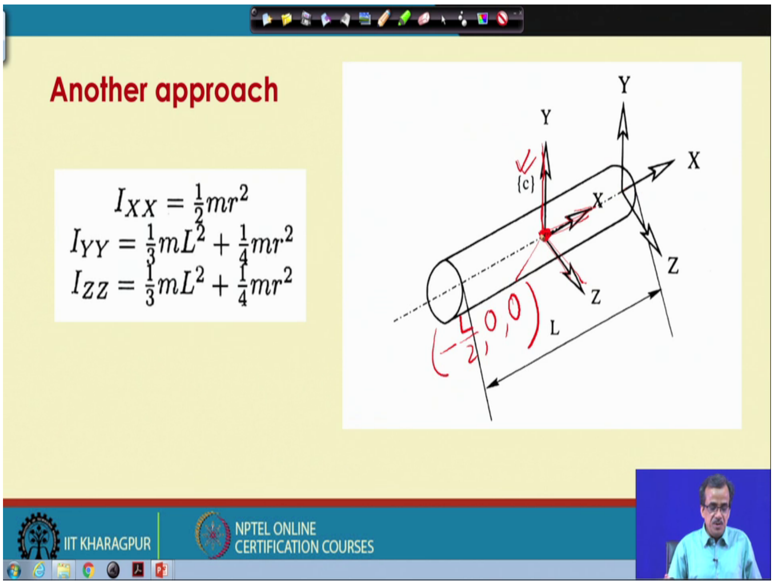
Robotics

Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 29
Robot Dynamics (Contd.)

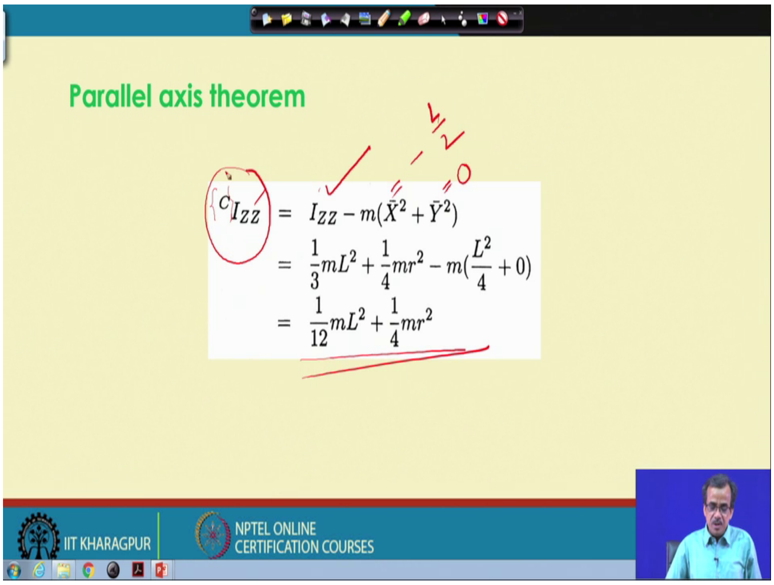
(Refer Slide Time: 00:27)



Now, I am going to discuss another approach using the Lagrangian method to derive that particular expression for the joint torque that is theta 1, theta 2 for a say 2 degree of freedom serial manipulator. Now, here actually 1 modification will have to do. Now, till now, if this is the robotic arm having the length L and the coordinate system is attached here, so this is your X, Y and Z, this is the coordinate system. But the motor is connected at this particular the joint. And as I told several time that we are trying to find out the reacts and torque.

Now, what you will have to do is we consider the moment of inertia with respect to your, so this coordinate system. For example, we got I XX equals to half m r square. So, m is the mass of this particular link, r is the radius. So, this is having circular cross section with radius r. Then I YY is one-third m L squared plus one-fourth m r square, so this expression we have already got, we have already derived. Then I ZZ is one-third m L square plus one-fourth m r square, so this expression we have already derived.

(Refer Slide Time: 01:50)

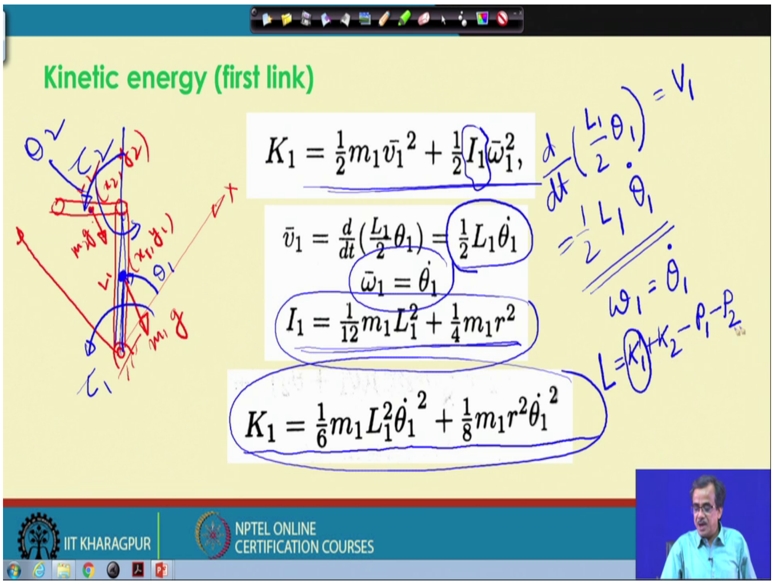


Now here, if we just go for this particular approach what will have to do is, so we will have to express the moment of inertia with respect to, so this particular coordinate system which is attached at the mass center. Now, this is the coordinate system which is attached at the mass center that is denoted by c. And what is the coordinate of this particular mass center, the coordinate of the mass center is nothing but minus so L by 2, Y equals to 0, Z equals to 0, so this is the coordinate of this particular mass center.

So, what I am going to do is, I am just going to represent the moment of inertia with respect to this particular coordinate system and I am just going to transfer from here to here using the parallel axis theorem. So, using the parallel axis theorem, so I can find out that moment of inertia about ZZ with respect to C; C is the coordinate system, which is attached to the mass center is nothing but I ZZ, so this is I ZZ minus m X bar square plus Y bar square.

Now, here this X bar is nothing but, actually minus L divided by 2 and Y bar is is equal to 0. And so this can be written as one-third m L square plus one-fourth m r square minus your m L square by 4 plus 0 and if you simplify, so you will be getting this particular the expression. So, this is nothing but the expression for, this I ZZ about the coordinate system which is attached to the your the mass center, so knowing this actually what I can do is, so we are going to derive the same the tau 1 and tau 2.

(Refer Slide Time: 03:57)



Now, if I just draw this picture once again, the same manipulator like your the same manipulator having 2 degrees of freedom. So, this is your X, this is Y, so here I have got say L 1, this is L 2, this is L 2. So, length of this particular is L 1, this is L 2. Now, here so this is actually your the mass center here m 1 g will be acting, this is the mass center where m 2 g will be acting.

Now, here this mass center is having the coordinates say x 1, y 1; this is having the coordinate say x 2, y 2. And our aim is to find out our aim is to derive the expression for so this particular tau 1 and the tau 2, so that is our m. The problem is the same; I am using a slightly different approach.

Now, here let us first try to concentrate on the first link, that is your whose length is L 1. Now, here we try to find out the kinetic energy of the first link and by definition the K 1 is half m 1 v 1 square plus half I 1 omega 1 square. So, I 1 is the moment of inertia omega is nothing but angular velocity and v 1 is a linear velocity and m 1 is a the mass. So, what I will have to do is, the mass is assumed to be concentrated at the mass center. So, I will have to find out what should be the linear velocity of this particular the mass center.

Now, to find out the linear velocity what will have to do is, so this is the joint angle say theta 1; and this is nothing but, the joint angle that is your theta 2. So, if this is theta 1 and up to this is actually L 1 by 2, then how much is the arc, arc is nothing but, is your L

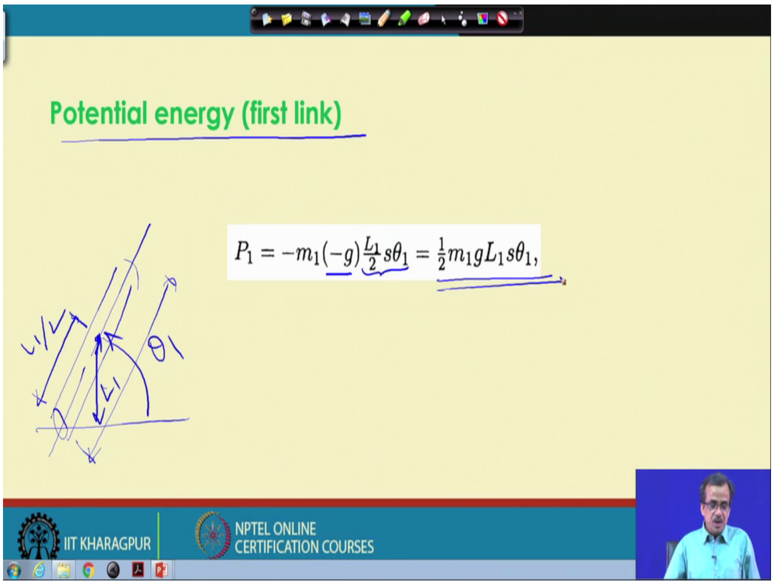
1 by 2 multiplied by theta 1 and the rate of change of this with respect to time is nothing but is your V 1 so that is nothing but the linear velocity at this particular point.

So, this L 1 theta 1 by 2 d dt of that, so this is actually I have determined here d dt of L 1 by 2 theta 1 and that is nothing but, is your half L 1. So, d theta 1 dt that is nothing but theta 1 dot, so this is nothing but the expression. And here, so this particular omega 1 is the angular velocity and that is nothing, but theta 1 dot. Are you getting my point? And these particular I 1 the expression for this particular I 1 is nothing but, these 1 12 m 1 L 1 square plus one-fourth m 1 r r square, so this I have already derived.

Now, if I substitute here m 1 is the mass m 1 will remain same as m 1 V 1, so this is the expression for V 1 then I 1, so this is the expression for I 1 and omega 1 is nothing but theta 1 dot. So, if I substitute all the terms here and if I simplify then I will be getting. So, this particular expression for the kinetic energy of the first link that is k 1 is nothing but, one-sixth m 1 L 1 square theta 1 square plus one-eighth m 1 r square theta 1 dot square. So, this is nothing but the expression for this particular the kinetic energy, this is the kinetic energy for the first link.

Now, I am just going to derive the potential energy for the first link and then I am going to derive the kinetic energy for the second link and potential energy for the second link and if I can find out the kinetic energy and the potential energy for both the links I can find out, what should be the Lagrangian for this particular the robotic system. For example, say the Lagrangian for the robotic system L will be nothing but kinetic energy of the first link plus kinetic energy of the second link minus potential energy of the first link minus potential energy of the second link. So, till now I have derived only this particular k 1so, I am just going to derive k 2 p 1 p 2.

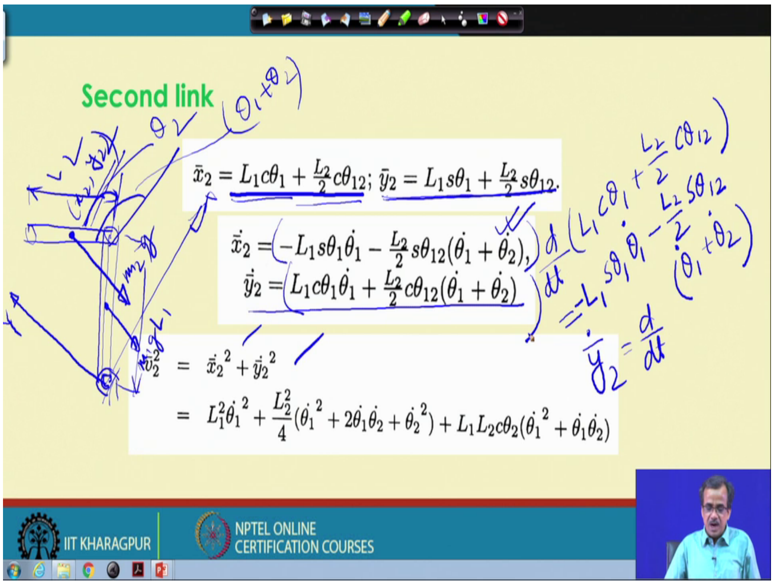
(Refer Slide Time: 09:42)



So, let us see, how to derive this particular the p 1 that is the potential energy for the first link. Now, the potential energy for the first link that is p 1 is nothing but, minus m 1 then multiplied by minus g once again g is acting vertically downward opposite to the that Y direction and this particular the height. If you see, so that was nothing but if this is the first link, if this is actually the first link the total length was say L 1 and this particular angle, angle was actually with theta 1 and this is nothing but L 1 by 2, so up to this is your L 1 by 2. So, L 1 by 2 sin theta 1 is actually this particular the height, so this is nothing but L 1 by 2 sin theta 1.

And, if you just simplify, so you will be getting this as the expression for the potential energy. Now, till now, we have derived the expression for the kinetic energy and the potential energy for the first link. Now, I am just going to derive the kinetic energy and potential energy for the second link.

(Refer Slide Time: 11:03)



Now, once again if you go back like if you just once again draw this particular picture once again, so this is actually your. So, I have got first link here and I have got the second link here this is your X, this is your Y ok. So, the length of this particular link is your say L 1 and this particular link the length is your L 2 and it is mass center is here whose coordinate is nothing but x 2, y 2 and here m 2 g is acting and here m 1 g is acting.

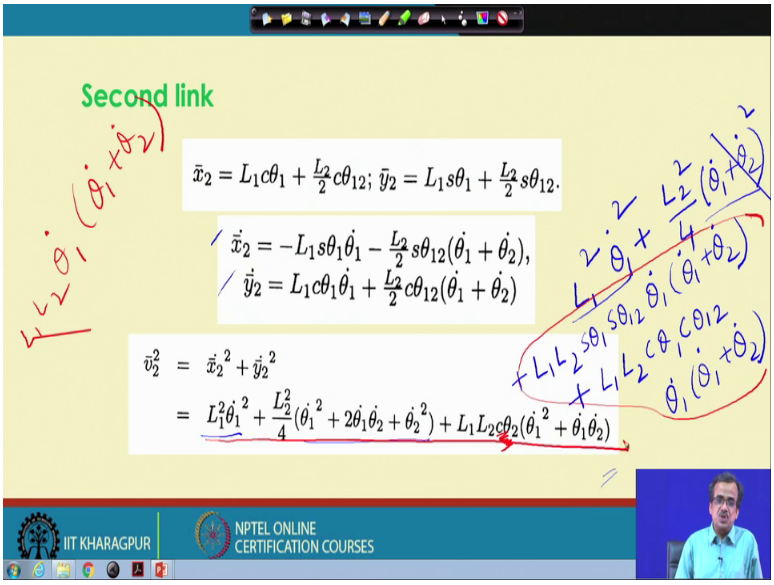
Now, here this x 2, y 2 is the coordinate of the mass center for the second link. Now the general expression for the coordinate of the mass center for the second link that is, x 2 is nothing but L 1 cos theta 1 plus L 2 by 2 cos of theta 1 plus theta 2, because with respect to this is your theta 2, this particular angle is theta 2, but with respect to x this is theta 1 plus theta 2 ok. Then, y 2 bar is L 1 sin theta 1 plus L 2 by 2 sin of theta 1 plus theta 2.

Now, we can find out the time derivative that is x 2 dot that is d dt of this now d dt of this. So, d dt of your L 1 cos theta 1 plus L 2 by 2 cos of theta 1 plus theta 2 ok. So, this can be written as this can be written as your this can be written as for example, this is 1 term and separately we consider these 2 terms will understand. So, these can be written as dd theta d theta d 1 that type of thing, so if I consider so here, I will be getting L 1 minus L 1 sin theta 1 d theta 1 dt that is nothing but theta 1 dot.

Similarly, here we will be getting minus L 2 by 2 sin of theta 1 plus theta 2 or multiplied by theta 1 dot plus theta 2 dot. So, I will be getting this particular the expression. By following the same method, so y 2 I know the general expression for y 2 is L 1 sin theta 1 plus L 2 by 2 sin of theta 1 plus theta 2. So, y 2 dot that is nothing but, y 2 dot is nothing but is your d dt of this particular expression and you will be getting L 1 cos theta 1 theta 1 dot plus L 2 by 2 cos of theta 1 plus theta 2 theta 1 dot theta 2 dot, so we will be getting this particular y 2 dot.

Now, v 2 square is nothing but x 2 dot square plus y 2 dot square. So, square of this, so square of this plus square of this and if you just add them up, then you will be getting the expression like your. So, from here I will be getting L 1 square then comes your theta 1 dot square.

(Refer Slide Time: 15:00)



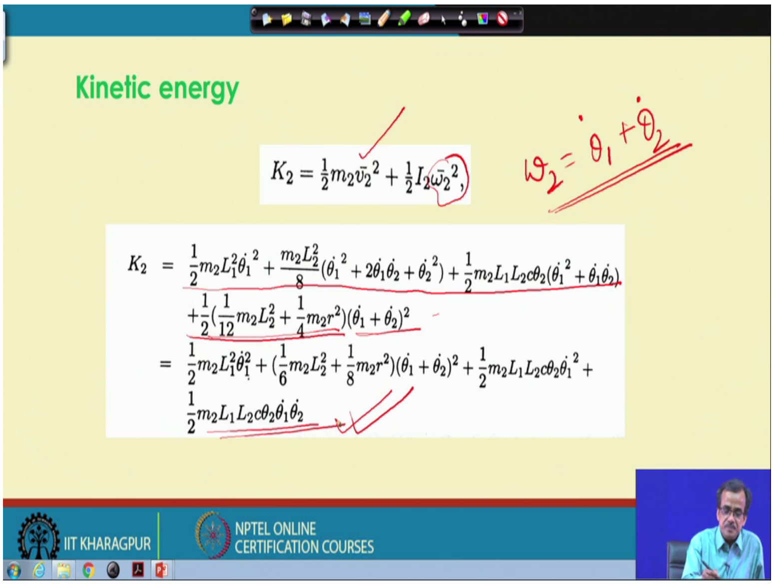
So, I am just squaring and adding, so these 2 squaring and adding, so I will be getting L 1 square theta 1 dot square and these sin square theta 1 plus cos square theta 1 will give rise to 1 plus here I will be getting your L 2 square by 4 ok, L 2 square by 4 then comes I will be getting. So, here sin square of theta 1 plus theta 2 plus cos square of theta 1 plus theta 2 will give rise to 1, so here there will be theta 1 dot plus your theta 2 dot square.

So, this type of things will be getting plus 2 into this, so I will be getting L 1, L 2 then comes sin of theta 1, sin of theta 1 plus theta 2 then theta 1 dot multiplied by theta 1 dot plus theta 2 dot plus, I will be getting L 1, L 2 then comes cos theta 1 cos of theta 1 plus theta 2 multiplied by theta 1 dot theta 1 dot plus theta 2 dot.

So, using this actually I can if you simplify further. So, I will be getting L 1 square theta 1 dot square. So, these I am getting L 2 square by four then this is expanded theta 1 dot square plus 2 theta 1 dot theta 2 dot plus theta 2 dot square. So, I am getting this also; now here we will have to simplify, now to simplify this part actually what I can do is we can take L 1 L 2 then theta 1 into this as common. So, I can write down like this particular part I am just going to concentrate on say this particular part.

This particular part if I want to make it simple, so this will become L 1 L 2 this will become L 1 L 2 then comes your theta 1 dot into theta 1 dot plus theta 2 dot ok, then comes your cos of theta 1 plus theta 2 cos theta 1 plus sin of theta 1 plus theta 2 sin theta one, so cos of theta 1 plus theta 2 minus theta 1. So, this is nothing but cos theta 2. So, I will be getting this particular cos theta 2 term ok. So, I am getting this particular expression for v 2 square and once you have got the expression for v 2 square.

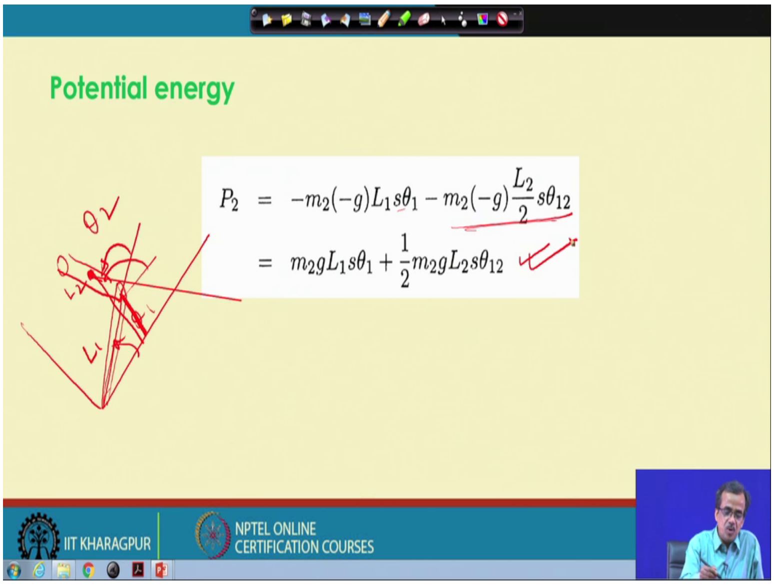
(Refer Slide Time: 18:13)



So, very easily I can find out the kinetic energy for the second link k 2 that is half m 2 v 2 square. So, I know the expression of v 2 now ok. Then I 2 I know the expression moment of inertia for the second link and this omega two, so this omega 2 will have to be careful, so this omega 2 will be nothing but theta 1 dot plus your theta 2 dot not only theta 2 dot. So, we will have to be careful.

Now if we just write down the expression v 2 square we have already derived and we just put it here I 2 I know the expression of I 2 I know, so this is the expression of I 2. In fact, this is the expression of half m 2 v 2 square then comes your this particular half I 2. I 2 is nothing but this 1. Then omega 2 omega 2 square that is nothing but, theta 1 dot plus theta 2 dot square and if you simplify. So, you will be getting this particular the expression ok, only thing you need some practice to find out whether you are getting the same expression or not. So, you will be getting this particular the expression for the kinetic energy for the second link.

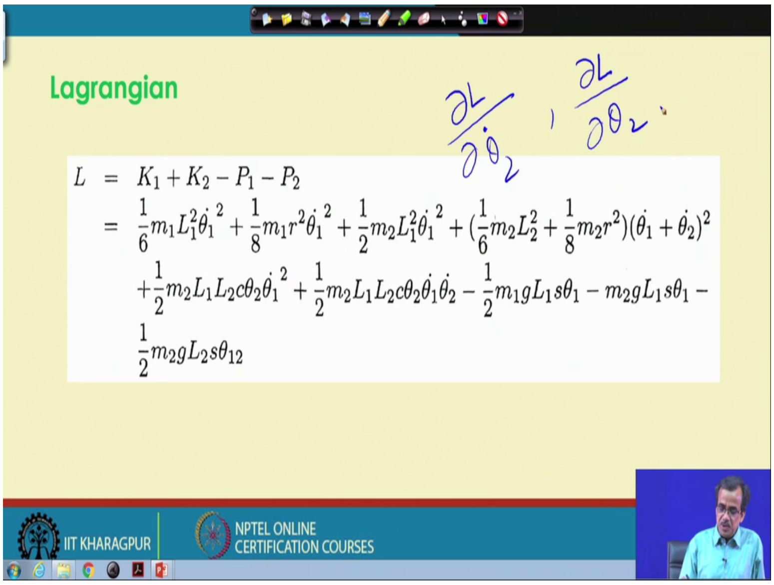
(Refer Slide Time: 19:50)



And once I got the kinetic energy for the second link, now we are in a position to determine the expression for the potential energy for the second link. Now, for the second link actually once again if you draw this particular picture, so this is 1 1 link and this is another link. So, your minus m 2 minus g L 1 sin theta 1 because this particular angle is your theta 1 and this is your theta 2 ok, this length is L 1, this is L 2.

So, here I will be getting L 1 sin theta 1 for the whole link, so this is L 1 sin theta 1 and here minus m 2 minus g L 2 by 2. So, this is total L 2 its midpoint is L 2 by 2. So, L 2 by 2 sin of theta 1 plus theta 2. So, this is actually your, the total height will be getting. So, this is the way actually you can find out the potential energy. So, this is the expression for the potential energy, now as I told that we have got the expression for the kinetic energy of the two links and potential energy of the two links.

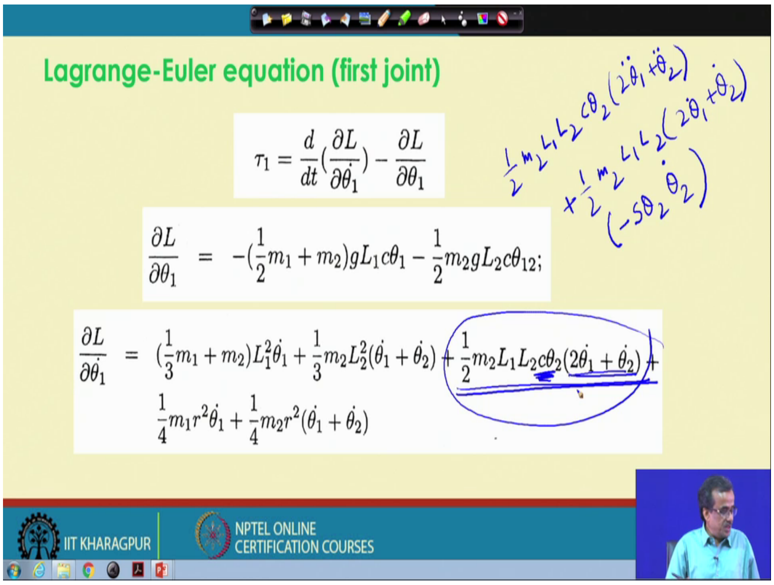
(Refer Slide Time: 21:11)



Now, we are in a position to find out the expression for this particular Lagrangian. So, this Lagrangian L is nothing but k 1 plus k 2 that is kinetic energy of the first link plus kinetic energy of the second link minus potential energy of the first link minus potential energy of the second link.

So, whatever expressions you got you just write down and then you do little bit of arrangement rearrangement rather rearrangement in terms of your theta 1 dot square theta 1 dot square theta 1 dot square in terms of theta 1 dot theta 1 theta 2 dot square. So, in terms of that you try to arrange this particular term and these are all sin theta 1 sin theta 1 sin theta sin of theta 1 plus theta 2. So, in this particular way you try to arrange these particular the terms and this is the expression for this particular the Lagrangian for the whole robotic system. And once I have got this particular Lagrangian now you know the expression you know how to determine actually the your the tau 1 and tau 2.

(Refer Slide Time: 22:40)



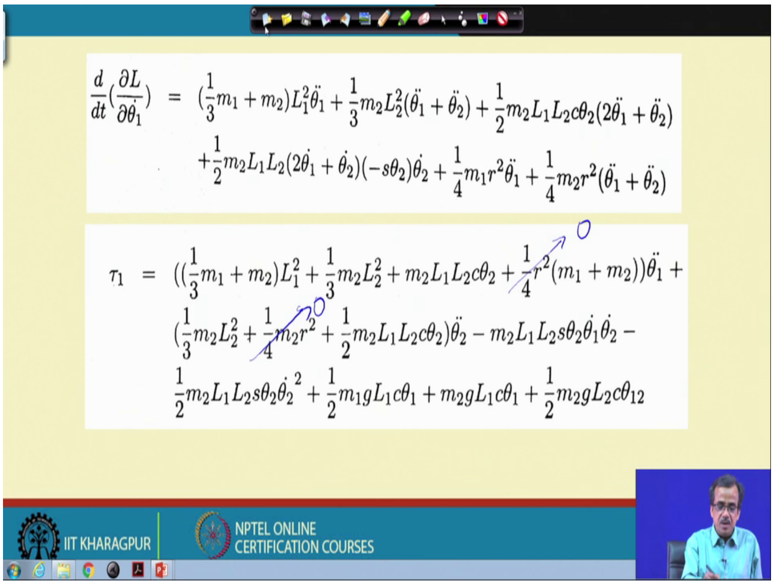
So, using the same formula the tau 1 is d dt a partial derivative of Lagrangian with respect to your theta 1 dot minus partial derivative of L with respect to this particular the theta 1. So, we know the expression for this particular your L that is the Lagrangian. So, once again let us go back to the expression of Lagrangian and here the derivative which we will have to find out is nothing but the derivative which we will have to find out is nothing but your partial derivative of L with respect to your theta 1 dot this is 1 to determine tau 1 another is partial derivative of L with respect to your theta 1 ok.

Now, you see you concentrate on this particular expression. So, here we have got 1 theta 1 dot square term, I have got another theta 1 dot square term theta 1 dot square term and here I have got theta 1 dot term theta 1 dot square term theta 1 dot term. So whenever I am just going to find out this particular partial derivative. So, starting from here up to this we will have some contribution. Are you getting my point? But these terms will not have no contribution.

Similarly, whenever we are going to find out your partial derivative of L with respect to theta 1, now theta 1 dot square will have no contribution, here there will be no contribution, no contribution, no contribution ok, and this is theta 2, but this is with respect to theta 1. So, there will be no contribution here. Here also no contribution, but the contribution will come from here and it will come from here in this particular partial derivative ok, so that we will have to understand.

Now, exactly the same thing whatever I told, so the partial derivative of the Lagrangian with respect to theta 1. So, this is the thing which we will be getting. And this particular partial derivative with respect to theta 1 dot, so these are the terms which you will be getting. And once I have got it, now we are in a position like now we will have to find out the time derivative of this. Now, the time derivative of this if we try to find out ok, now to find out the time derivative of this what will happen, so this particular theta 1, that means your just let me see most probably it is done here, yes.

(Refer Slide Time: 25:53)



So, the d dt of the partial derivative of L with respect to theta 1, so this particular previous term it was what theta 1 dot, now it will become theta 1 double dot. This was theta 1 dot, theta 2 dot, so this will become theta 1 double dot, theta 2 double dot. So, here it was theta 1 dot, theta 2 do, so this will become theta 1 double dot theta 2 double dot and next this particular term, So, once again it will take the form like this, and your so this particular term has got has got two terms here you can see just a minute. So, this term we will be getting from here and this particular term will be getting from here, but these actually this particular term if you want to find out, the time derivative.

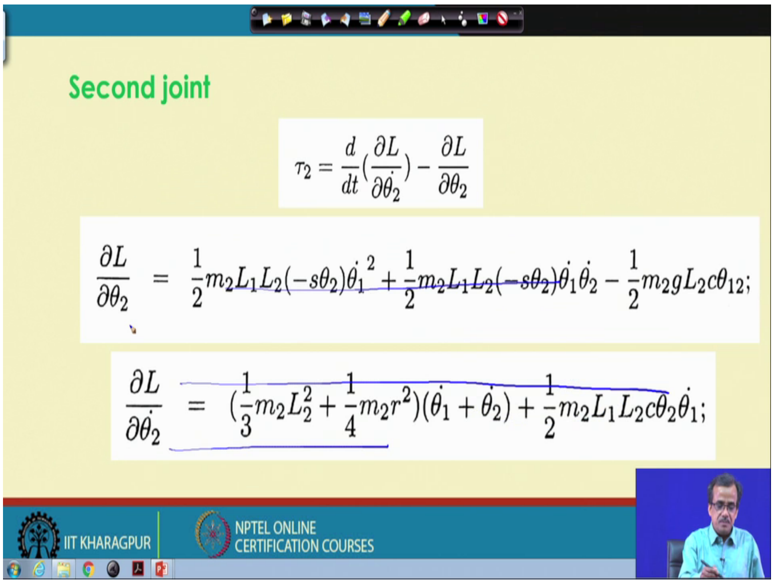
So, finding this particular term the time derivative, so these terms and these terms you will have to consider separately. So, once you will have to consider this as constant, and you will have to find out the time derivative. For example, if I concentrate only here, so its time derivative will be something like this like half m 2 L 1 L 2 cos of theta 2 like 2 theta 1 double dot plus theta 2 double dot plus half m 2 L 1 L 2 2 theta 1 dot plus theta 2 dot. Now, you will have to find out d dt of cos theta 2 that is nothing but d d theta 2 that is minus sin theta 2 of d theta 2 d 2 that is your d theta 2 dot ok. This is multiplied.

So this particular thing you will have to do. And if you do that, then you will be getting, so this particular expression for this joint torque that is tau 1. And once again you can see that, so this is nothing but this is nothing but is your the inertia terms multiplied by theta 1 double dot this is inertia terms multiplied by theta 2 double dot. This is the centrifugal terms the centrifugal terms and you will be getting some gravity terms something like this.

Thank you.

We need just 10, 10-15 minutes.

(Refer Slide Time: 29:13)



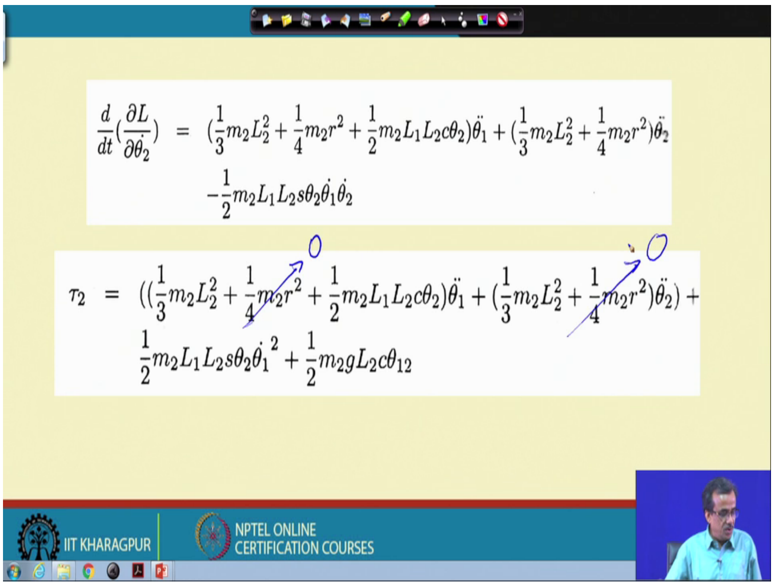
Now, let us try to find out the expression for the torque for the second joint. Now, here this tau 2 is nothing but d dt of partial derivative of Lagrangian with respect to theta 2 dot minus partial derivative of L with respect to theta 2. Now, exactly in the same way, so we will have to find out the partial derivative of L with respect to theta 2.

Now, if I see the expression of that particular your if I see the expression of this particular Lagrangian, so this is the expression of the Lagrangian. Now, we will have to find out actually the partial derivative of L with respect to theta 2 dot; this is one partial derivative another partial derivative with respect to your theta 2.

Now if you see with respect to theta 2 dot here there is no such theta 2 dot here. So, here the theta 2 dot comes here, so it will have some contribution. Then theta 2 dot here it has got theta 2 dot, so it will have some contribution ok. On the other hand, this theta 2 theta 2 here there is no theta 2 here no here cos theta 2, so it will have some contribution it will have some contribution this is theta 1 theta 1 and here we have got theta 2. So, it will have some contribution.

So, this is the way actually we will have to find out the partial derivative of this Lagrangian with respect to theta 2 ok and if you find out the partial derivative of this particular Lagrangian with respect to theta 2. So, we can find out like partial derivative of L with respect theta 2, so this is the expression which will be getting this is the expression which we will be getting. And partial derivative of L with respect to theta 2 dot, so this is the expression which I will be getting. And now we will have to find out d dt of this.

(Refer Slide Time: 31:34)

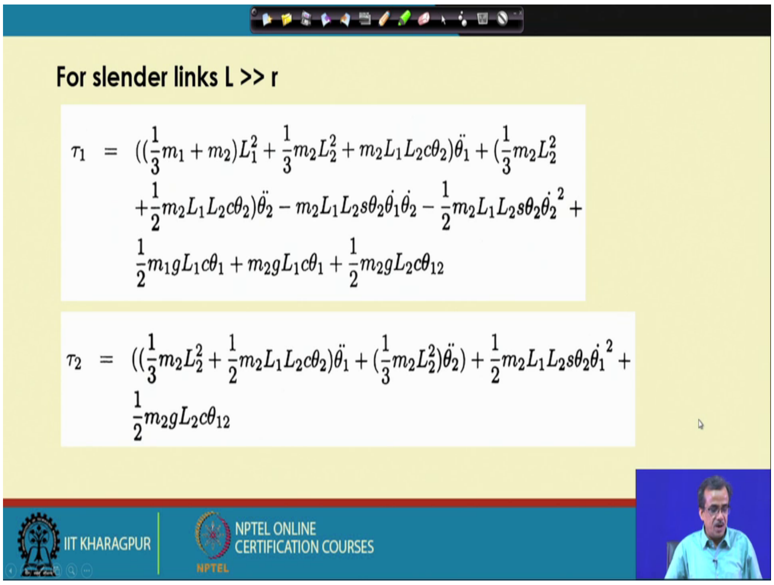


And if you find out the d dt of this then what will happen is your, so this theta 1 dot term will become your theta 2 dot term theta 1 double dot. So, this will become theta 1 double dot theta 2 double dot. And here also dot will become double dot sort of thing. And then if we just arrange you will be getting this particular your the expression. This is the expression which we will be getting for this. And then you substitute you will be getting the expression for the joint torque that is tau 2. And this particular tau 2, if you see once again, so this is nothing but is your the inertia term this is also inertia term, this is your the centrifugal term, and this is your the gravity terms.

So, we have got this particular expression for this particular tau and tau 2 for the same problem using two different methods. And if we compare, so we are getting exactly the same expression what we got earlier. So, the same expression I will be getting like if we compare these particular tau 2 for the same problem whatever tau 2 I got a few minutes ago exactly the same expression I got. The same is true for your tau 1. The expression which I got using this particular method and the expression which I got a few minutes ago exactly the same expression we got ok. And it proves that both the methods are correct and we are getting exactly the same expression for joint torque and tau 1 and tau 2.

Now, if I consider the slender link exactly in the same way. So, what you can do is this r squared term we can neglect for the slender link this particular r square term we can neglect. Similarly, what you can do is from this particular tau 2, so you will be we can neglect this particular the r 2 terms will you become 0, here also r 2 terms will become 0, and you will be getting this particular the final expression for tau 1 and tau 2 for the slender link.

(Refer Slide Time: 33:54)



Now, we have got the expression for this particular the joint torque. And as I told once I have got the expression next task will be your how to implement in the robot in the robotic joint, so that the motor can generate this particular the joint torque which will be discussed in the next class. But before that one thing I just want to mention.

Now, today I discussed actually two approaches two methods to determine the joint torques. The first method is more structured but the second method is very easy to implement; but if we just compare for a manipulator having two degrees of freedom, three degrees of freedom, we can go for the second approach.

But for a manipulator having say 6-5 or more than 6 degrees of freedom so my recommendations would be the first approach. And using this you can find out the joint torque, then we will see how to control in future like how to control these particular motors to generate that particular the torque and to generate the joint angle as accurately as possible.

Thank you.