Robotics

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Lecture – 29
Robot Dynamics (Contd.)

Now, I will try to derive the expression for the other h terms.

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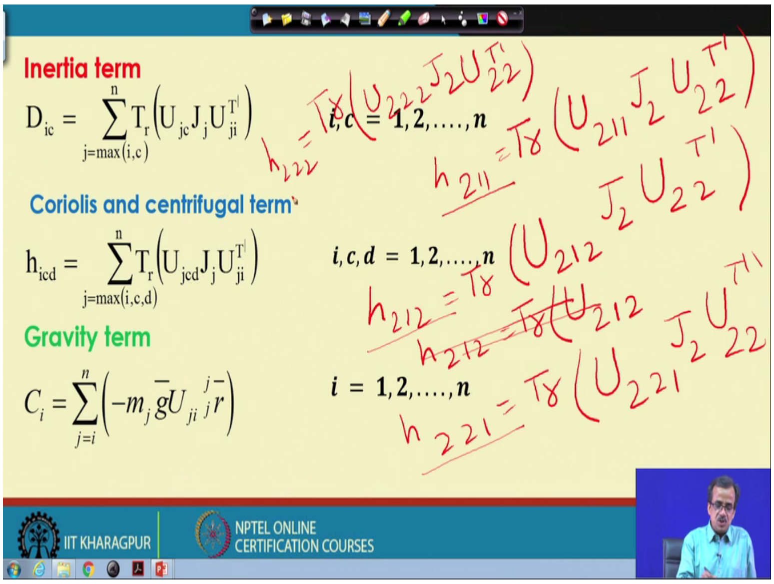

That is your h 2 1 1; now, this h 2 1 1 that is i equals to 2 C and D they are equal to 1. So, j equals to maximum between your 2 1 2, 2 1 1. So, this is 2 2 2. So, there will be only one term. So, this is nothing, but trace off like U j equals to 2 c, d is 1 1 then comes your J 2 then comes your U j; j equals to 2 and your this particular i i is also equal to your 2. So, U 2 2 transpose. So, this is the way you can find out the expression for this particular h 2 1 1.

Similarly, h 2 1 2 so, that is nothing, but so, once again 2 1 2 the maximum is 2. So, 2 2 2 there will be only one term then comes your U j is 2 here and c, d 1 2 , then comes your J 2 then U, j is nothing, but equal to 2 and i is also equal to your 2. So, i is also equal to your 2 and I will be getting this particular the expression.

Now, then comes your h 2 1 2, 2 1 2 that is nothing, but once again. So, 2 1 2 maximum is 2 2 2 2. So, I will be getting trace of U j equals to 2 here then c, d 2 1 2 sorry. So, I am just trying to find out actually h 2 2 1. So, so, this is nothing, but U. So, j equals to how much j equals to 2 and c, d c, d is nothing, but 2 1, then comes your J 2 then comes U, j equals to 2 and i is equals to your 2 i is 2 here. So, this is the expression then comes your I can also find out what is h 2 2 2.

Now, this h 2 2 2. So, here once again there will be only one term. So, is nothing, but trace of U then j equals to 2 then comes c, d equals to 2 2 then J 2 then comes your U, j equals to 2 and i is also equal to 2 and transpose of that. So, this is the way actually we can find out the expression for h 2 1 1, h 2 2 2, h 2 2 1 and h 2 2 2. So, using this actually I can find out the expression for your the final expression for this particular the h terms.

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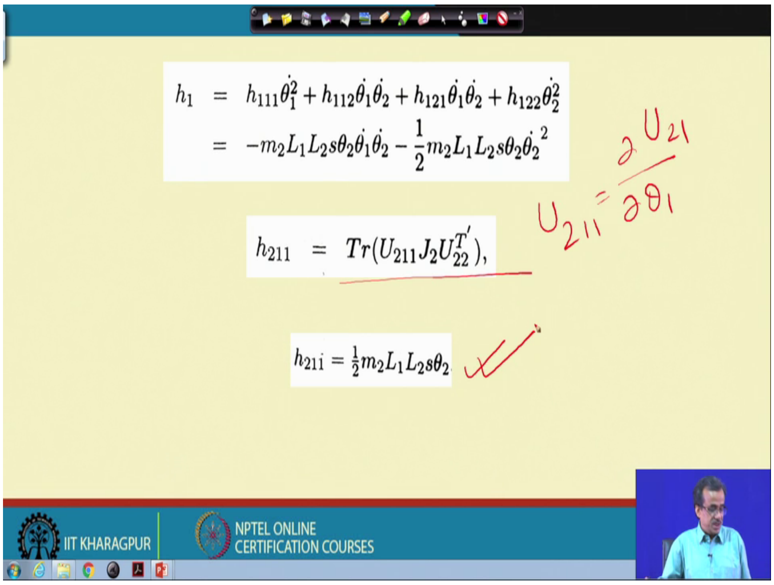

So, this is your h 2 1 1. So, this is the expression as we have already seen. Now, once again we know that this U 2 1 1 that is nothing, but U 2 1 1 is nothing, but actually partial derivative with respect to theta 1 of U 2 1, ok. J 2 you know U 2 2 transpose we know. So, these matrices we can multiply and we can find out the trace and the trace will become equal to this. So, this is nothing, but the expression for your this h expression for h 2 1 1.

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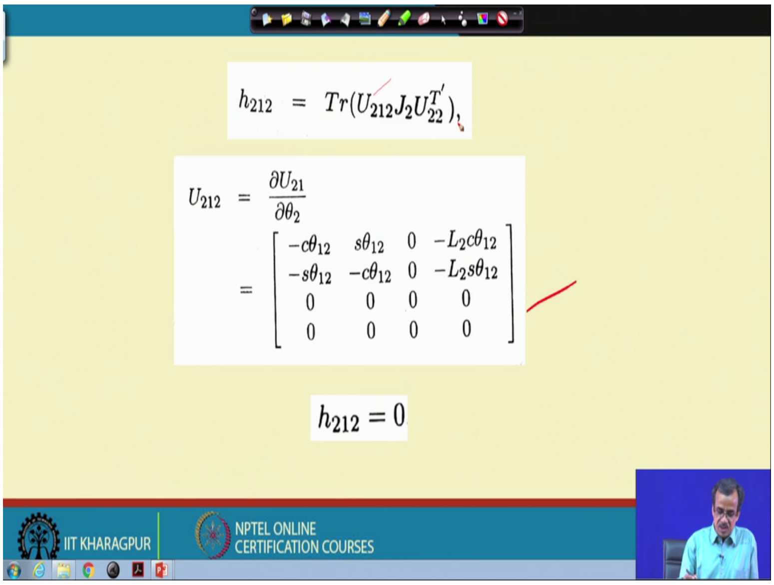

Now, h 2 1 2 so, this is the expression and once again you can find out what is U 2 1 2 that is nothing, but the partial derivative of U 2 1 with respect to theta 2. So, this is the expression. So, this is known all the terms are known and if you just multiply these three matrices and if you find out the trace of that this will become equal to 0.

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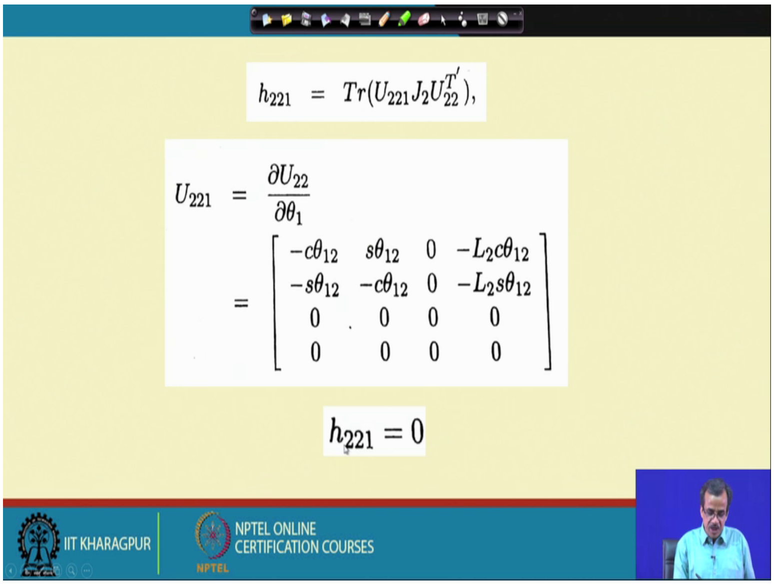

So, what I do is so, we can find out this particular h term following the same method we can find out what is h 2 2 1, the same method and we will be getting h 2 2 1 is equal to 0.

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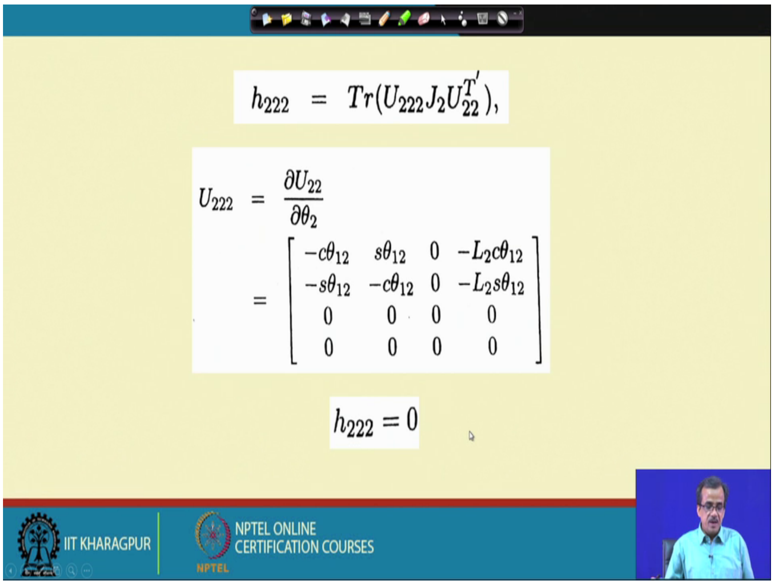

Then comes your h 2 2 2. So, this is nothing, but this particular expression which you have already derived and here so, this U 2 2 2 is nothing, but the partial derivative of U 2 2 with respect to theta 2 and if you just find out so, you will be getting this particular matrix and the other matrices are also known. So, these three 4 cross 4 matrices if you multiply take the trace value you will be getting that is equal to 0.

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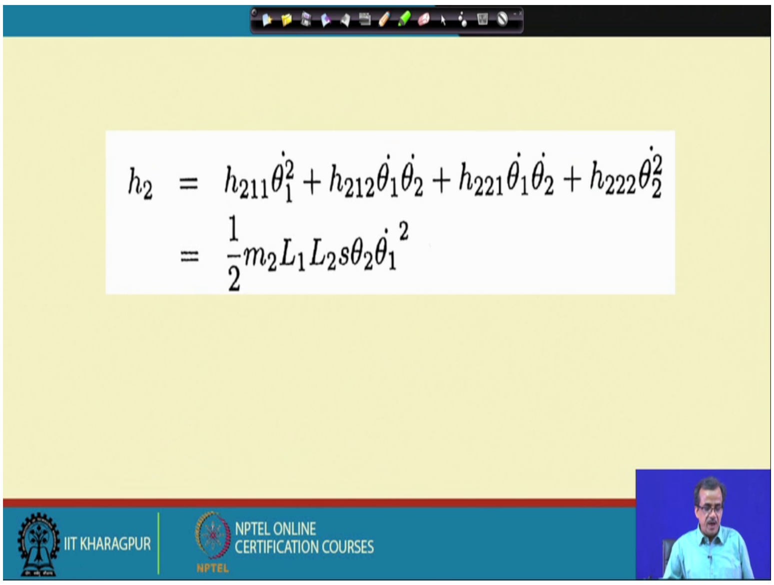

So, following this particular method so, we can find out actually your all the h values so, the only thing, which is left if you see this particular expression for your tau 1 and tau 2 so, this is the expression for this tau 1 and tau 2.

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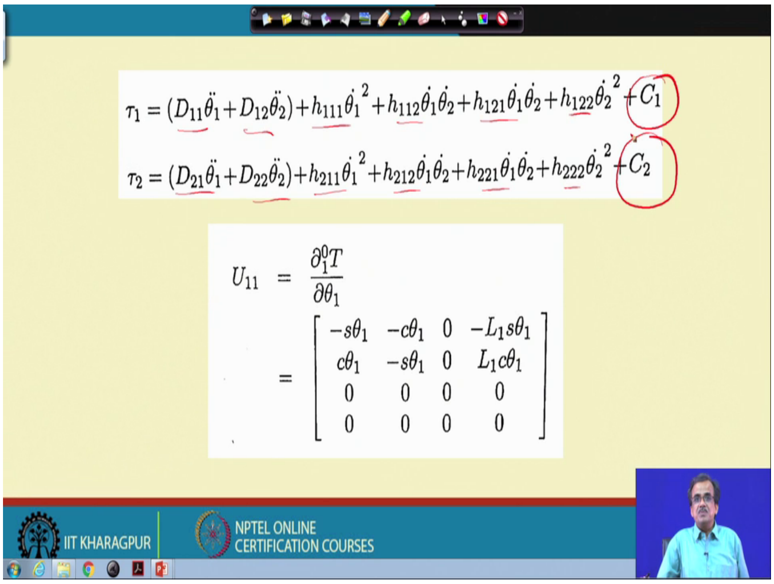

So, all such D terms all the D terms all the h terms all eight h terms we have determined only thing which is left is your C 1 and C 2; that means, your these two gravity terms are yet to be determined. Now, let us see how to determine. So, this particular the gravity term now, to determine the gravity terms actually I am just going back to this particular expression and let us try to find out the expression for C 1 and C 2 from here so, this is C i, so, let me try to find out the expression for C 1.

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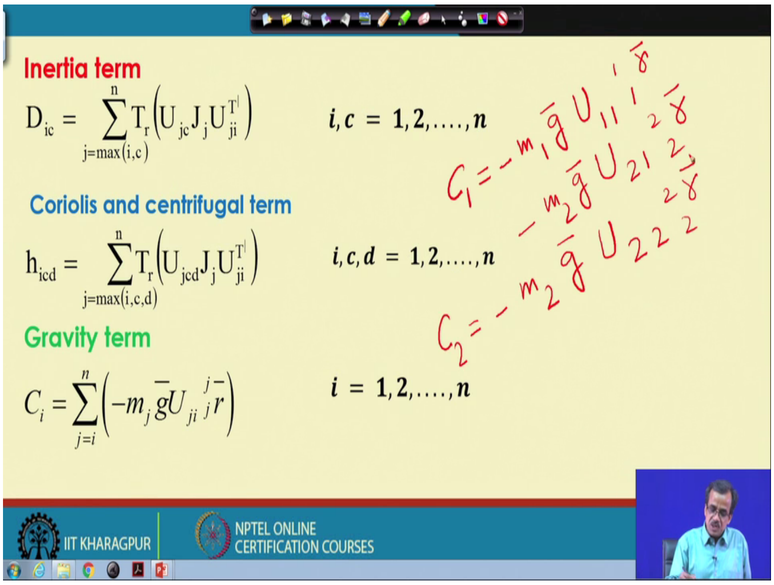

So, i equals to 1 there is j equals to what 1 2 2. So, there will be two such terms. So, I can write down like m minus m j equals to 1 here then comes g bar then comes U, j equals to 1 here and i equals to 1 then comes your r 1 with respect to 1 bar then there is another term that is your j equals to your j equals to 2 now. So, m 2, then comes your g bar then comes U then j is equals to 2. So, it is 2 1. So, U 2 1 then comes your j equals to 2 that is 2 2 bar.

Now, let us see how to determine and I can also find out the expression for C 2 also. So, let me write down the expression for C 2 also. So, i equals to 2. So, j equals to 2 2 2. So, there will be only one term that is minus your m 2 g bar then comes U then j equals to 2 and your i is also equal to 2 then comes your r 2 with respect to 2 bar. So, these are the expression for C 1 and C 2.

Now, let us see how to derive further using this particular expression. So, how to derive further?

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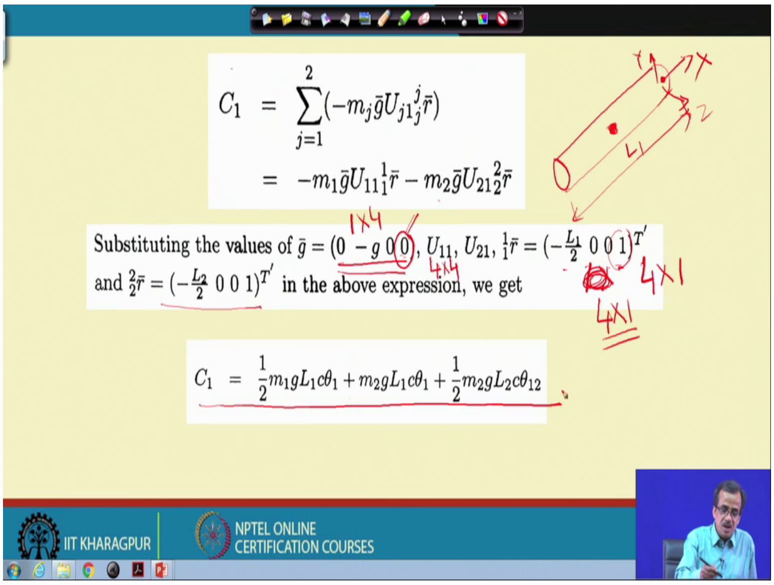

Now, to derive phi the further actually let us see this particular C 1. So, exactly the same expression I got whatever I wrote. So, this is actually or C 1.

Now, here m 1 is the mass of this you are the first link, but g is the acceleration due to gravity. Now, here if you see as we discuss the g has three components like X, Y and Z component. Now, here actually the way the coordinate system has been considered actually if you see the coordinate system let us see the coordinate system once will understand if you see the coordinate system.

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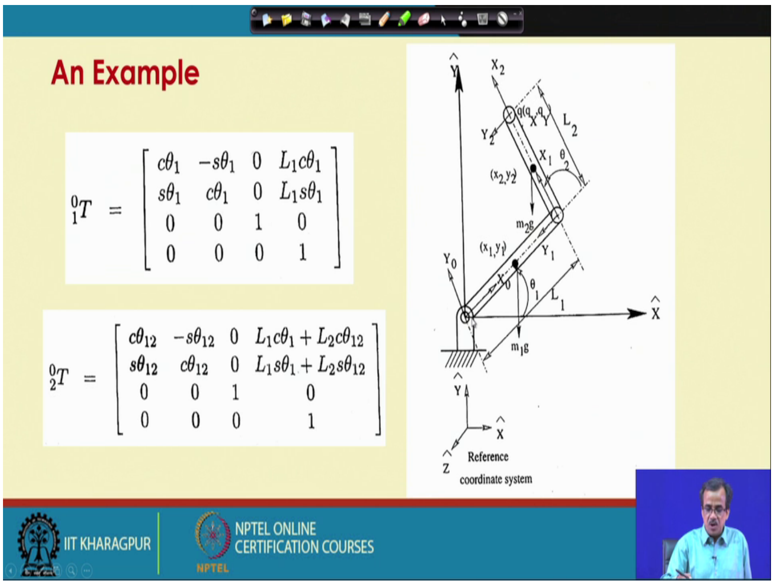

So, this is actually this is the Y direction. So, g is acting opposite to the Y direction vertically downward and that is why actually here will have to put so, this particular expression 0 minus g in place of y I have written minus g and Z is 0, but here one extra 0 I have put the reason I will tell you.

Now, here this U 1 1 we have already determined and this is nothing, but a 4 cross 4 matrix, ok, then comes your this r 1 with respect to 1 how to determine r 1 with respect to 1. So, if I consider that this is my link 1 and for this particular link supposing that the total length is nothing, but L 1, ok. So, its mass center is here, and the coordinate system as I told because we are interested to determine how much will be the reaction torque. So, the coordinate system is attached here, ok. So, with respect to this say this is the X axis with respect to these what will happen is your, this will be minus L by 2. So, this is Y and this is your Z. So, this will be minus L by 2.

So, here this r 1 with respect to one bar is nothing, but the coordinate of the mass center that is your minus L 1 by 2 0 0 that is Y is 0, Z is 0 corresponding to this particular point and this one is actually just below the position vector we put 1 so, that particular the 1. Now, you check the dimension. So, U is having 4 cross 4 and this particular r 1 with respect to 1. So, it is having 1 cross 4 this is here there is a transpose so, this will become your then 4 cross 1 matrix. So, this is the 4 cross 1 matrix and this is your 4 cross 4 matrix, so, if you multiply ultimately I will be getting one 4 cross 1 matrix.

Now, these particular g matrix has to be 1 cross 4, otherwise we cannot multiply and that is why actually we have made it 1 cross 4. So, this is nothing, but the 1 cross 4. So, g has got three components and that is why this particular 0 has been added as an extra, ok, just to make it that particular 1 cross 4. So, this 1 cross 4 can be multiplied by this 4 cross 1, ok. That is why so, this particular extra 0 has been considered here exactly in the same way here also we can determine.

The only thing is the expression of this U 2 1 is different and this r 2 with respect to 2 is nothing, but this because in place of link 1, now will have to consider link 2. For this link 2 your mass center is minus L 2 by 2, 0 0 1 transpose and if we just put all such things here and multiply then I will be getting the expression for C 1 and this is nothing, but this one. So, this is the way we will be getting the expression for this particular the C 1.

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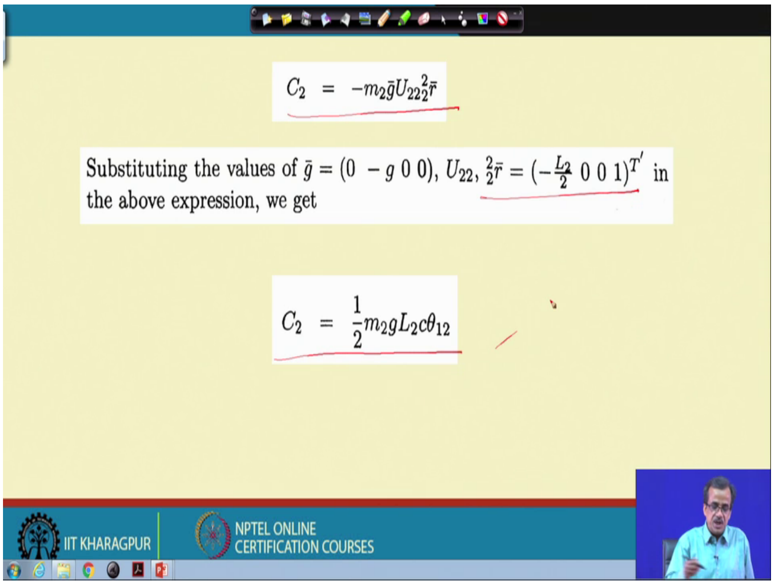

Now, let us see how to determine the expression of C 2 following the same method. C 2 expression I have already got and exactly in the same way g has to be written U 2 2 is known r 2 with respect to 2. So, this is also known and if you multiply then I will be getting this your C 2 that is half m 2 g L 2 cos of theta 1 plus theta 2. So, this particular expression will be getting for your this C 2.

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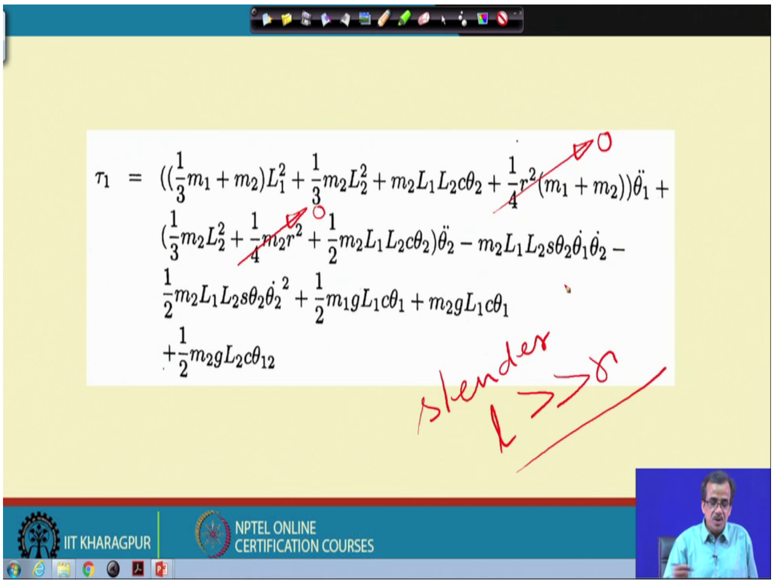

And, once I have got this particular C 2 So, now we are in a position to write down the expression for this particular the joint torque, ok. Now, in this particular expression so, these particular terms if you just add them up whatever we got because all like two values of D, four values of h and one value of C we have got, if we just write them up and if we just arrange then you will be getting this type of expression. So, this big expression multiplied by theta 1 double dot that is you see one third m 1 plus m 2 into L 1 square, plus one third m 2 L 2 square plus m 2 L 1 L 2 cos of theta 2 plus 1 4 r square into m 1 plus m 2 and you can see that all such terms are related to the mass, the length of the link and your mass and length of the link and there could be some radius term here also, but here there is no radius term, but yes, radius term is there. So, these are all actually related to the geometric. Are you getting my point? The geometry and this is nothing, but the inertia term. These are all inertia terms multiplied by this theta one double dot.

Similarly, so, these particular terms are multiplied by theta 2 double dot; theta 2 double dot is nothing, but angular acceleration of the second joint. Now, here once again you can see m 2 L 2 then comes L 1 and we have got r that is your the radius of the second the link and or the or the first link. So, here the radius terms are also there, ok. So, this is once again actually the inertia terms. Now, then comes your these particular terms like theta terms involving theta 1 dot theta 2 dot then theta 2 theta 2 dot square. So, these are nothing, but is your the coriolis and centrifugal term sort of thing and this particular part your cos theta cos of theta 1 and theta 2 these are nothing, but is your the gravity term.

So, we have got inertia term. So, these up to these actually we have got the we have got the inertia term. So, these are nothing, but the inertia terms then we have got your the coriolis and the centrifugal term and here so, these terms are nothing, but is your the your the gravity terms and another observation we should we should have a look see we are trying to find out the expression for the joint torque at joint 1, ok.

Now, here if you see carefully there are a few terms related to your theta 2. For example, I have got cos theta 2, I have got theta 2 double dot cos theta 2 then your theta 2 dot theta 2 dot square then cos of theta 1 and theta 2; that means, although you are trying to find out the joint torque that is torque at joint 1 the second joint angle has got some contributions towards this particular the tau 1.

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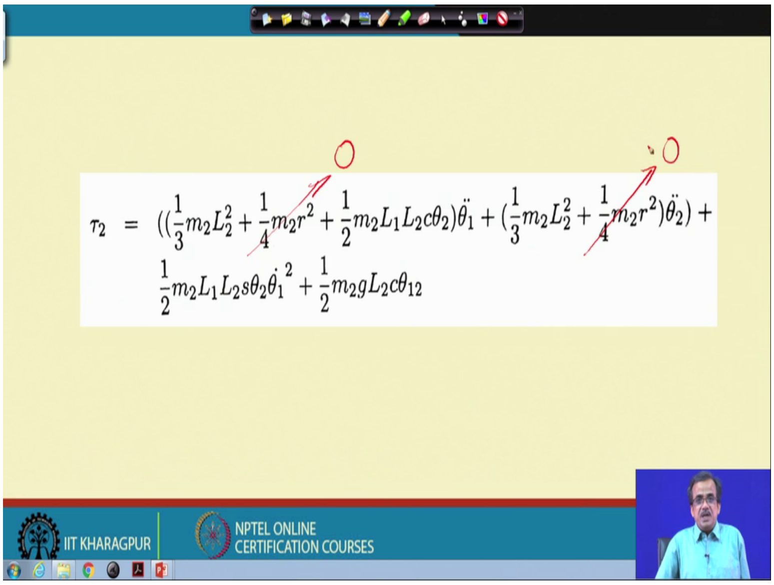

Now, we are just going to see the expression for the second joint term that is your so, these particular terms will be nothing, but the inertia terms like one third m 2 L 2 square plus one fourth m 2 r square plus half m 2 L 1 L 2 cos theta 2 into theta 1 double dot plus one third m 2 L 2 square plus one fourth m 2 r square theta 2 double dot. So, these are all inertia terms, ok. So, this is multiplied by theta 1 double dot this is multiplied by theta 2 double dot and then we have got actually your another term here so, we can see so, this term is actually the centrifugal term involving theta 1 dot square and we have got the gravity terms, ok.

So, this particular expression for theta 2 once again contains three terms and once again if we look into this we are trying to find out the expression for the joint torque that is theta that is tau 2 and here this theta 1 double dot has got significant contribution then here we have got cos of theta 1 plus theta 2. So, theta 1 has got significant contribution on the joint torque 2 and that is why actually this particular the contributions are coupled contribution and due to this particular coupled contribution the better ways should be to determine this particular joint torque like to consider the multi body dynamics and show that the coupling terms we can consider very efficiently and you will be getting very good expression for this particular tau 1 and tau 2.

But, this particular method the method which I have discussed it has got one advantage I should say because if you use this particular method there is a possibility that you will be getting very structured very structured form of this particular expression for the joint torque and which may not be available in other method, but this method gives a very structured form.

Now, another thing I am just going to mentioned like if I consider the slender link; that means, your for slender link. So, slender link your L is very large compared to your r. So, the terms involving r square r small. So, those terms actually we can neglect. For example, from this particular the expression here there is one r square term, ok. So, this particular term you can neglect and it will tend to 0 if we consider the slender link.

Similarly, here there is another r square term. So, this will also tend to 0 if we consider the slender link, and the expression for tau one will become simpler, but here you will not find any such terms involving r or that type of thing. So, this is the way we can make it simple by considering that the links are slender.

Similarly, here if we consider the slender link so, this term will tend to 0. Similarly, here there is another term who will become equal to 0. So, the expression for this particular tau 1 and tau 2 will become simpler. Now, in robotics actually what we do at each of the robotic joint we just put the DC motor and the motor is going to provide this particular torque, and this particular torque has to be this torque is going to generate the joint angle and will have to very accurately generate that particular the joint angle. Now, how to generate that very accurate joint angle that I will be discussing after some time while discussing the control scheme.

Now, here this particular once you have got the variation of this torque as a function of time now, we can think about what should be the power requirement for a particular joint and if you know the power requirement so, we can specify we can prepare the specification of the motor, which we are going to put at that particular the robotic joint. So, that the robotic joint will be able to provide that particular torque and it will be able to generate that particular your the rotation very smoothly.

Thank you.