one is called the path planning problem and another is called the velocity planning problem.

So, in path planning problem we consider that at time t equals to t 1. So, this is nothing, but a find path problem; that means, the robot will have to find out a collision free path in the presence of some static obstacle. So, considering the obstacles to be stationary; so, it will try to find out a collision free path and next we just go for the velocity planning; that means, the robot is going to follow the path which it is goes through this path planning; that means, in the first stage.

Now, the velocity of this particular robot has to be adjusted so that it does not collide with your the moving obstacle. So, once again let me repeat the dynamic motion planning problem is converted into two sub problems; one is called the path planning problem that is PPP another is the; the velocity planning problem that is VPP ok. So, we first plan a path, the collision free path considering the obstacles are stationary and the robot will try to follow that predetermined path by adjusting its velocity so, that it does not collide with the moving obstacle. So, this is the way actually they implemented the path velocity decomposition method and it became very popular.

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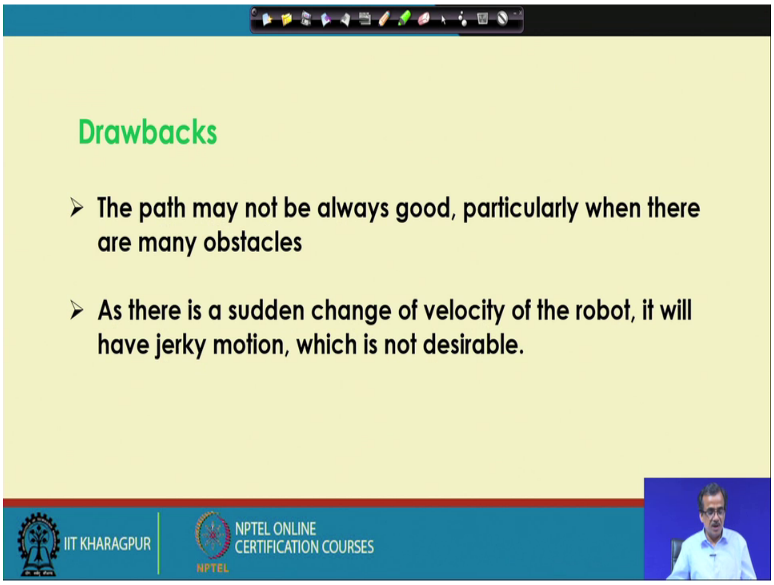

This particular approach became very popular, but this method has got a few drawbacks for example, say if there are so many obstacles, there are so many moving obstacle. So, this particular method may not find a feasible path; the robot may not be able to find out a feasible path by following this path velocity decomposition method.

Now, here as I told that in the second stage; we will have to plan the velocity of the robot. So, the velocity of the robot is going to vary with time and there could be sudden change of the velocity of the robot. And consequently there could be some sort of jerky movement of the robot which is not desirable. So, these are actually the drawbacks of this particular the path velocity decomposition.

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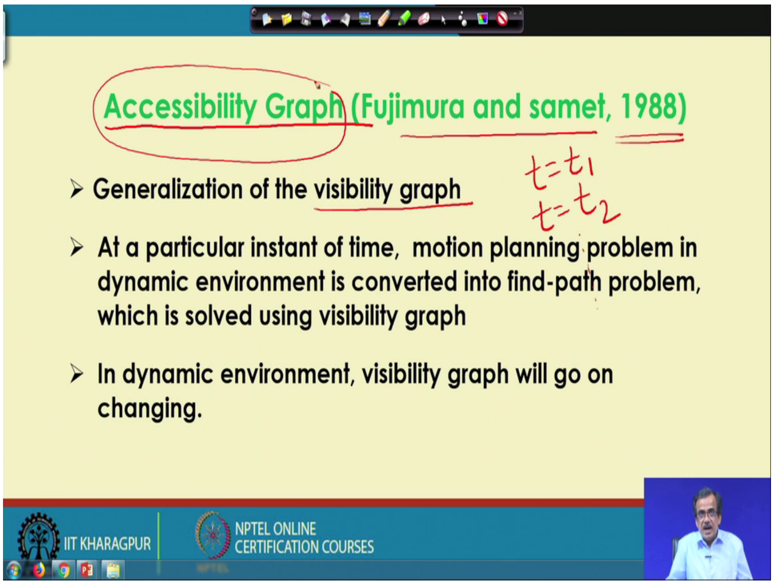

Now if you see the literature there are some other algorithms which have been proposed to solve the dynamic motion planning problem and out of those the methods the accessibility graph also reach some popularity. Now let us try to explain the principle of this particular the accessibility graph. The accessibility graph this concept was proposed by Fujimura and samet in the year 1988.

Now, here actually what he do is this is the modified version of the visibility graph which you have already discussed and which was proposed by Nilsson in 1969. Now if I just consider a dynamic motion planning problem as I discussed that at time t equals to t 1. So, this particular dynamic motion planning problem will become a find path problem; that means, that is the path planning problem for a robot in the presence of static obstacle.

Now, if it is a find path problem; so, I can use the concept of the visibility graph to find out a path, to find out a collision free path which I have already discussed. Now at time t equals to t 1 we consider that this is a find path problem. So, we will be getting some visibility graph collision free path; next at time t equals to t 2. So, once again I will be getting another scenario; so, another find path problem I will be getting. So, I will be getting another visibility graph; so, with time I will be getting a number of visibility graphs ok.

Now, these particular visibility graphs will go on changing with time. So, this accessibility graph is nothing, but the modified version of this particular the visibility graph. As if we have added one more dimension that is time to the visibility graph and this particular visibility graph is going to vary with time. And that is nothing, but the concept of the accessibility graph, but the main drawback of this particular accessibility graph is the computational complexity.

So, it is computationally very complex and it cannot be implemented online. So, what is our aim? Our aim is to determine one collision free path time optimal path, but at less computational time so that we can implement so this particular motion almost online. So, this particular accessibility graph as I told is computationally very expensive and which may not be suitable for your online planning.

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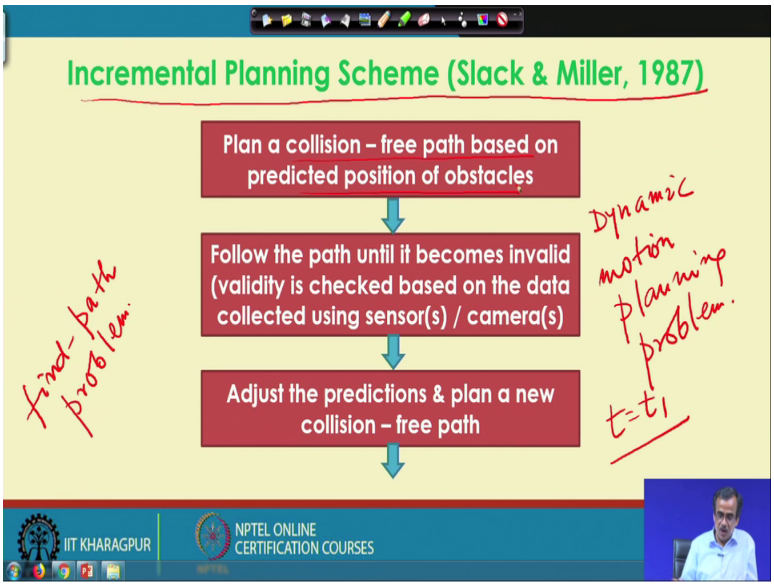

Now, another concept that is the concept of the incremental planning that was proposed by Slack and Miller in the year 1987; Now this concept is very simple very simple concept they used, the problem is the dynamic motion planning problem; that means, your the obstacles are moving and your the robot is also moving. Now the robot will have to find out the collision free time optimal path; so, this is dynamic motion planning problem.

Now, how to solve this particular problem? Now what I do is once again at time t equals to t 1, we consider that this particular dynamic motion planning problem is nothing, but a find path problem. So, if it is a find path problem; so very easily actually we can find out what should be the collision free path? So, we plan a collision free path based on predicted position of the obstacle ok. Next we follow that particular path until it becomes invalid, the moment it is found to be invalid ok. So, what we do is we replant and we try to find out actually another collision free path and we follow that particular collision free path.

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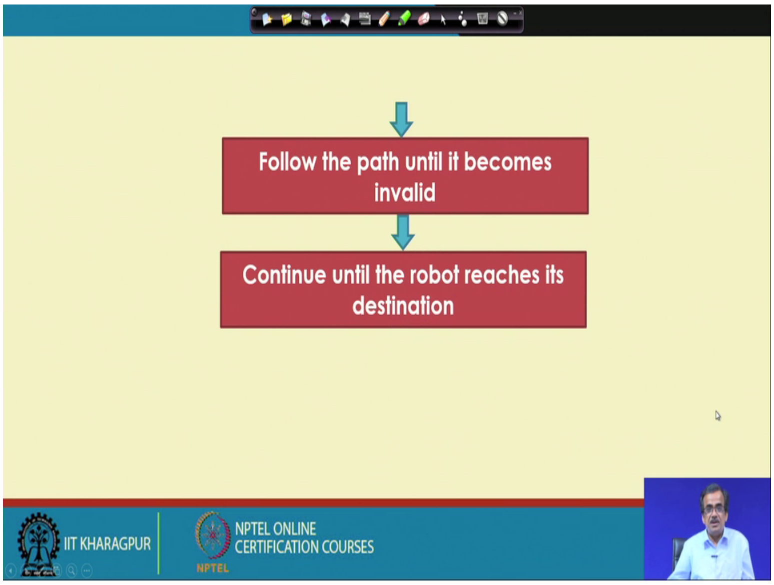

So, till it becomes invalid and the moment it becomes invalid once again we replant and this particular process will continue till the robot reaches the destination. So, this is the way actually we can implement your the incremental planning. Now, incremental planning actually could not reach much attention of the researcher; it is due to the fact that your several times will have to find out the collision free path we will have to re-plan and that is actually not that was not very interesting. So, it could not reach much attention of these researchers working in the field of robot motion planning.

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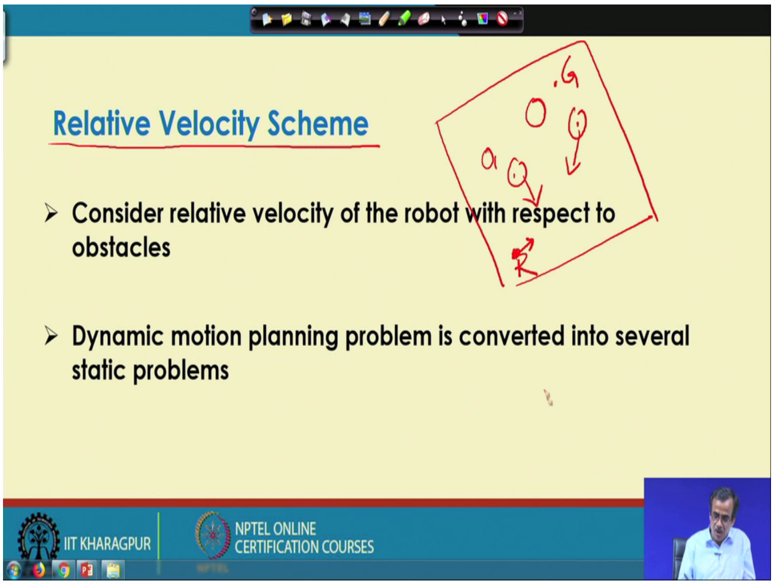

Now, then came actually the concept another concept that is the concept of your the relative velocity scheme. Now supposing that the robots are moving, the obstacles are also moving; so, we can find out the relative velocity of this particular robot with respect to the moving obstacle. Now let me just draw it here little bit say if this is the field; so, the robot is here the robot is moving and obstacles are also moving and say this is the goal. So, what you do is as both the robots and the obstacles are moving. So, we can consider the relative velocity of this particular robot with respect to the different obstacle. The moment we consider the relative velocity of this particular robot with respect to obstacle O 1, we consider as if obstacle O 1 is stationary and we try to find out the relative velocity.

Now, this is the concept of relative velocity. So, like your two bodies are moving with different velocities; so we try to find out the relative velocity of body 1 with respect to body 2; as if we consider the body 2 is kept stationary and we try to find out the relative velocity. Now, the same principle is copied here both the robot and the obstacles are moving. So, we try to find out the relative velocity of the robot with respect to O 1, then relative velocity of the robot with respect to O 2, relative velocity of the robot with respect to another obstacle O 3 and so on.

So, for the same robot there will be different relative velocities with respect to the different obstacles. And its implementation actually becomes little bit difficult and by following that so we could convert the dynamic motion planning problem into several static problems. And once you have got that particular; the matrix of the relative velocity of a robot with respect to the different obstacle; so, we try to implement just to find out the collision free path.

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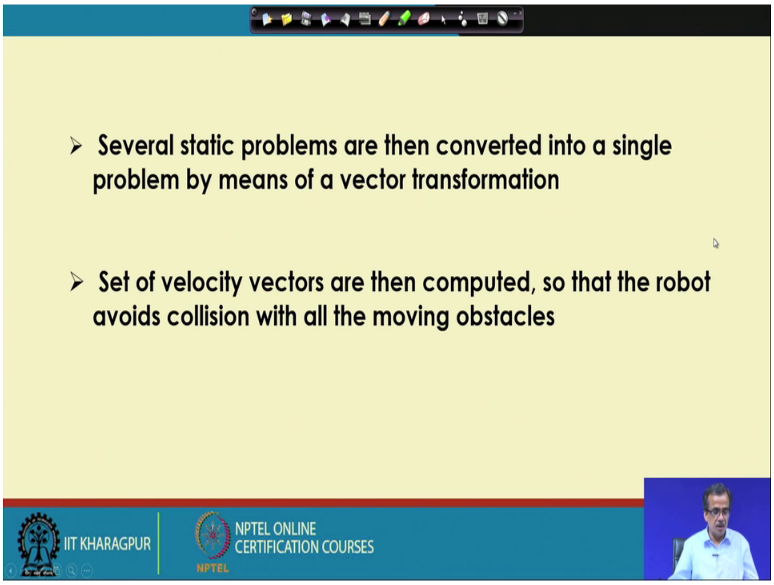

So, the set of velocity vectors we try to find out so that the robot avoids collision with all the moving obstacles. So, this is the way actually we can implement this particular you are the relative velocity scheme just to find out the collision free path the collision free the time optimal path.

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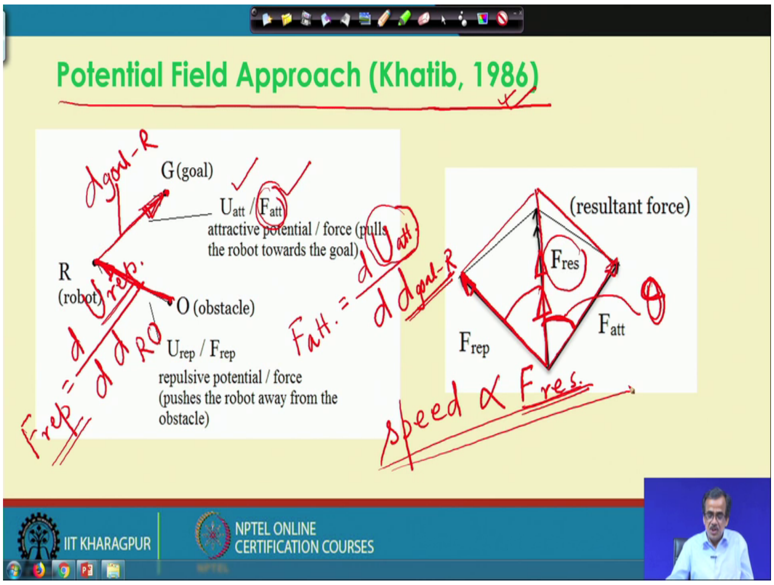

Then came actually the concept of the potential field approach and that was proposed by Khatib in the year 1986. And out of all the traditional motion planning algorithms; so, this particular algorithm could reach the maximum popularity. Now here actually in this potential field method, the robot will move under the combined action of attractive and repulsive potential or attractive and repulsive forces.

Now, let me consider that this is the goal for the robot and this is the present position of the robot and at time say t equals to t 1. So, this is nothing, but this is nothing but the distance between the robot and the goal. The goal is going to attract that particular robots towards it and there will be some attractive potential that is U attractive or there could be some attractive force.

Now, this is nothing but the attractive potential or the attractive force with which this goal is going to attract the robot towards it. Similarly, there could be repulsive potential or the repulsive force between the robot and this particular obstacle. And due to this repulsive force actually the obstacle is going to repel that particular the robot. So, it is going to repel; so here there will be some attraction, but here there will be some repulsion.

Now, this robot will be under the combined action of this particular attraction and this repulsion ok. Now here, so before I go for this how to find out the resultant of this attraction and attractive force and the repulsive force. Let me tell you how to find out this attractive potential; attractive force from this particular the attractive potential. It is very simple now this attractive force that is F attractive is nothing, but the derivative of this particular; your the potential attractive potential with respect to your; if this is the distance; so this is dd goal R that is the distance between the goal and the robot. So, this is nothing, but d goal R; so, this is your d goal R that is the distance between the robot and the goal.

So, what we do is we try to find out the derivative of this particular the attractive potential with respect to dd goal; that particular distance; so we will be getting the attractive force. Similarly from this repulsive force, if you want to find out; so this repulsive force that is F repulsive is nothing, but the derivative of your repulsive force U rep with respect to the distance between the robot and this particular obstacle. So, this is the distance between the obstacle and the robot. So, we try to find out the derivative with respect to your d R O and that is nothing but the repulsive force.

Now, as I told that the robot is subjected to both attractive as well as repulsive force. Now this I am just going to draw it here; so, here I have got the attractive force in this particular direction. So, I am just drawing it here; so this is nothing, but the attractive force and here there will be a repulsive force. So, here I am just going to draw the repulsive force ok; so, I have got the attractive force I have got the repulsive force.

So, very easily I can find out what should be the resultant force and this resultant force is denoted by F res; so, I can find out so this particular the resultant force. And once you have got this particular the resultant force; so, what do you do is; the robot actually as I told that it is moving under the combined action of attractive and repulsive, now it will try to follow this particular the resultant force. That means, the speed of the robot or the acceleration of the robot will be proportional directly proportional to the magnitude of this particular the resultant force.

And moreover so this is the angle of the resultant force with respect to the attractive; supposing that this is angle theta. And this particular angle theta with respect to the attractive; so, this will be the angle of deviation for this particular the robot.

So, the robot is subjected to the attractive force and repulsive force and due to this attractive and repulsive force. If you try to move with a speed which is proportional to the magnitude of this particular resultant force and its angle of deviation will be this particular angle that is theta; that is the angle between the resultant force or attractive or we can find out; so, this particular angle that is the angle between the resultant force and this particular the repulsive force. So, we need two information for the movement of the robot; one is the speed another is the angle of deviation. So, this is the way actually we are determining the speed and the angle of deviation for this particular the robot

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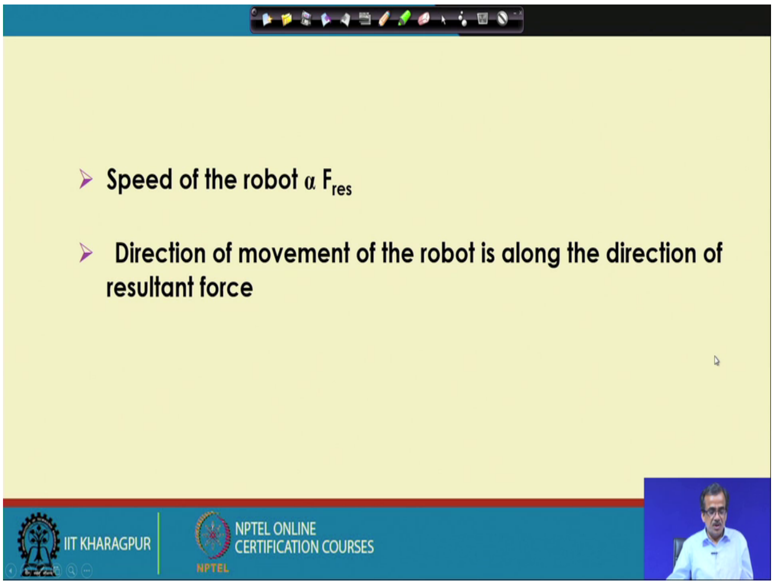

Now here whatever I mentioned the same thing I have written it here.

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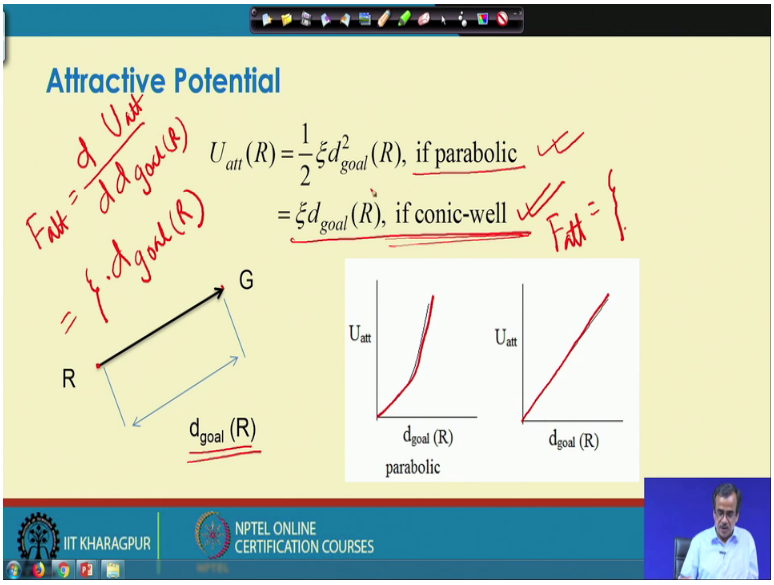

So, what I am going to do is; so, I am just going to assign some attractive potential and repulsive potential, some mathematical expression. And try to see how to derive that particular thing like you are how to derive that attractive potential and the repulsive potential.

Now, as I told that this is nothing, but the position of the robot and this is the goal and this is nothing, but the distance that is d goal R. Now this U attractive is here I have considered half zeta d goal R square; if it is considered to be parabolic that is second order curve. And if I consider that U attractive is nothing, but zeta d goal R; so, this is nothing, but a straight line.

Now, if I just draw it here; so this U attractive as a function of d goal R; so this particular thing. So, I will be getting; so these type of plot for the attractive potential; if I consider this type of expression, so this is nothing, but the variation of U attractive with d goal R. And as I told how to find out this particular F attractive; so this F attractive will be nothing, but so d d goal R of this particular thing. So, that will be your d d goal R of U attractive potential and this will be nothing, but; so here there is a square. So, 2 will come; so this will become zeta multiplied by d goal R; so, this will be the attractive force.

Similarly, if I consider; so this type of distribution for the attractive potential then your F attractive will be nothing, but is your constant and that is nothing, but this zeta ok. So, by differentiating actually we can find out the attractive force. So, we can use different types of function for this attractive potential and accordingly we can find out like what should be your the expression for attractive force.

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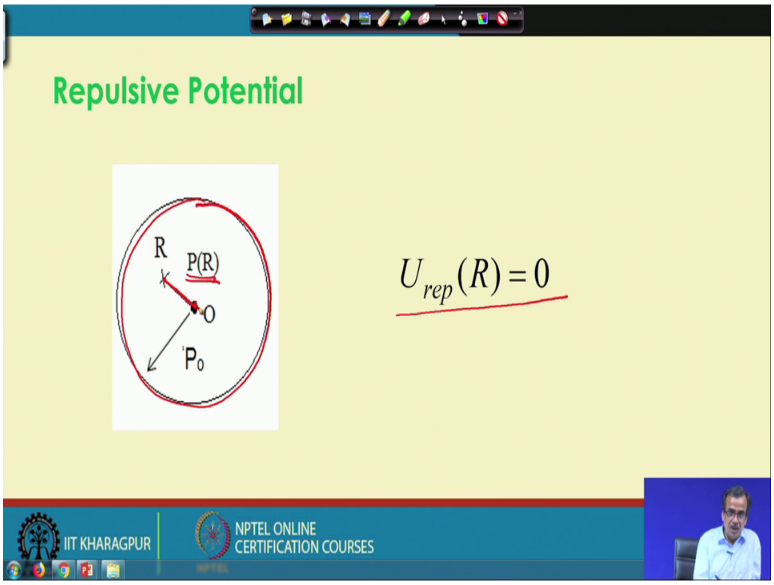

Next we try to concentrate on the repulsive force.

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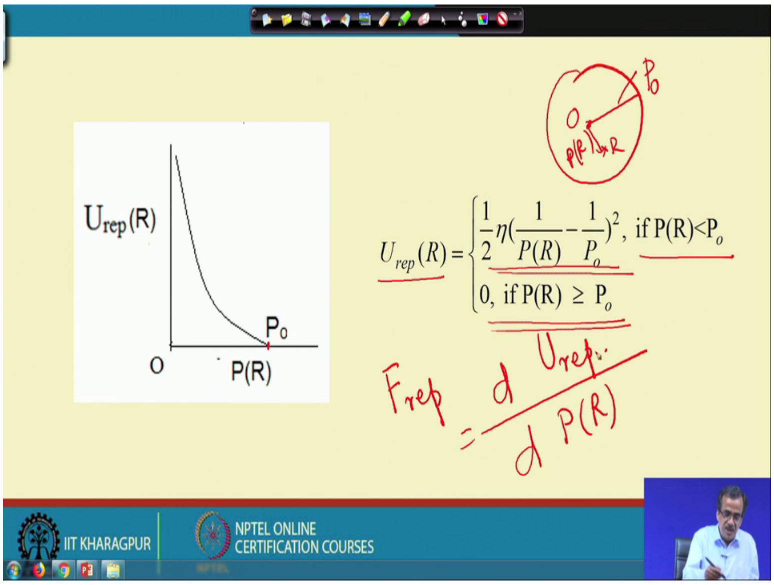

Now here this repulsive force is actually is defined in such a way; supposing that say this is actually the obstacle and the robot is here. And this is the distance between the robot and the obstacle that is denoted by P R. Now surrounding this particular the obstacle, we define one circle and on the boundary of the circle and beyond the boundary of the circle the repulsive potential will become equal to 0, but inside this particular circle there will be some repulsive potential. And this particular the repulsive potential will be maximum when the robot comes very close to this particular the obstacle.

So, when the robot is very close to the obstacle; the repulsive potential will be more and whenever the robot reaches this particular; the boundary of this particular circle then the repulsive potential will tend to 0. And beyond which outside this particular circle the repulsive potential will become equal to 0. Now, the same thing I have just plotted in here; so this is actually the plot of the repulsive potential. So, when P R is small, the repulsive potential is very large and this repulsive potential will become equal to 0 when P R becomes equals to P 0 ok.

So, when P R becomes equal to P 0 that is actually the radius of that particular circle. So, this repulsive potential becomes equal to 0; now this is the mathematical expression like repulsive potential is equals to half eta 1 by P R minus 1 by P naught square. If P R is found to be less than P naught; so, if I just draw it here. So, this is the obstacle say center of the obstacle; so this is your P naught and supposing that the robot is here. So, this is the robot and this is your P R.

So, if P R is less than P naught; that means, it is inside this particular circle. So, this is the expression for this particular; the repulsive potential otherwise; so this particular repulsive potential becomes equal to 0. And once you have got this particular repulsive potential; so by differentiating with respect to this particular P R P R is nothing, but the distance between the obstacle and the robot. So, we can find out what should be your F repulsive. So, F repulsive is nothing, but d d P R of your U repulsive; so, this is nothing, but your F repulsive ok. So, this is the way actually we define this particular attractive and the repulsive potential.

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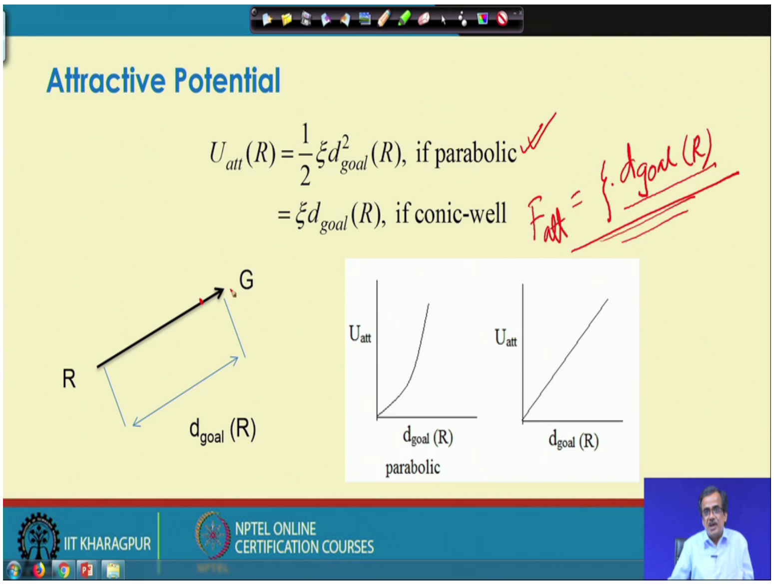

Now, here actually for this particular attractive potential there is another point to be considered that is your; as I told that if I consider say this particular thing. So, the F attractive becomes nothing, but zeta; so d goal R ok. So, here there is another important point to be discussed like say zeta is having some fixed and numerical value. Now if d goal is small; that means, your the robot has reached very near to this particular goal might be the robot is here. So, that so if d goal decreases; so this particular attractive potential is going to be reduced.

Now, this is done very purposefully otherwise the robot will not be able to stop at the goal with 0 velocity. So, when R is; so when this d goal R is more then the attractive potential is more. So, when the robot is at far distance from the goal; so, the active force will be more, but whenever it comes very close to the goal, the attractive potential is going to be reduced. And so that the robot can stop at the goal with 0 velocity; so this has been actually done very purposefully; so, this is the way actually we can determine attractive and repulsive potential attractive and repulsive force.

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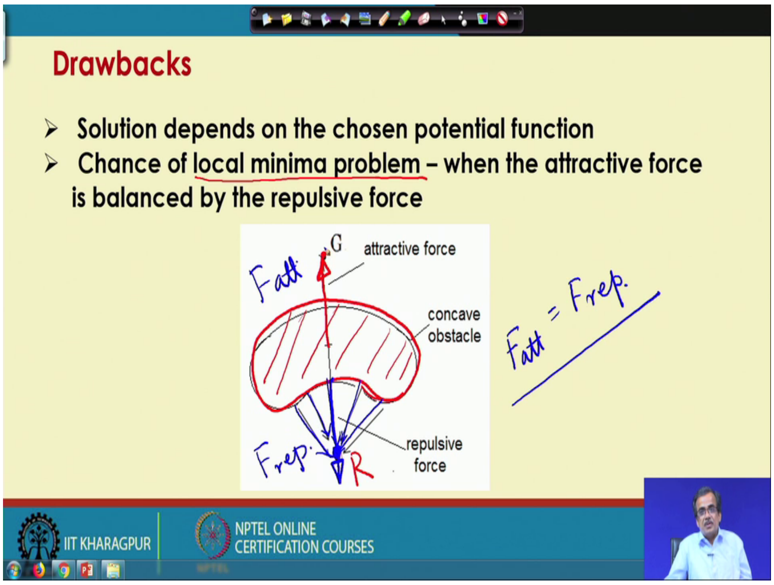

Now, as I told that out of all the traditional tools for your the motion planning; this potential feed method is the most popular one, but it has got a few drawbacks. For example, say here the solution depends on the chosen potential function this is called the artificial potential function. And depending on the nature of this artificial potential function; so, we will be getting this solution for the robot. The robot will try to find out a path the collision free path depending on the chosen potential function.

Now, here there is another very big problem which we are going to face that is actually called the local minima problem. Now this is a very typical scenario and this happens for the concave obstacle. Now supposing that; so, this is the concave obstacle; so this is the concave obstacle sort of thing, it is a very hypothetical situation. And supposing that this is the goal for the robot and fortunately or unfortunately this is the present position for the robot ok.

Now, if this is the situation; here we have got the concave obstacle the goal will try to attract this particular robot. So, there will be some attractive force in this particular the direction, but this particular your the obstacle the obstacle boundary is going to put some sort of repulsive force on this particular the robot. For example, say it is going to; so some sort of repulsive force here, some sort of repulsive force here, repulsive force here; so, it is subjected to the repulsive forces.

Now, all such forces are passing through a particular point. So, all of us we know how to find out the resultant of this particular the forces, the set of forces. So, graphically we can find out what should be the resultant of these particular forces. And supposing that the resultant your repulsive force is something like this. So, this is the resultant and impulsive; so, this is F repulsive and this is your F attractive ok.

Now fortunately or unfortunately if F attractive becomes equal to your F repulsive then what will happen? So, attractive force becomes equal to the repulsive force; the robot will become stationary here. So, there will be no movement of this particular robot and the robot will not be able to reach this particular the goal. So, this type of the problem we may face in your the potential field method.

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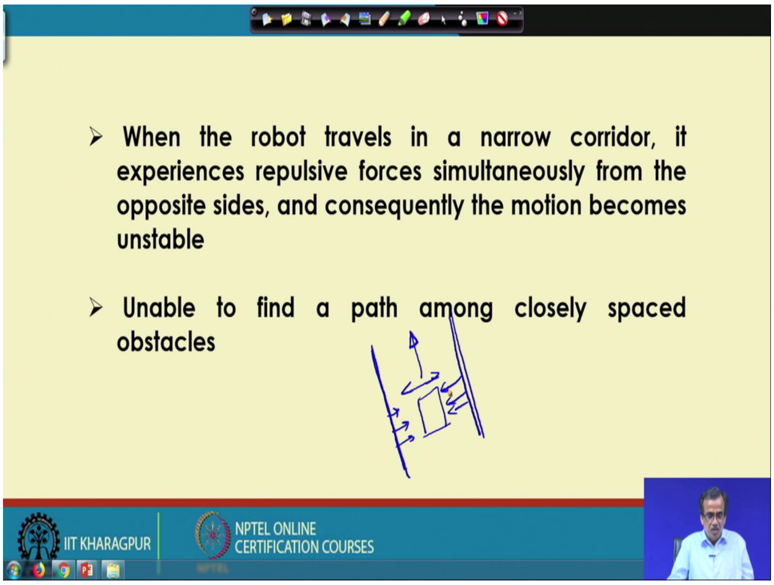

There are some other disadvantages, supposing that I have got one narrow corridor. And I am just going to find out a collision free path for this particular robot say I have got a robot here and I am just going to make a plan for this particular the robot.

And this is one wall, this is another wall. So, there will be some repulsive force here, there will be some repulsive force here and consequently so this particular robot will have some sort of oscillatory movement something like this; so, there will be oscillation and which is not desirable. Moreover like in the space if there are so many such obstacle; large number of obstacle there is a possibility that it may not be able to find out the optimal, the time optimal and collision free path.

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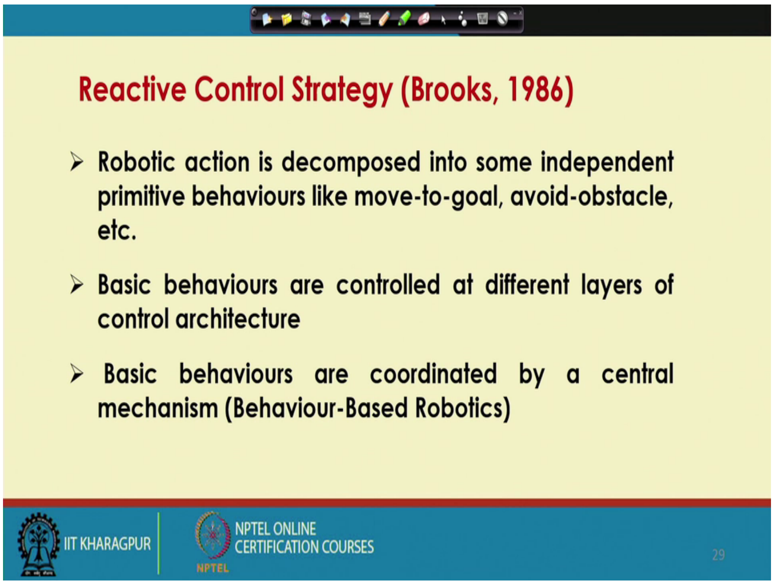

Now, here there is another very popular scheme that is called the reactive control strategy that I will be discussing later on.

Thank you.