Robotics
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Lecture - 27
Robot Dynamics (Contd.)

Now let us see how to determine the joint talk for the different robotic joints, I using the principle of the Lagrangian method.

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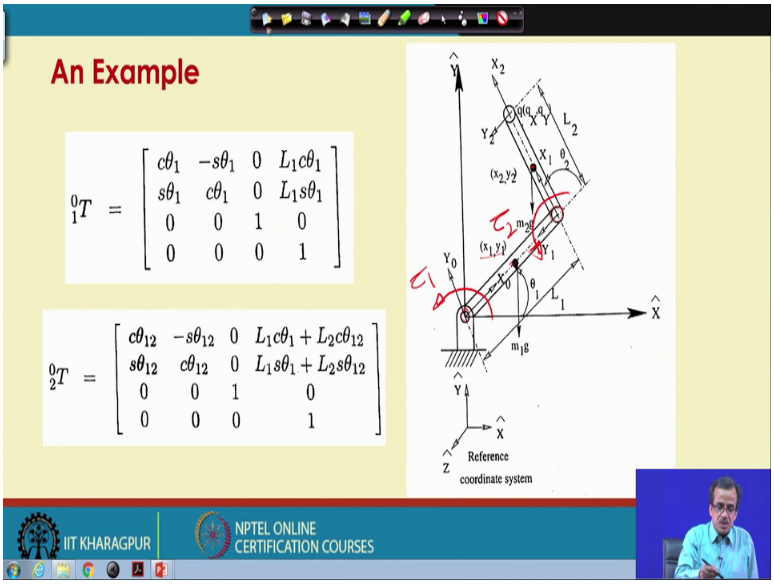

So, we are going to take 1 example the example of 1 2 degrees of freedom serial manipulator. So, this is a 2 degree of freedom serial manipulator this is the first joint, and the link 1 the second joint, and link 2 the length of the first joint is L 1, and the length of the second joint is l 2, the joint angles are theta 1 and theta 2.

Now, here the link 1 is having the mass m 1. So, m 1 g that particular force is acting here at the mass center and x 1, y 1 is the coordinate of the mass center similarly for the second link the mass center is x 2, y 2 and this m 2 g is acting here vertically downward and g is nothing but the acceleration due to gravity.

Now, here our aim is to determine what should be the joint talk here that is tau 1 and what should be the joint talk here that is your tau 2. So, tau and tau 2, we will have to find out we will have to derive the mathematical expression for the joint talks tau 1, tau 2.

Let us see how to proceed with this now to determine. So, this particular joint talk actually what will have to do is first will have to assign the coordinate system at the different joints according to the dh parameter setting rule now according to the dh parameter setting rule. So here, this is nothing but x naught and this is y naught and z naught is perpendicular to the board and away from the board. So, this I have already discussed, so at the joint two so this is my x 1 this is y 1 and z 1 is perpendicular to the board similarly here at the end. So, this x 2 y 2 and z 2 is perpendicular to the board.

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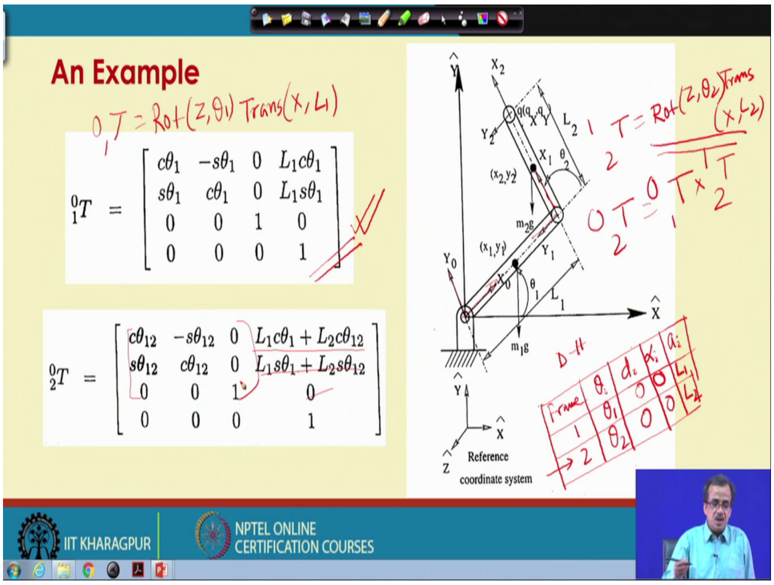

Now, if you just draw this dh parameter that table. So, it looks like this for example, say if I prepare the dh parameter stable. So, this is nothing but the frame then we will have to consider the screw z that is the rotation about z. So, by an angle theta I then translation along z that is di then comes your rotation about that x that is alpha I and translation along x that is ai and here. So, for this particular the first one, so this is the joint angle. So, this particular joint angle is the variable theta 1 then d 0 alpha is 0 and your the length of the link that is nothing but l 1; so l 1 for the second one, so the joint variable is theta 2 0 0 l 2.

Now, this is the dh parameter table now if I know this particular dh parameter table. So, very easily we can determine, what is the transformation metrics that is t 1 with respect to 0? That is nothing but rotation about z. So, this t 1 with respect to 0 is nothing but rotation about z by angle theta 1 then comes your translation along x by l 1. So, we can express 4 4 metrics which we have already discuss and if you multiply. So, this will be the final metrics the 4 cross 4 metrics which will be getting.

Similarly, this t 2 with respect to one; that means, I am here. So, I can find out like t 2 with respect to 1 is nothing but rotation about z by an angle theta 2, then comes translation along x by l 2 and these to 4 cross 4 matrices if you multiply. So, we will be getting another 4 cross 4 metrics and this t 2 with respect to 0 like t 2 with respect to 0 is nothing but t 1 with respect to 0 multiplied by t 2 with respect to 1.

So, this particular metrics and that particular metrics if you multiply then I will be getting the 4 cross 4 metrics that is nothing but t 2 with respect to 0 that is this particular the point with respect to the base coordinate system. And, as we know that these indicates the position terms and this is nothing but the orientation term that 3 cross, 3 cross 3 this we have already discussed. So, let us start with here and then let us see how to determine the joint talk.

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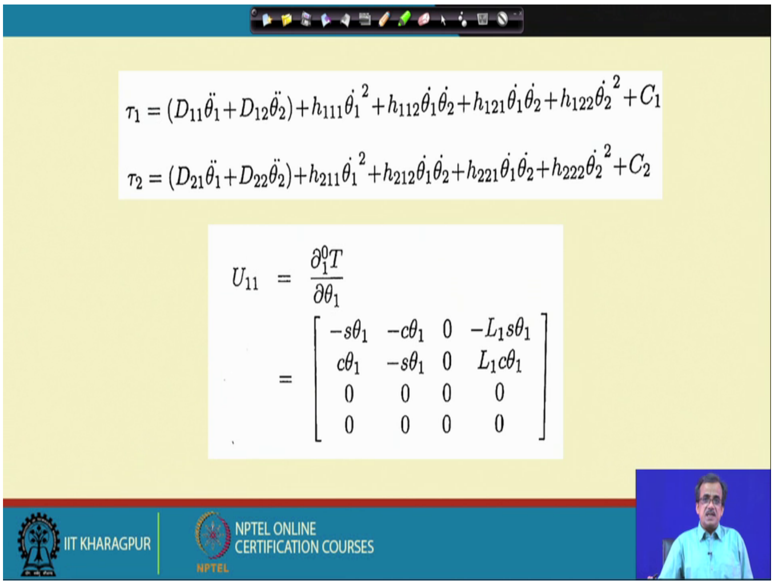

Now, if you see the expression like the expression for the joint talk. So, will be getting such a big expression for this particular tau 1 and such a big expression for this particular the tau 2. Now these expression we can derive from the general expression like if you just go back a few slights.

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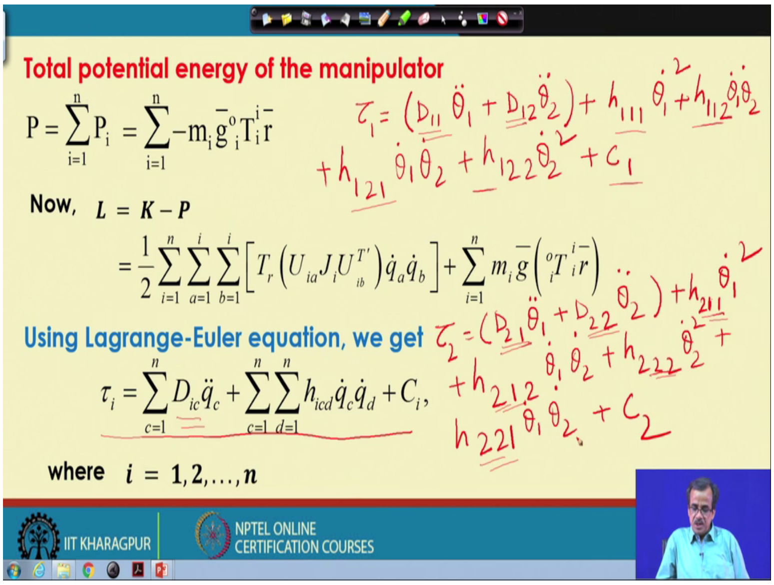

Then form here we can, in fact, derive the expression for tau 1 and tau 2. So, let us try to concentrate here, so let us try to concentrate on this particular the equation and see how to determine that particular tau 1 and tau 2, now here there are 2 joints. So, c varies from 1 to 2.

So, let me try to find out the expression for tau 1, s, tau 1 ; that means, I equal to 1 and c varies from 1 to 2. So, c equals to 1 c equals to 2, so if I take c equals to 1. So, I will be getting d 1 1 and qc double dot c equals to 1 in place of q I will be using theta, so theta 1 double dot. So, theta 1 double dot plus I equals to 1, so you will be getting d and, but c equals to 2, so d 1 2 theta 2 double dot. So, this I will be getting form this particular the expression.

Now, I concentrate here, so I equals to 1, so you put I equals to 1. So, I will be getting h 1 then you consider c equals to 1 to 2 d equals 1 to 2 you first consider c equals to 1. So, I will be getting 1 here and d varies from 1 to 2, so it is 1 and I will be getting theta 1 dot square then comes your h I equals to 1 c equals to 1 d equals to 2. So, I will be getting theta 1 dot theta 2 dot plus then I consider like c equals to 2. So, h 1 2 1 then you will be getting theta 1 dot theta 2 dot then comes your h 1 like c equals to 2 and will be getting d is also equals to 2. So, I will getting theta 2 dot square plus I mean I will be getting c 1.

So, this is the expression for this particular the joint talk tau 1 similarly we can write down the expression for tau 2 also. So, tau 2; that means, I equals to 2 and c is varying from 1 to 2. So, I will be getting d 2 1 like theta 1 double dot then comes your d 2 2 then comes your theta 2 double dot plus I will be getting here h then 2 c equal to 1 d equals to 1. So, I will be getting theta 1 dot square then comes your h 2 like c equals to 1, but d equals to 2. So, I will be getting theta 1 dot theta 2 dot plus h 2 2 2. So, I will getting h 2 1 one h 2 1 2 h 2 2 2, so I will be getting here theta 2 dot square and another term h 2 then 2 and I will have to consider this 1 also.

So, I will be getting theta 1 dot theta 2 dot and your c 2. So, this is the way h 2 1 1 h 2 1 2 h 2 2 1 h 2 2 2 theta 2 dot square and here theta 1 dot and so this is. So, this is the way actually will be getting the expression for tau 1 and tau 2. So, there will 2 d terms and there will be 4 such h terms and 1 c 1 similarly here there are 2 d terms and there will be 4 such h terms there will be 4 such h terms and will be getting your 1 c terms. So, this is the way actually we can find out the expression for this particular the tau 1 and tau 2 the same expression I have written it here.

So, this is exactly the same expression which I have written it here like tau 1 and tau 2 now I will have to concentrate on. So, this particular term that is d 1, but before I go for this particular d 1 actually what I will have to do is, so I will have to find out another term that is called u 1 1 and u 1 1 is nothing but the partial derivative of the transformation metrics t 1 with respect to 0 and this partial derivative with respect to theta 1.

So, if we remember the expression for t 1 0 for example, if you see the expression for t 1 0. So, this is the t 1 0, so the partial derivative of t 1 0 that is your if you want to find out the partial derivative of t 1 with respect to 0 that is t 1 with respect to 0.

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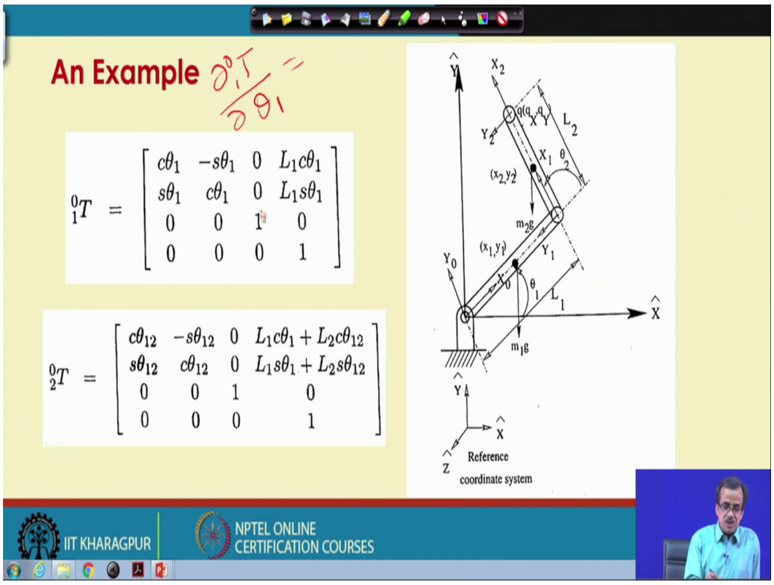

And if this partial derivative is with respect to theta 1, so here in place of cost theta 1 I will be getting minus sine theta 1. So, here I will be getting minus cos theta 1 then this is 0 and here I will be getting minus l 1 sine theta 1.

Similarly, this will be cos theta 1 this will be minus sine theta 1 0 and this will be l 1 cos of theta 1 and this will be 0 0 this 1 will also become 0 because this is the partial derivative 0 0 0 0 0. So, this type of expression you will be getting for your u 1 1. So, this u 1 1 as I told is minus sine theta 1 minus cos theta 1 0 minus l 1 sine theta 1 cos theta 1 minus sine theta l 1 0 l 1 cos theta 1 and here will be getting all 0 terms.

So, this is the way actually we can find out. So this u 1 1 this is actually the rate of change of these particular transformation metrics with respect to your only theta 1.

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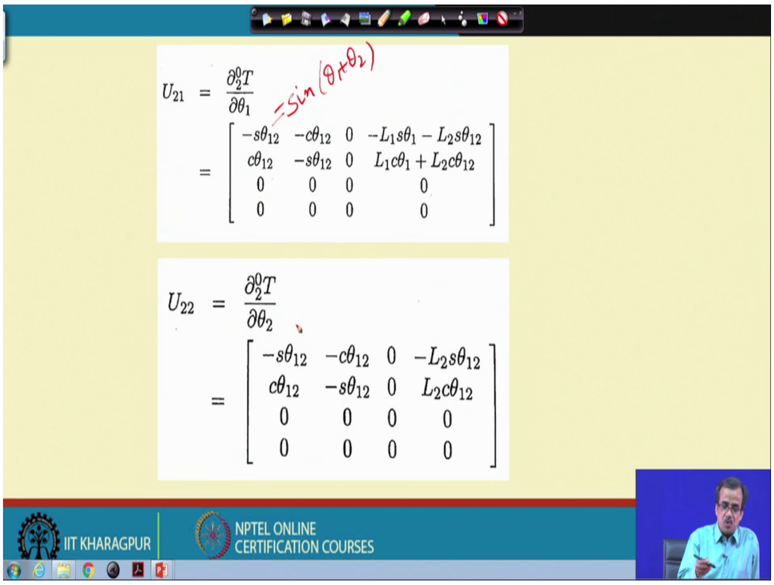

Similarly, we can also find out u 2 1 and u 2 1 is nothing but the partial derivative of t 2 with respect to 0 with respect to your theta 1. So, t 2 with respect to 0 like if you see this particular expression like t 2 with respect to 0 like your t 2 with respect to 0. So, with respect to theta 1, so we will have to find out its partial derivative.

For example here I will be getting in place of cos n theta 1 plus theta 2, I will be getting sign of theta 1 plus theta 2 here I will getting minus cos of theta 1 plus theta 2 and so on. So, this is the way actually we can find out the partial derivative and this is nothing but minus sign of theta 1 plus theta 2. So, this is nothing but sine of minus sine of theta 1 plus theta 2 then minus cos of theta 1 plus theta 2 and so 1 and these particular terms all terms will become 0 and this will also become is equal to 0.

Now, by following the same method I can also find out the u 2 2 that is nothing but the rate of change of t 2 with respect is 0 with respect to theta 2. Now with respect to theta 2 only if you determine then you will be getting something like with respect to theta 2 if you determine. So, then here will be getting like d d theta 2 of cos of theta 1 plus theta 2, so will be getting minus sine of theta 1 plus theta 2.

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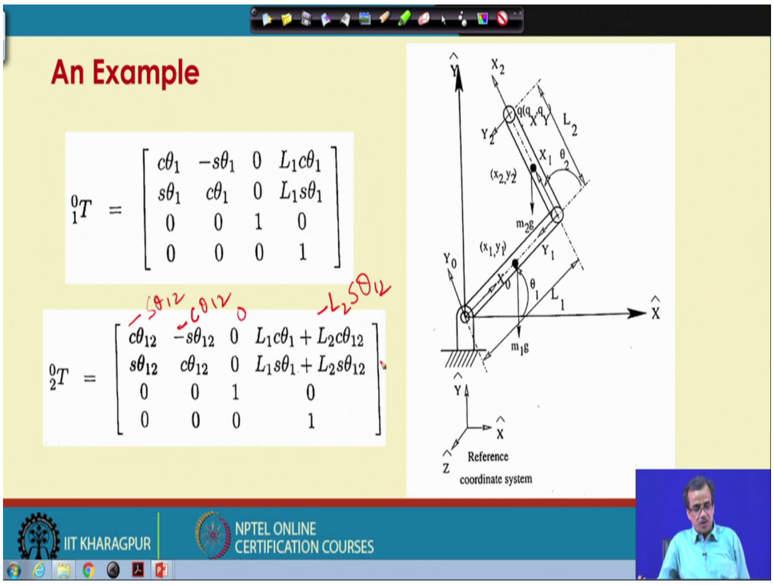

And that means you will getting here minus s theta 1 2 here will be getting minus cos theta 1 2 then this will become 0. And here, there will be no contribution because here there is no theta 2 the only contribution will come here and this will become minus l 2 sine of theta 1 plus theta 2.

Similarly, the other terms also you can find out, and by following the same method actually we can find out what is your u 2 2. So, this is nothing but is your u 2 2 and once I have got this particular u 2. So, let us try to concentrate on the inertia tensor. So, these are nothing but the inertia tensor inertia tensor for the link 1 and link 2.

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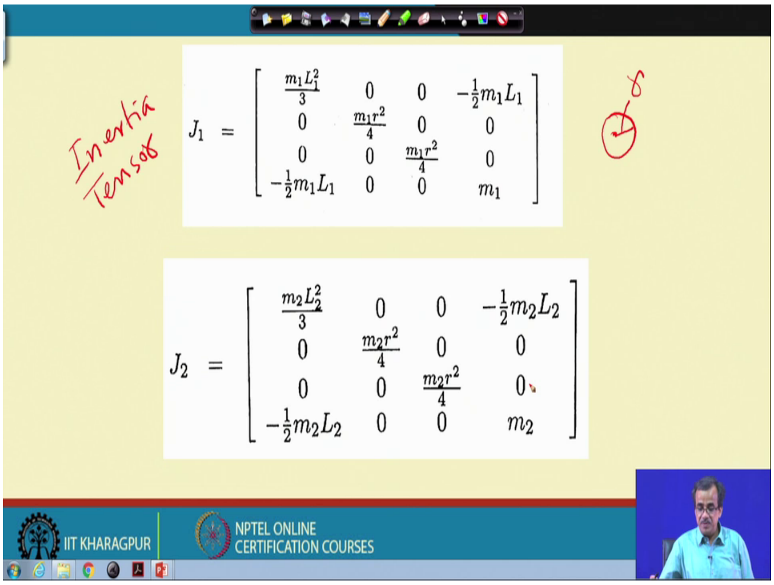

Now, if we remember you have already derive this particular expression. So, this particular j 1 is the inertia tensor for the first link. And we have consider that this particular link is having actually your circular cross section with radius r and for that will be getting the inertia tensor like this like m 1 1 1 square by 3 0 0 minus half m 1 1 1 0 m 1 r square by 4 0 0 0 0 m 1 r square by 4 0 minus half m 1 1 1 0 0 m 1.

So this, I have already derive this particular inertia tensor. Now similarly for the link 2 I can find out what is j 2 and exactly in the same way I can find out this 4 cross 4 metrics. And this I have already discussed in much more details in 1 of the previous classes how to determine that this particular the inertia tensor.

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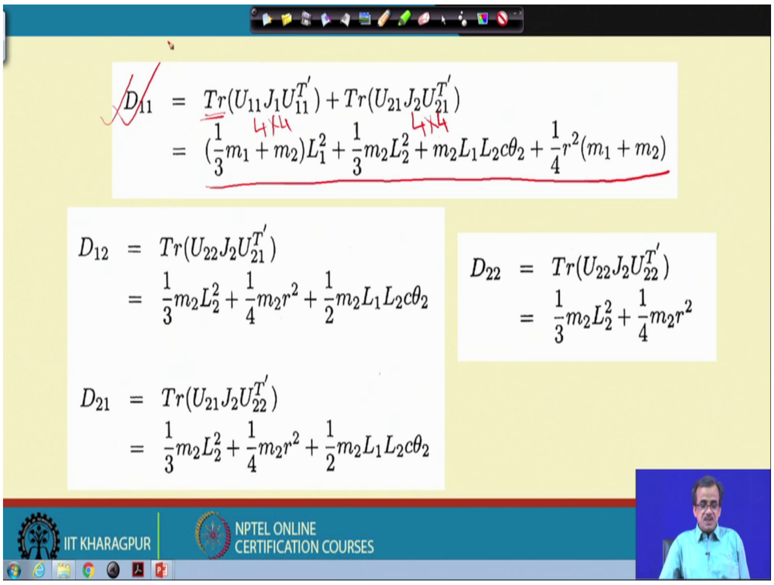

Now, I am just going to derive that particular the d 1 1 term. Now if you see, this particular d 1 1. So, this is nothing but actually this particular d 1 1 term I am just going to find out. So, I am just going to derive what should be the expression for this particular d 1 1.

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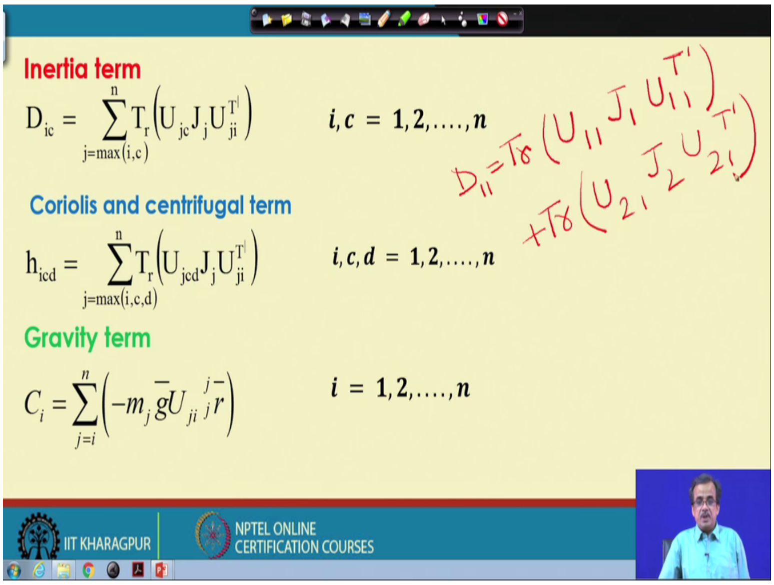

Now, to derive this particular expression let me try to go back to this particular the expression for Dic. Now if you concentrate on this particular Dic there is a inertia term and our aim is to determine what should be your the expression for d 1 1 ; that means, I equals to 1 c equals to 1 and j is the maximum between 1 coma 1 that is 1. So, j will vary from 1 to 2 and now with the help of this I can write down. So, when j equals to 1, so it is nothing but trace of tr is the trace of u the j equals to 1 here and c c is 1 here then comes your j 1 then comes u j equals to 1 I equals to 1. And this is the symbol for the transpose plus like t r that is the trace.

Now, I am just going to use j equals to 2, so this will become u 2 1 j 2 u 2 1 transpose of that. So, this is the expression for this particular d 1 1 now the same expression I am using here. So, the same expression I am using here for this particular d 1 1, that is trace of d u 1 1 j 1 u 1 1 transpose plus trace of u 2 1 j 2 u 2 1 transpose.

Now you see all the terms you know to us for example, say this u 1 1, we have already derived j 1 you have derived. So, you know u 1 1 transpose of that then comes u 2 1 we have derived j 2 you have derived u 2 1 transpose is also known to us. So, each of this matrices are 4 cross 4 metrics ok. So, if you multiply 2 times.

So, then you will getting finally, 4 cross 4 metrics. So, you will be getting on 4 cross 4 metrics here and here also you will be getting 1 4 cross 4 metrics and here actually this is a trace. So, for this 4 cross 4 metrics, so what will have to do is you will have to consider only the diagonal elements. So, by trace we mean the sum of the diagonal elements. So, we consider the sum of the diagonal elements here and some of the diagonal elements here and if you just add them of. So, will be getting this particular the final expression for this particular d 1 1.

So, d 1 1 the expression for d 1 1 is known, now the same method we will have to follow for the other for example, say you find out d 1 2 once gain we go back to the expression, so this particular the expression.

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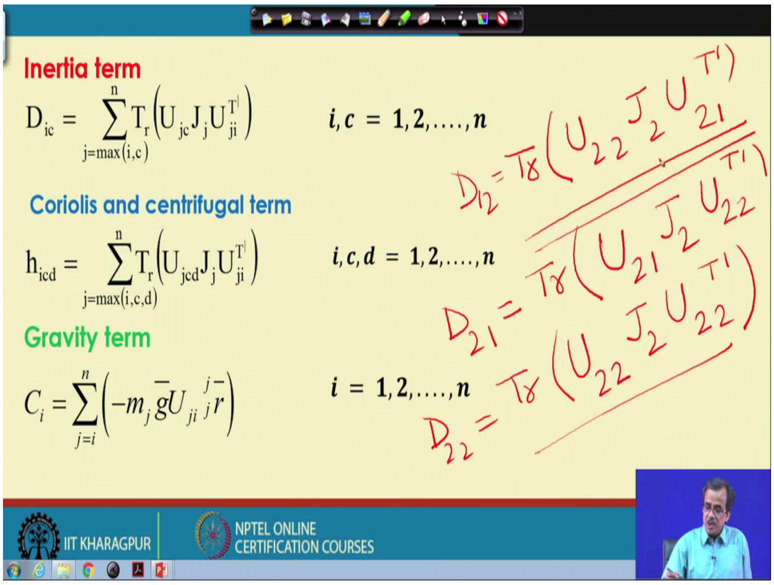

So, here d 1 2; that means, your d 1 to I equals to 1 and c equals to 2 and maximum between 1 and 2 is 2. So, 2 2 2, so there will be only 1 term here, so this is nothing but the trace of. So, j equals to 2, so u 2 and what about c c is also is equal to 2. So, your u 2 2 then comes j 2 then comes u j j equals to 2 and what about i i is nothing but 1 ok, so i is nothing but 1.

So, this particular 1 so u 2 1 transpose of that, so this is the expression for your d 1 2. Now similarly we can also find out the expression for d 2 1 now this d 2 1; that means, i equals to 2 and c equals to 1 and the maximum between 2 and 1 is 2. So, there will be only 1 term because j varies from 2 2 2. So, you will be getting the trace of like u j is equals to 2 here and what about c c is nothing but your 1 c is 1 then comes your j j j j; that means, j 2 then u j equals to 2 and i i is nothing but 2 and transpose of that.

So, this is nothing but your d 2 1 and I can also find out the expression for d 2 2 now this particular d 2 2, i equals to 2, c equals to 2 j equals to 2 to 2. So, trace of your u j equals to 2 here and c is also 2 then comes your j 2 then comes j equals to 2 and i is also 2 and transpose of that. So, I can find out the expression for this d 1 2 d 2 1 and d 2 2, and all the terms I know and very easily actually we can find out the expression for this particular the d term the way I discuss.

So, this is the expression for your d 1 2 and once again you write down the 4 cross 4 metrics here 4 cross 4 metrics for j 2 4 cross 4 for u 2 1 transpose, you multiply them consider the sum of the diagonal elements. So, will be getting this is the expression for your d 1 2.

Similarly, your d 2 1 the expression we have already seen and if you just follow the same method like metrics multiplication and then if you consider your the sum of the diagonal element. So, we will be getting the same expression has d 1 2. So, d 2 1 becomes equals to d 1 2, but d 2 2, so once again you follow the same method write down the expression metrics multiply consider the trace, so will be getting this is the expression.

So, till now all the d values are calculated. now if I have got all the d values; that means, you’re in this particular expression like your all d values are known now will have concentrate on the h value that is your h 1 1 1, h 1 1 2 1 2 1 and 1 2 2. Similarly we have got 4 more, so there are 8 such h terms and that I will have to derive ok. Now, how to derive, so what a do is once gain we go for the expression for this h icd, so here if you write down like your h 1 1 1.

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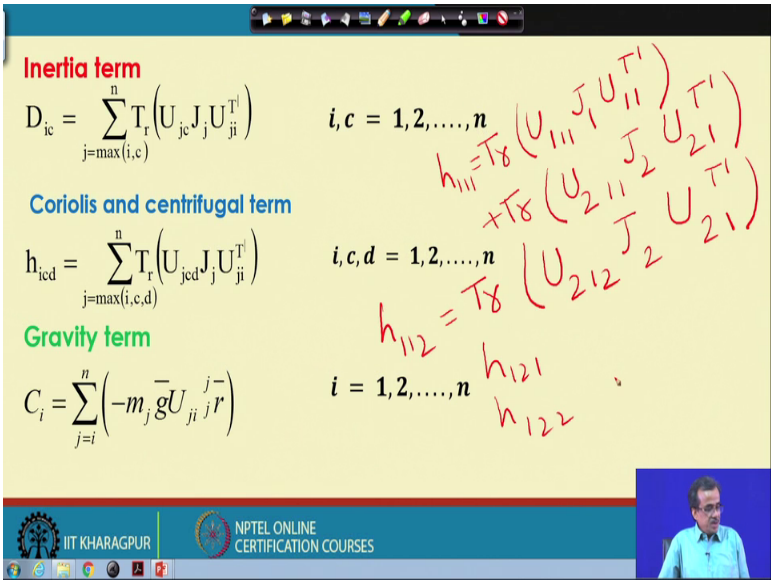

So, h 1 1 1 for example, here icd all are 1, so j varies from 1 to 2, so there will be 2 such terms. So, this is nothing but trace of u j equals to 1 here and cd is 1 1, then comes your j 1 then comes u j equals to 1 then comes your i equals to 1 transpose of that plus trace of j equals to 2 here. So, u 2 1 1 then comes j 2 then comes u j equals to 2 here 1 and then this is you’re the transpose of that.

So, this is nothing but is your h 1 1 1 similarly I can also write down the expression for h 1 1 2, now h 1 1 2 is what I equals to 1 c equals to 1 d equals to 1. So, j is the maximum of icd; that means, 2 2, so there will be only 1 term here. So, this is nothing but the trace of your u. So, j equals to 2 here and cd that is 1 2 then comes your j j is 2 then comes u then j j is nothing but is your 2 here and i i is nothing but 1 here and the trace of that.

So, similarly I can find out the other your h terms like your say h 1 2 1 h 1 2 2 and other terms we can find out. So, we can we can determine your other terms of this particular h terms, now if you see this particular h 1 1 1.

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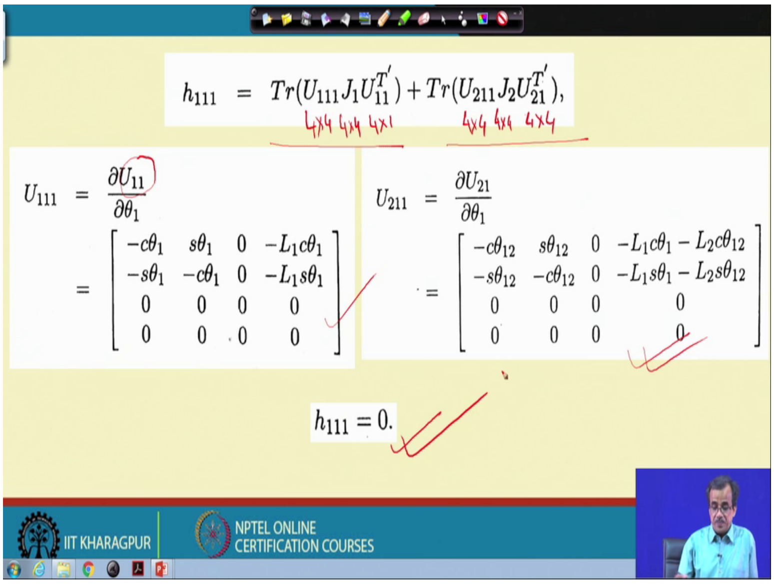

Let us see how to derive now this h 1 1 is nothing but as we have seen trace of this particular thing plus trace of this particular thing now this u 1 1 is nothing but the partial derivative of u 1 1 with respect your theta 1.

We know the expression of these particular the 4 cross 4 metrics of u 1 1. So, again will have to find out the partial derivative with respect to 1 ok, then you will be getting this u 1 1 1 once again the 4 cross 4 metrics. Similarly you know this u 2 1 we have already seen that 4 cross 4 metrics and that metrics we will have to find out the partial derivative with respect to 1 and that will become u 2 1 1. And will be getting this particular the partial derivative this type of 4 cross 4 metrics.

Now, u 1 1 is a 4 cross 4 metrics j 1 1 4 cross 4 metrics and u 1 1 4 cross 4 metrics. Similarly u 2 1 1 4 cross 4 metrics j 2 4 cross 4 metrics and this is also 4 cross 4 metrics. So, ultimately I will be getting 1 4 cross 4 metrics another 4 cross 4 metrics, we consider the trace value and if we just add them up fortunately we will be getting this h 1 1 1 is equal to 0.

So, this is the way actually we can find out the other h values by following the same method like h 1 1 2 the expression I have already seen.

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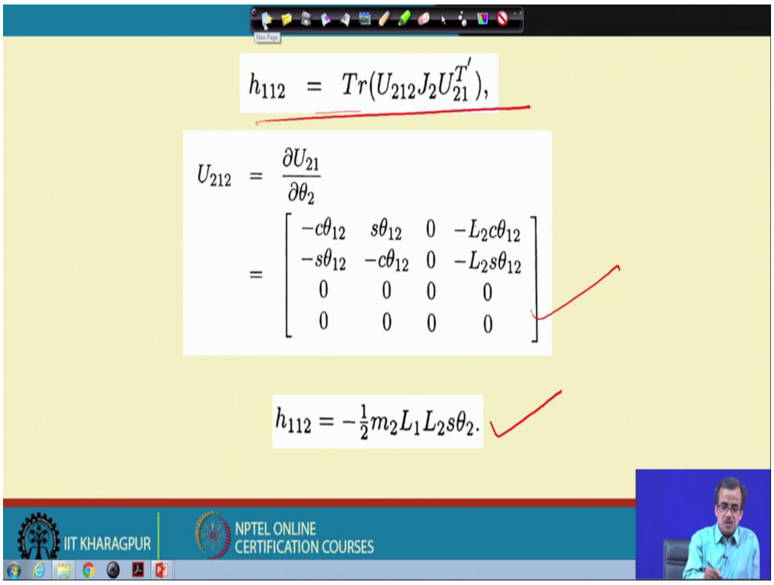

So, this is the expression for h 1 1 2 and this u 2 1 2 is nothing but the partial derivative of u 2 1 with respect to theta 2. So, this is the expression and if you just substitute the metrics says multiply find out the sum of the principle diagonal elements. So, you will be getting h 1 1 2 is nothing but this, so this is actually the expression for h 1 1 2.

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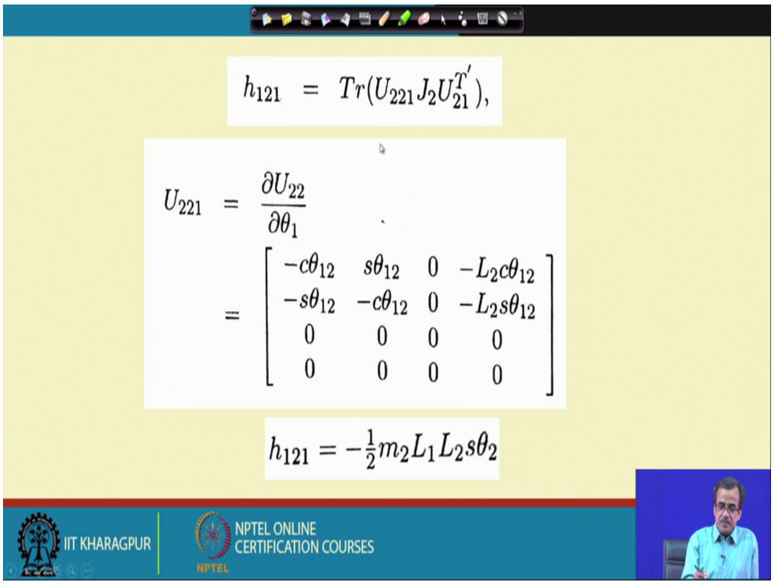

Then h 1 2 1 so this is the expression and if you just follow the same method and this u 2 2 1 is nothing but partial derivative of u 2 2 with respect your theta 1. So, this particular metrics we will be getting and following the same principle I can find out h 1 2 1 is nothing but this.

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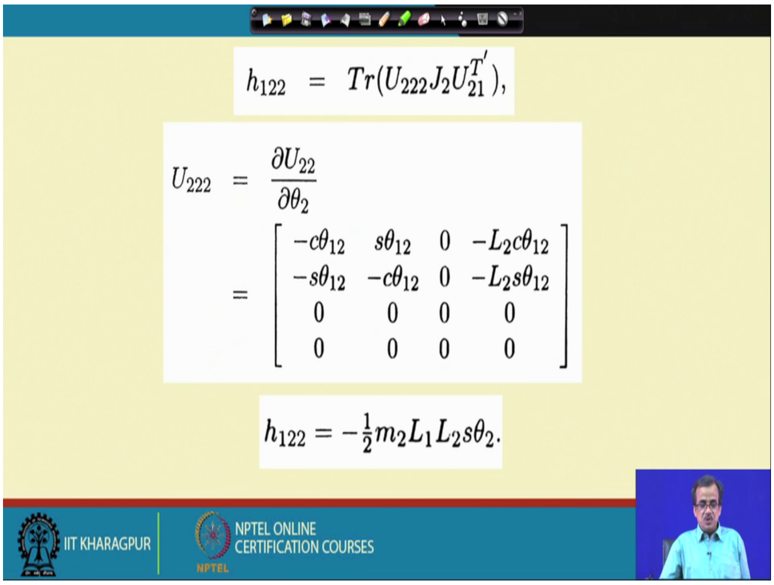

Now, here actually once again let us try to recapitulate our purpose is to determine the expression for the joint talks like tau 1 and tau 2. So, we have following that then comes your h 1 2 2. So, this is the expression follow the same method and u 2 2 2 is nothing but partial derivative of u 2 2 with respect to theta 2. So, this is the metrics you will be getting and this is your h 1 2 2. So, this is the expression of this h 1 1 2. So, till now 4 h values we have calculated and we will have to determine the remaining 4 h values.

Thank you.