Robotics
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 26
Robot Dynamics (Contd.)

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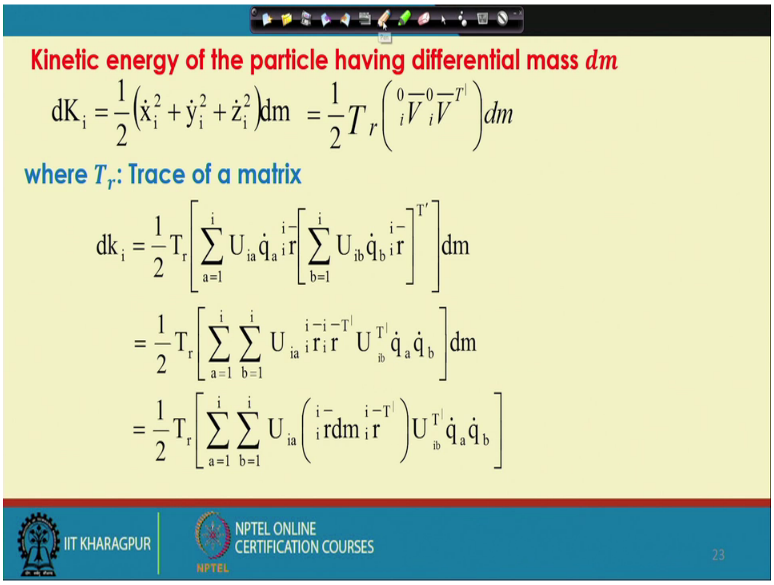

So, this is the expression of this particular your the kinetic energy of the differential mass that is dk i.

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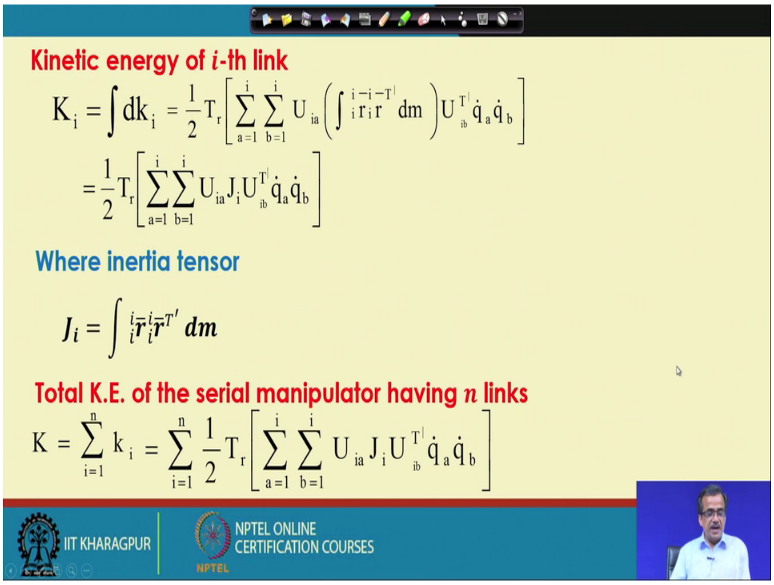

Now once we have got it, so I can find out the kinetic energy of the ith link, that is k i and that is nothing but integration dk i and that is equals to half trace, then I am just going to write down the expression only thing I have done is I have put one integration sign here.

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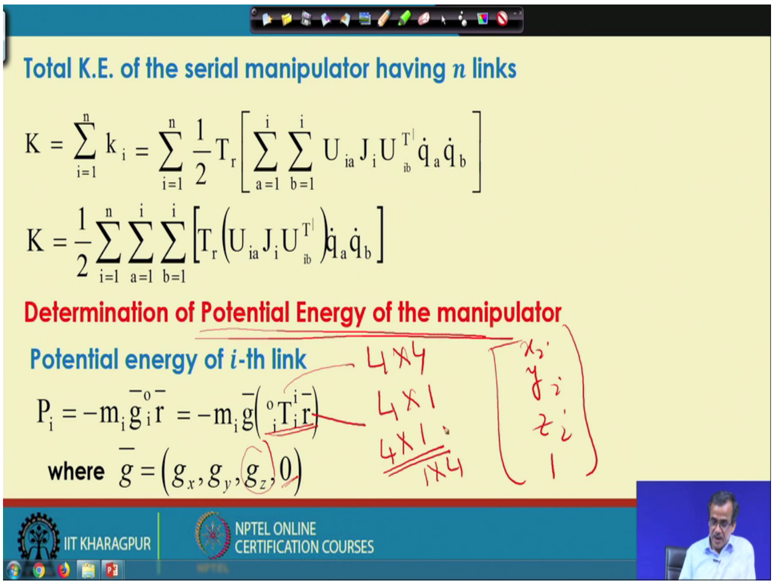

So, this particular integration sign, I have put one integration sign here other things are the same. So, this can be written as half trace summation a equals to 1 to I, b equals to 1 to i U ia. Now, this particular expression is nothing but Ji and this Ji is nothing but the moment of inertia, which I have already derived. Next is U ib transpose q a dot q b dot ok, where the moment of inertia J i is nothing but this particular the expression.

Now, the total kinetic energy for the whole manipulator is nothing but k is nothing but summation i equals to 1 to n k i that is summation i equals to 1 to n half trace, summation a equals to 1 to i b equals to 1 to i U ia J i U ib transpose q a dot q b dot something like this. So, this is the expression for your the kinetic energy for the whole manipulator.

Now, the kinetic energy for this particular the manipulator having n links is nothing but this particular the expression. So, this can be rearranged and it can be rewritten in a slightly different way. So, here so so we can write down this K is nothing but K is nothing but half summation i equals to 1 to n summation a equals to 1 to i summation b equals to 1 to i trace of U ia J i U ib transpose q a dot q b dot. So, this is the way actually we can find out the expression for the total kinetic energy for the whole robot.

Now, we are going to find out the potential energy of the manipulator. How to determine the potential energy? Now, the potential energy P i is nothing but minus m i g bar r i with respect to 0. Now, this particular g is nothing but the acceleration due to gravity and truly speaking this is a vector having component g x, g y, g z. And here I have put this 0 very purposefully that I am going to tell. Now, this g x, g y, g z are nothing but the three components of the acceleration due to gravity. Now, at a particular place so this particular g x and g y are negligible and that is why we generally consider only your g z which is acting vertically downward.

Now, if it is a acting vertically downward so we will have to put so here accordingly we will have to sign correction we will have to do. And this T i with respect to 0 is the transformation matrix and r i with respect to i. So, this T i with respect to 0 multiplied by this r i with respect to i, so this is nothing but the height of this particular the link that is r i with respect to 0. So, I can find out the expression of this particular the potential energy.

Now, if you see the dimension of this T, T i with respect to 0, so this is nothing but a 4 cross 4 matrix. If I see r i with respect to i, so this is nothing but a 4 cross 1 matrix, because x i y i z i x i y i z i and then I put a 1 here, so this is nothing but a 4 cross 1. So, if I multiply 4 cross 4 and 4 cross 1. So, I will be getting 4 cross 1 matrix. And this 4 cross 1 matrix, I will have to multiply by 1 cross 4 matrix then only I will be able to do this particular multiplication and that is why in place of g x, g y and g z, I put one 0 here just to make 1 cross 4 matrix, so that I can multiply with this 4 cross 1 matrix.

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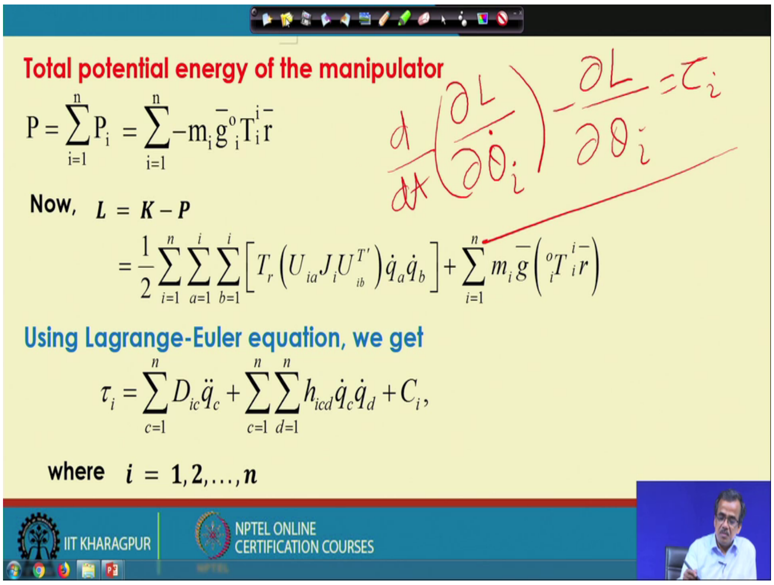

Now, if you multiply then we will be getting the expression for this particular the potential energy. Now, P is nothing but summation i equals to 1 to n P i. So, summation i equals to 1 to n minus m i g bar T i with respect to 0, r i with respect to i. So, this particular Lagrangian is nothing but the kinetic energy minus potential energy. So, we know the expression for kinetic energy and this is the expression for the potential energy.

So, I can write down kinetic energy minus potential energy. So, this is the expression for the whole Lagrangian for the robotic system. And now actually what we do is we try to go back to that particular expression that is your that Lagrangian equation d dt of partial derivative of L with respect to your theta i dot minus the partial derivative of L del theta i is nothing but is your tau i. So, this particular expression I am going to use. Now, here in place of theta i, we can put this particular your q i. So, this is actually the same expression which I which I showed.

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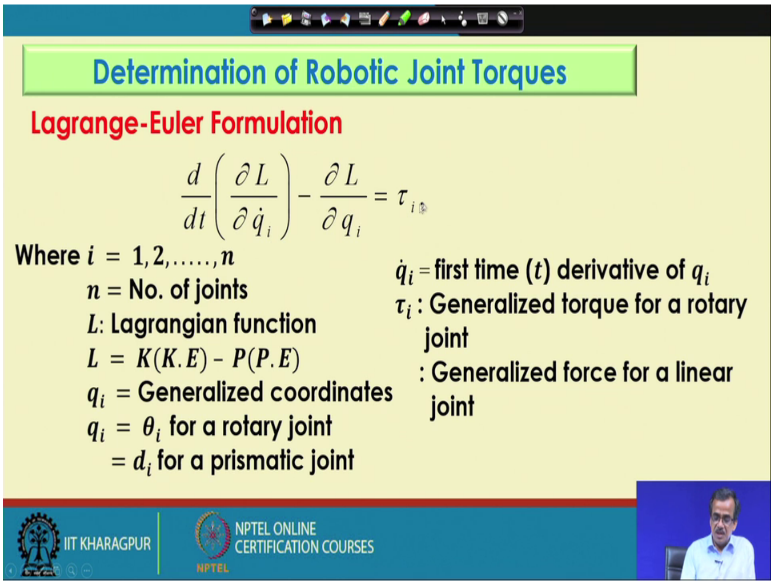

So, let me try to go back to that particular expression once again. So, this is actually the expression. So, I am just going to use this particular the expression.

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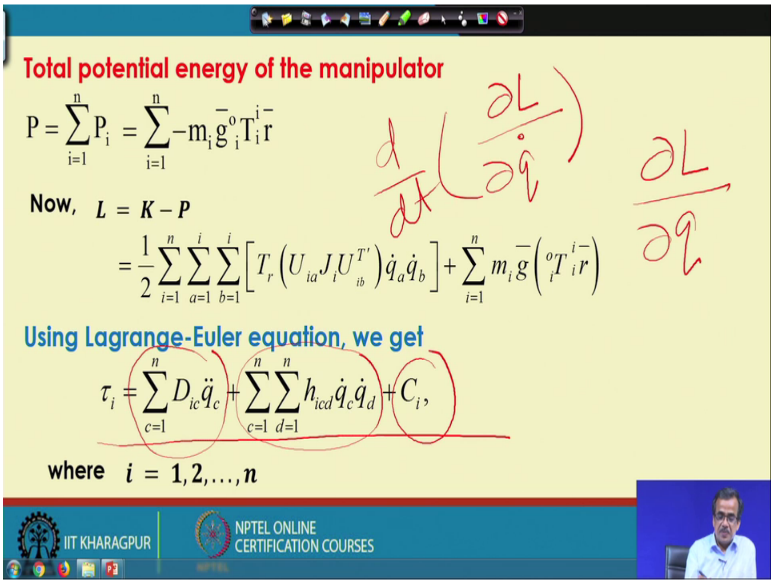

And now if I just substitute your this particular the Lagrangian and if I just find out, if I just find out the partial derivative of L with respect to this q dot, and if you find out d dt of that and if you find out separately the partial derivative of Lagrangian with respect to q then we will be able to find out the final expression of this particular joint torque which is nothing but this. Now, here in this particular expression for the joint torque, we have got three distinct component. So, this is nothing but the inertia term, which depends on the mass distribution of the link. This is your coriolis and centrifugal term; and this is nothing but the gravity term.

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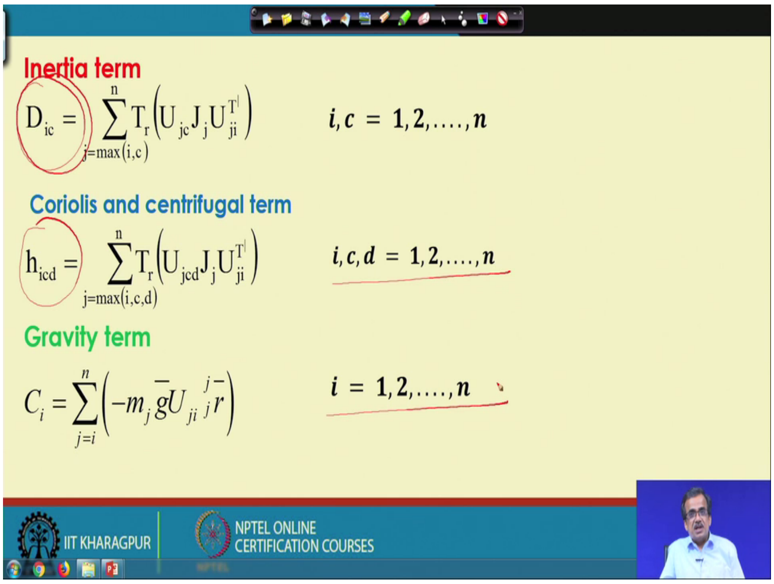

Now here so this particular terms like your inertia term D ic is nothing but summation j equals to the maximum between i and c to n trace of U jc J j U ji transpose where i c varies from 1, 2 up to n. So, this is nothing but the inertia tensor that we can find out this D ic. Then the Coriolis and centrifugal term is h icd that is summation j equals to the maximum among icd to n trace of U jcd J j U ji transpose where icd varies from 1, 2 up to n. And the gravity terms C i that is equals to summation j equals to i to n then comes your minus m j g bar U ji r j with respect to j where i varies from 1 to n.

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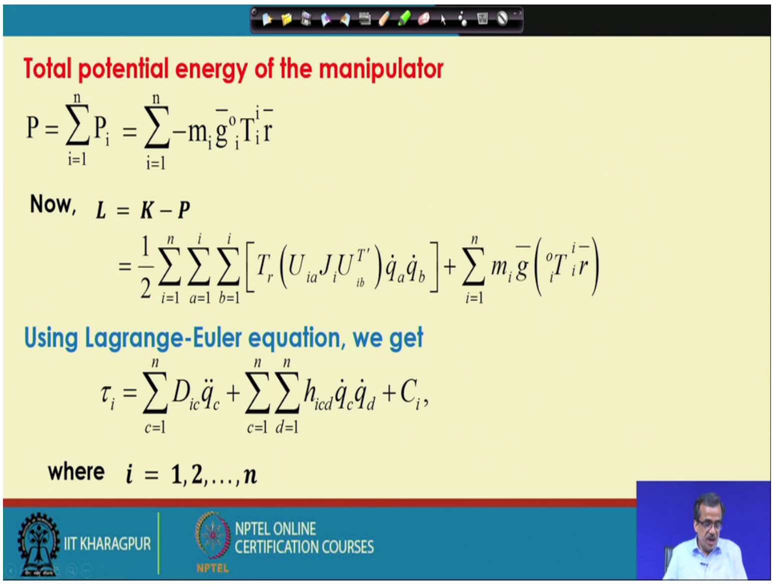

Now, using this particular expression, so what you can do is so we can find out the expression for the joint torque or the force ok. Now, actually what I am going to do using this particular expression, we will try to derive the expression for the joint torque or the force which is required at the different joint. And we will take some example numerical example and with the help of this particular numerical example I am just going to find out the big expression for this particular the joint torque.

Thank you.