Robotics
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Lecture - 24
Robot Dynamics

Now I am just going to start with another new topic that is topic 4 and it is on Robot Dynamics. Now the purpose of dynamics is to determine the amount of force if it is a linear joint or the amount of torque if it is a rotary joint, which is the reason behind the movement of the robotic links and the robotic joint.

Now, let us see how to carry out so this particular the dynamics. Now to carry out the dynamics the prerequisite is the robot kinematics and trajectory planning those things we have already discussed. Now we are in a position to carry out the dynamic analysis, that means we are in a position to find out the expression for the joint torque or the joint force, which is going to create that particular the movement. Now let us try to find out the mathematical expression for the joint torque or the joint force.

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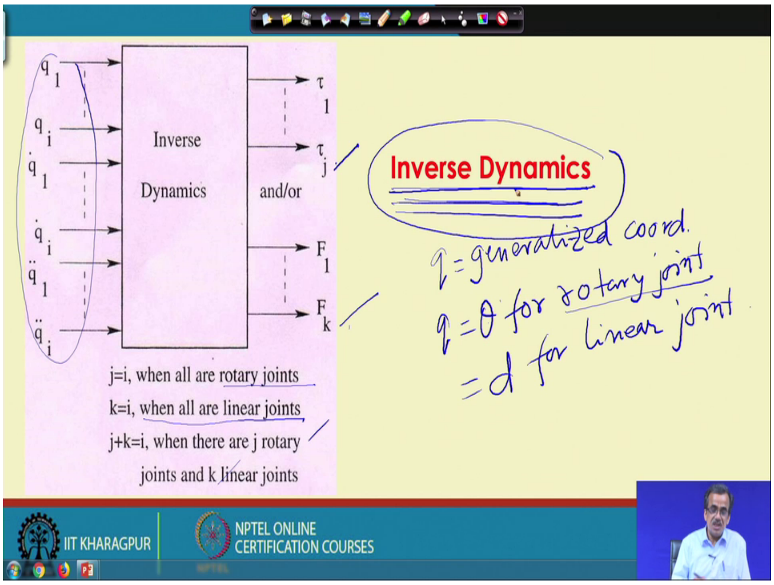

Now, before I proceed further, now I am just going to define 1 term, now that particular term is actually the inverse dynamics. Now while discussing the kinematics we have already discussed the meaning of the 2 terms one is called the forward kinematics another is the inverse kinematics. Now I am just going to start with another term that is called the inverse dynamics. Now let us see what do you mean by so this particular the term the inverse dynamics.

Now, let us concentrate on this particular the block diagram the input side and the output side. Now in the input side I have written like q 1 q 2 up to q i, then q 1 dot q 2 dot up to q i dot then q 1 double dot q 1 q 2 double dot up to q i double dot. Now this particular q actually represents the generalized coordinate generalized coordinate. And this particular generalized coordinate for a rotary joint this q is nothing but theta for rotary joint, and this is equals to d for the linear joint and this q dot is nothing but the first time derivative. So, if it is theta then that is the angular velocity and if it is d that is a linear velocity, then q double dot if it is theta then it is angular acceleration and if it is d that is the linear acceleration.

So, this if I consider is the rotary joint all are rotary joint all i joints are rotary for a particular manipulator, then this is nothing but theta 1, theta 2 up to theta i theta 1 dot up to theta i dot, theta 1 double dot up to theta i double dot. So, these are nothing but the inputs and what are the outputs? The outputs are tau 1 tau 2 up to tau j and or F1 F2 up to Fk. Now, here I put if j equals to i so that is the number of the torque values. So, if it is equal to i that means all the joints are rotary joints, then if I write that k is equals to i that means all the joints are linear joints. For example, say Cartesian coordinate robot all are linear joints where if j plus k, these j plus k equals to i that means we have got a combination and we have got j number of rotary joints and k number of linear joints ok.

So, here the inputs are the independent variables and outputs are nothing but either the joint torque or the force or a combination, so this particular problem of dynamics is defined as your inverse dynamics, but remember this is not the forward dynamics. So, this is in fact the inverse dynamics and I am just going to discuss in this particular course the inverse dynamics problem of this particular the robots and I am not going to discuss the forward dynamics. So I am just going to discuss the inverse dynamics. So, this is what we mean by the inverse dynamics problem which I am going to tackle in this particular the course.

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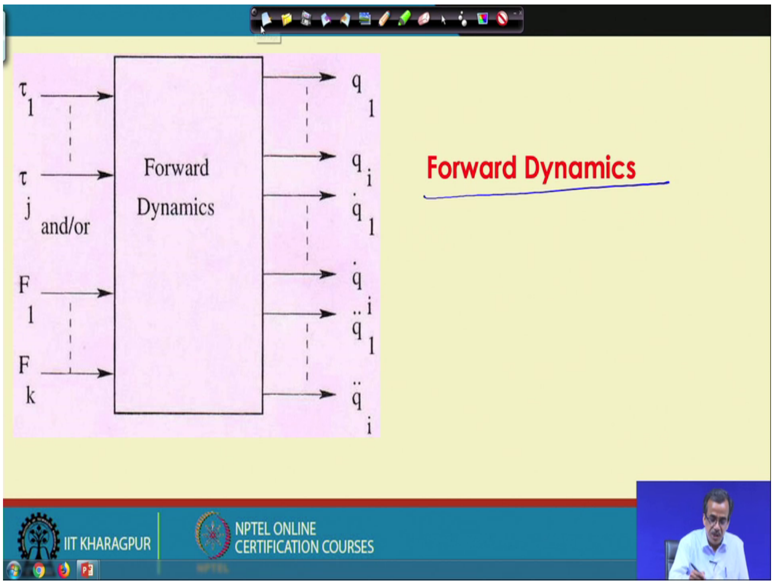

And as I told the reverse of that particular previous problem is the problem of the forward dynamics and here in forward dynamics as usual as I told. So the joint torque and force those will be the inputs and the output will be your joint angle that angular velocity angular acceleration ok, so this is nothing but the forward dynamics. Now here in this course as I told I am just going to consider only the inverse dynamics, but not forward dynamics. If we want to solve the forward dynamics in fact, you will have to take the help of the tools like neural networks fuzzy logic, which I am not going to discuss in this particular the course. So, I am just going to concentrate on your the inverse dynamics.

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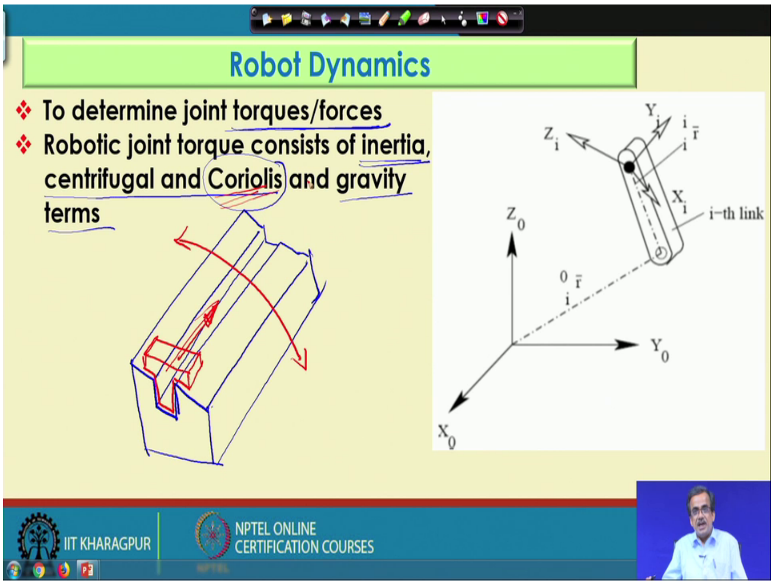

Now as I told the Purpose is to determine actually the joint torque or the joint force and if I concentrate on the joint torque, the joint torque consists of a few terms one is called the inertia terms, another is called the centrifugal and the Coriolis term another is called the gravity terms. So, gravity terms depends on the acceleration due to gravity, inertia terms depends on the mass distribution of the robotic link and that is expressed in terms of the moment of inertia that matrix and centrifugal force all of you know the concept and there is another thing that is called the Coriolis component or the Coriolis force.

Now, this Coriolis force actually requires some explanation, now this Coriolis component will come whenever there is a sliding joint on a rotary link, let me try to prepare one very rough sketch for that just to explain the concept of this type of the Coriolis force, let me try to prepare one very rough sketch. I am in fact going to draw 1 robotic link so this is nothing but a robotic link and this end is actually the connected to the motor and this is the other end and supposing that on this robotic link. So I have got 1 sliding component like this let me try to prepare 1 sketch for the sliding component. So, this is roughly the sketch rough sketch for 1 sliding component and this particular sliding component is going to slide here on this group. So this sliding component is sliding in this particular direction and this link is rotating and this part here we have got the connected to the motor.

So, roughly I think it can be visualized, so this particular sliding member so it is sliding and the link is rotating. So, now in that case so it will be subjected to some amount of force which is known as the Coriolis force this is the concept of the Coriolis force. And of course, if I want to consider that the friction, so that friction also we can consider. Now here, I am just going to concentrate on this particular figure, so these was that this is x naught y naught z naught is nothing but the base coordinate system of this particular the robot and I am just going to concentrate on a particular link that is nothing but the i-th link.

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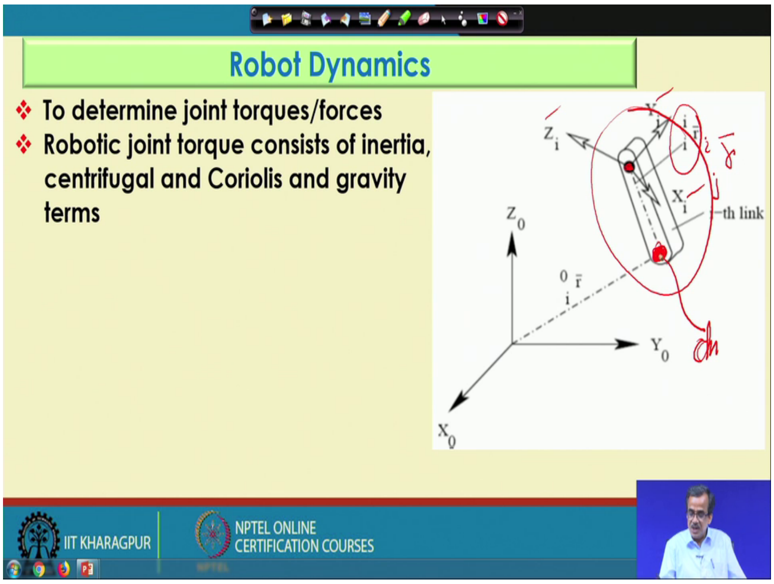

Now, here so this is the i-th link and for this particular i-th link the motor is connected at this particular end but very purposefully. So, I have put the coordinate system here but not here, the reason is very simple we mechanical engineers we always try to think about the reaction force reaction torque. So, whenever we calculate any force or torque that is nothing but the reaction force.

Now, here actually if I just want to put 1 motor, the motor is going to generate torque it is going to generate some angular displacement, velocity and acceleration sort of thing, now if I want to measure that what I will have to do is, I will have to measure the reaction torque. How to measure? To measure the reaction torque so, I will have to concentrate here that other end and that is why actually. So this particular the coordinate system is attached here but not here and here the coordinate system is denoted by x i y i and Z i.

Now, supposing that on this particular link I am just going to concentrate a particular point, say let me consider so this particular point and supposing that this point is having the differential mass say dm small mass dm. Now this particular the point lying on the i-th link can be represented in it is own coordinate system by a position vector, which is nothing but r i with respect to i; that means, here I am consider considering i-th point lying on the i-th link, I can also consider the j th point lying on the i-th link in that case the representation will be rj with respect to i bar ok. But here I am just going to use r i with respect to i; that means, the i-th point lying on the i-th link, I am just going to consider and it is position is denoted by r i with respect to i.

The same point can be represented in the base coordinate system of the robot by another position vector, which is nothing but r i with respect to 0. So, the same point in it is own coordinate system in it is base coordinate system. And we have studied in robot kinematics and we know how to relate this particular r i with respect to 0 and this r i with respect to i.

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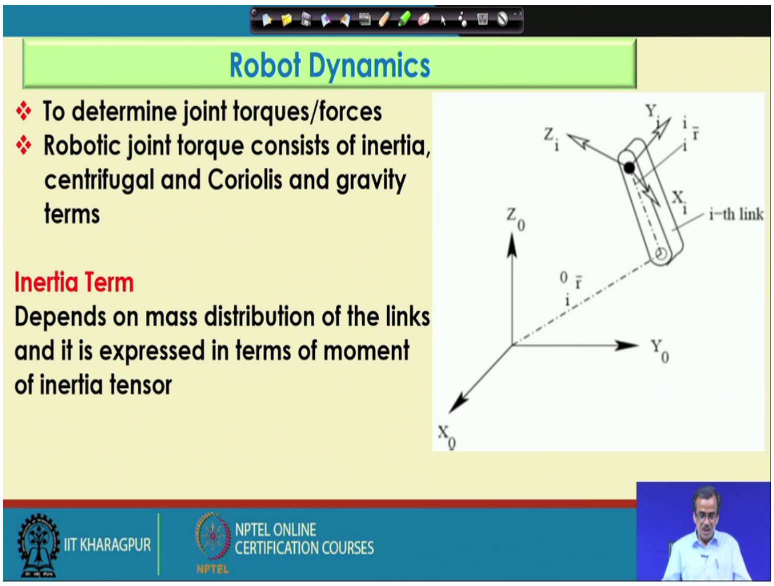

Now if you see that particular this particular your relationship, so we can find out that I am going to discuss in the next slide and in fact, I am just going to concentrate first on the inertia terms. Now here on the inertia terms, actually we try to find out your the mass distribution that is the moment of inertia term. So, this inertia terms I am just going to concentrate first.

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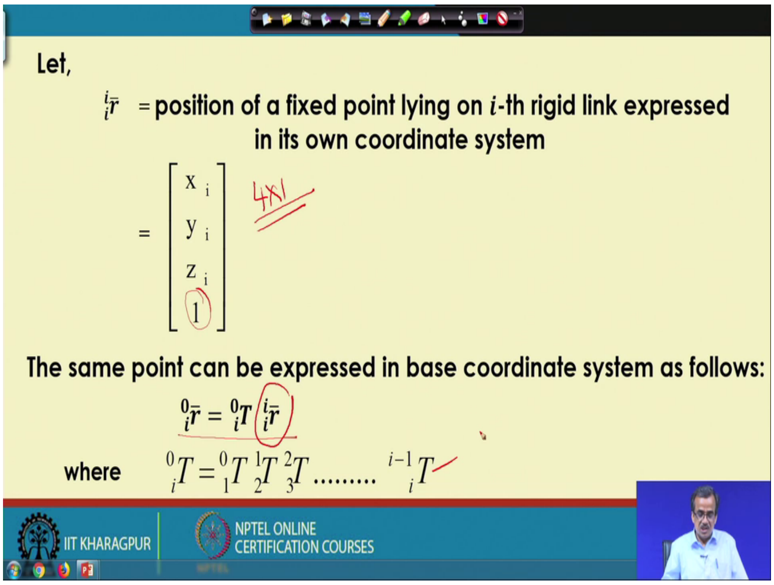

Now here, this r i with respect to i that is the position of the i-th particle lying on the i-th link is denoted by is your x i y i z i so this is the coordinate and this particular 1 actually I am just putting just to make this position vector as a 4 cross 1 matrix, I hope you remember this we followed while deriving the expression for homogeneous transformation matrix. So, at the bottom of this particular position terms we put 1 one the same 1.

Now, this r i with respect to i is nothing but x i y i z i 1 and this is nothing but a 4 cross 1 matrix, now this particular expression let us try to concentrate this is how to relate that position of i-th particle lying on the i-th link with respect to the base coordinate system provided I know this r i with respect to i.

So, what I will have to do is, I will have to multiply T i with respect to 0 that is the transformation matrix of i with respect to the base coordinate frame by r i with respect to i. And all of us we know and we have studied in kinematics that this T i with respect to 0 is nothing but T 1 with respect to 0 multiplied by T 2 with respect to 1 multiplied by T 3 with respect to 2 and so on and the last term is T i with respect to i minus 1. So, T stands for the transformation matrix and all of us we know how to derive and how to find out so this particular the expression.

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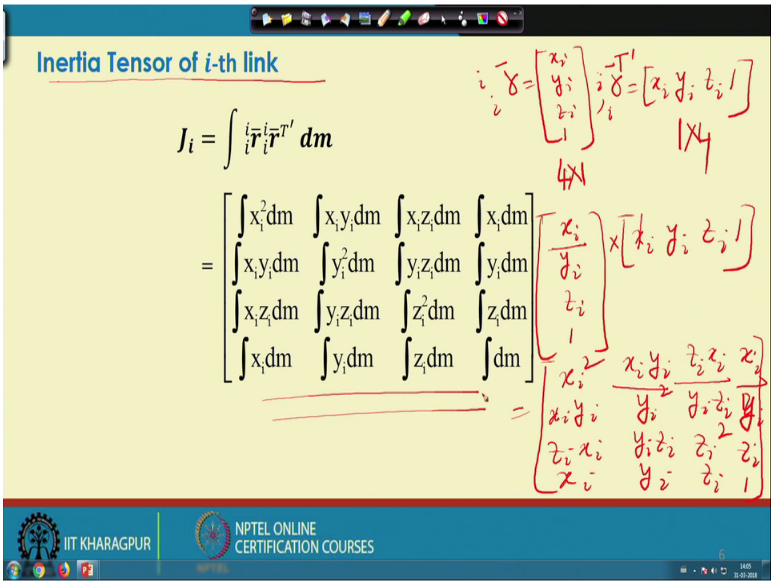

Now, I am just going to concentrate on the concept of this particular the inertia that is the moment of inertia term, how to define this moment of inertia and how to find out the inertia tensor for a particular the robotic link or the i-th link. Now moment of inertia all of us we know by definition that is m r square and for this particular the i-th link the moment of inertia that is denoted by ji that is integration r i with respect to I multiplied by r i with respect to i transpose dm.

So, here actually we will have to find out so this particular the tensor. Now let us see how to find out this particular the tensor. So, this r i with respect to I as we have seen is nothing but is your x i y i z i 1 and this r i with respect to I transpose is nothing but is your x i y i z i 1 ok. Now if I multiply so r i with respect to i by r i with respect to i transpose; that means, this is nothing but a 4 cross 4 cross 1 matrix and this is nothing but a 1 cross 4 matrix and if you multiply then you will be getting a 4 cross 4 matrix.

So, if I just multiply, so this is something like this x i y i z i 1 multiplied by your this particular matrix x i y i z i 1. So, first row first column so I will be getting your x i square x i square then first row second column is x i y i, first row third column z i x i first row fourth column is x i similarly y. So, y i x i so x i y i then comes here y i square y i z i and this is nothing but yi, next is your z i x i then comes your y i z i then comes z i square then comes z i then x i y i z i 1. So, this is the 4 cross 4 matrix which we will be getting and this is exactly same as this.

So, this J i is nothing but that is the moment of inertia tensor for i-th link is integration x i square dm, integration x i y i dm integration z i x i dm integration x i dm. So, this is actually nothing but the 4 cross for your inertia tensor.

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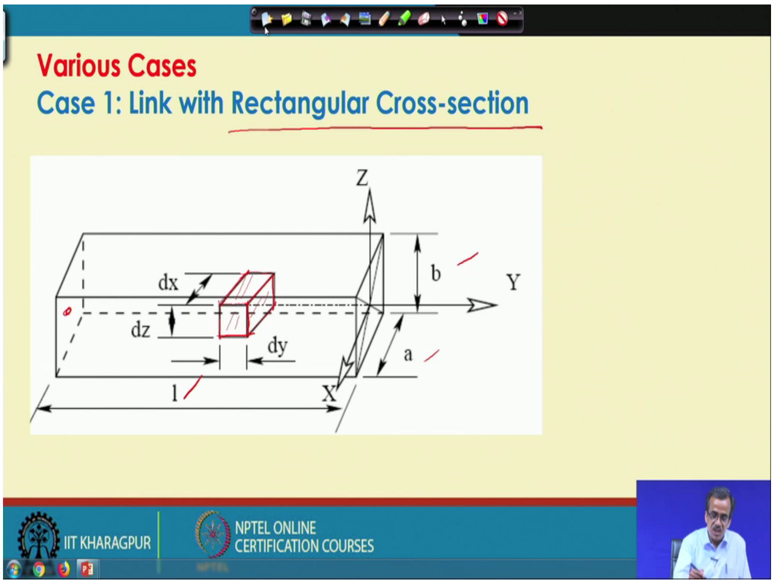

Now once you have got this particular inertia tensor, now what you can do is, we can consider the different cases like the different types of the links we can consider for example, say if I consider the robotic link with rectangular cross section. So, very easily we can find out what should be the expression of this particular the inertia tensor.

Now, let me consider 1 robotic link with rectangular cross section, truly speaking the robotic the motor the motor is connected here and I will have to put the coordinate system or the other end this is the length of the link that is l and it is having the dimension a and b the cross section. And I will have to find out the inertia tensor for this particular the robotic link and for simplicity. So, we are considering a robotic link with constant cross section and that 2 rectangular cross section with sides a and b.

Now, let us see how to find out the inertia tensor we concentrate on this particular differential mass. So, this particular differential mass the small mass having the dimension of the dx then comes here dy and this is dz, so we try to concentrate on this particular the element. Now if we concentrate on this particular element, so very easily can find out the differential mass, we are trying to find out the moment of inertia and that is by definition m r square so always it has to be positive.

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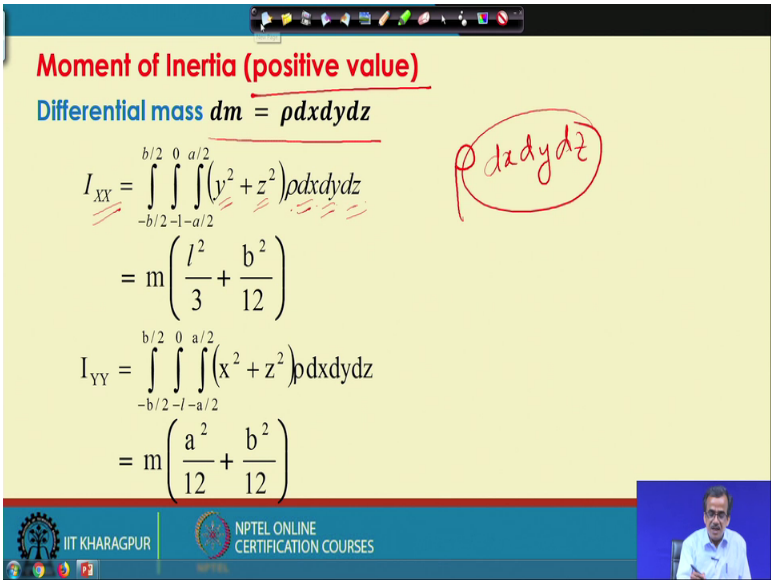

So, it could be positive and differential mass dm is nothing but the volume is dx dy dz. So, this is the volume of the differential mass and multiplied by the density rho this is nothing but the differential mass. Now moment of inertia about xx that is I xx is nothing but triple integration, so this is about I xx as so r square m r square. So, r square will be y square plus z square and rho dx dy dz and now we will have to decide the limit of integration first you concentrate on dx then dy and after that dz.

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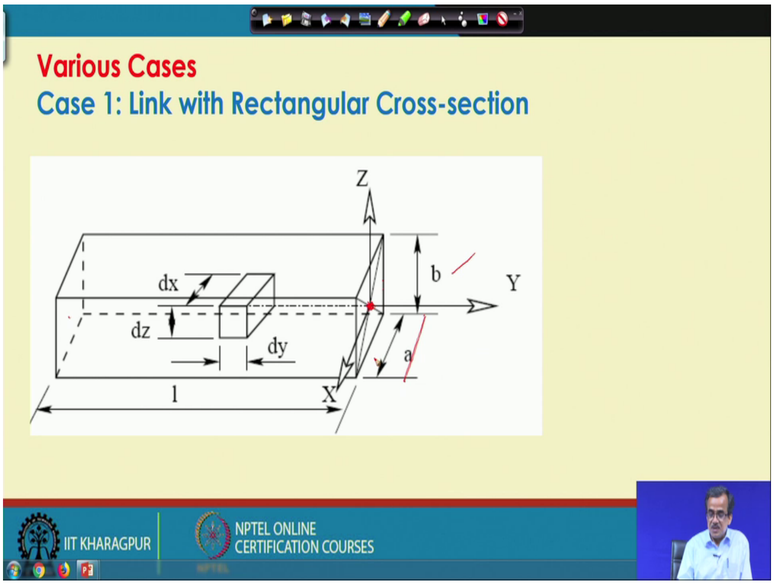

Now, let us go back to the previous slide now here and we are just going to find out the limit for dx, now this is the x direction and along this particular x direction the total dimension is a and this is at the midpoint. So, x will vary from minus a by 2 to plus a by 2 and what should be the range for the limit for y. Now the coordinate system is here so if it is here this corresponds to 0 value of y. So, this particular part is minus so from minus l to 0. So, the range for this particular y, it is minus l to 0 and the range for the z along the z direction the total dimension is b, so from minus b by 2 to plus b by 2.

Now, you see. So, what you can find out, we can find out the expression for this particular i the limit for this integration. And if we just solve this particular integration for example, x is minus a by 2 to plus a by 2 then dy is minus l to 0, then dz minus b by 2 to b by 2 and all of us how to know this carry out this particular integration, and all of you please practice to find out to derive this particular expression and if you carry out this particular integration.

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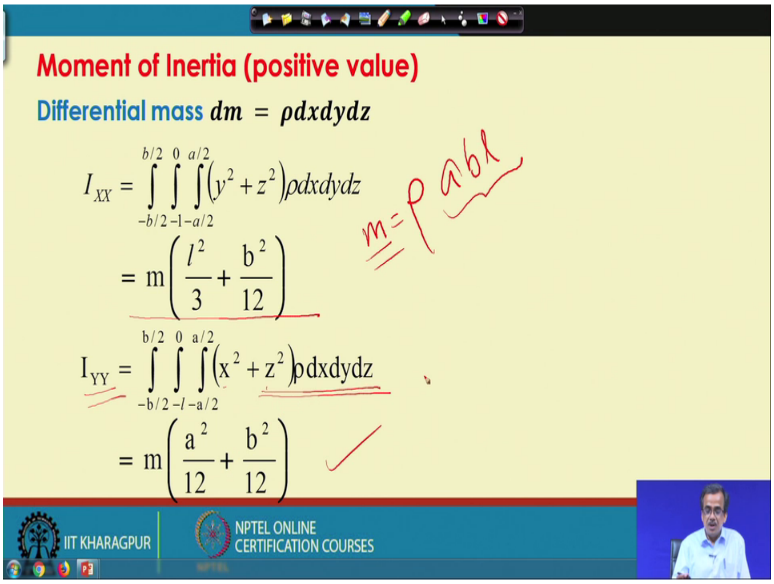

So, you will be getting the expression that is nothing but is your I xx that is m l square by 3 plus b square by 12, where m is nothing but the mass of this particular link that is nothing but rho multiplied by a b l. So, abl is nothing but the volume of this rectangular link multiplied by rho is nothing but the mass.

Now, by following the similar procedure, so we can also find out this I yy that is nothing but triple integration in place of r square we will have to write down x square plus z square rho dx dy dz and the limit for integration will be the same as I discuss earlier and if we carry out this particular integration, I will be getting m into a square by 12 plus b square by 12. So, all of you please try to derive this particular expression and you will be getting it.

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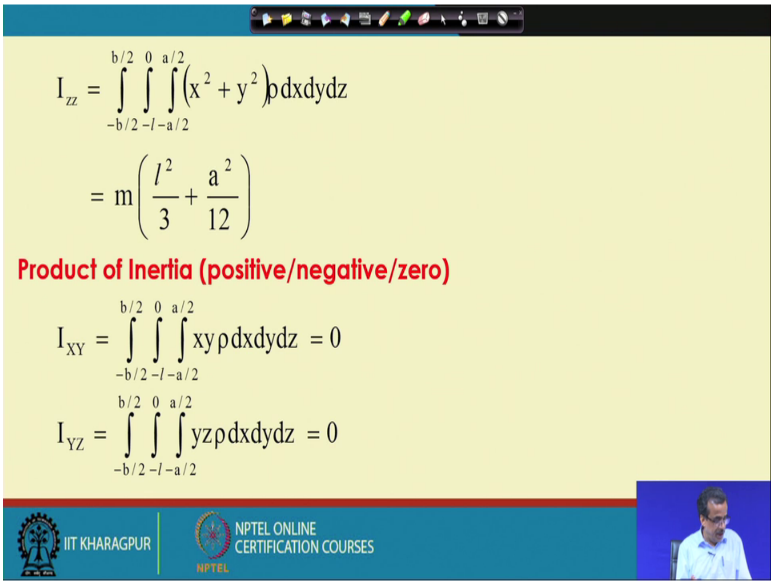

Similarly, by following this we can also find out what is your I zz and that is nothing but in place of r square, so I will have to write down x square plus y square and then if you carry out this integration. So, you will be getting m l square by 3 plus s square by 12, then comes the concept of product of inertia. Now this product of inertia in place of r square so I will have to write down so xy yz zx like this and this product of inertia all of you know it could be either 0 or it could be negative or it could be positive.

So, all 3 possibilities are there for example, if we calculate I xy that is triple integration xy rho dx dy dz and following the same limit of this integration and if you carry out this particular integration. So, we will be getting that is equals to 0. Please try to derive this and check then I yz. So, you will have to write down yz here, so I will be getting this is equal to 0.

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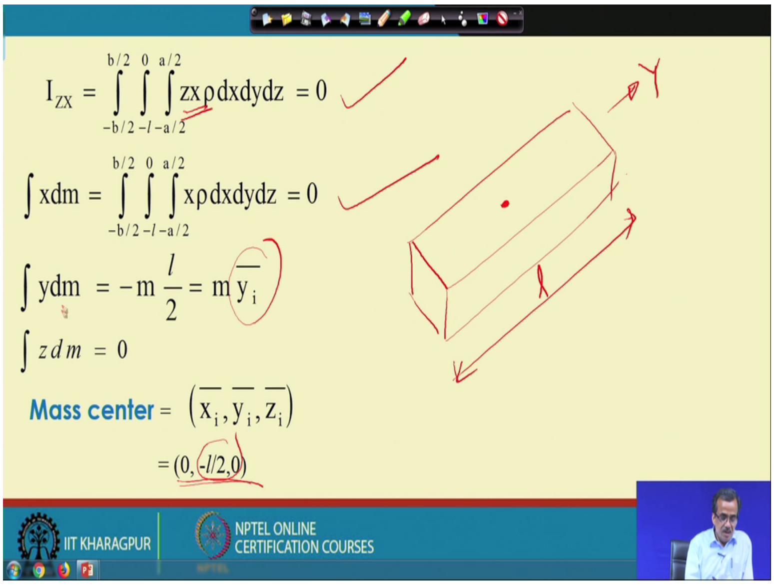

Now, similarly so I can find out, so this I zx is nothing but this and it will be getting this particular expression. So, I here will have to write down zx and if you carry out this integration we will be getting equals to 0. Similarly we will have to find out integration xdm. So, this is nothing but x rho dx dy dz same limit for integration and if you calculate we will be getting that is equals to 0, then comes your integration y dm so this will become minus m l by 2.

Now, if you see this particular you are if you if you just see this rectangular cross section link it is something like this, now here so the mass center is here ok. Now this is the y direction and the total length is your l ok. So, if you see the mass center so x coordinate of the mass center is 0 the y coordinate of the mass center is nothing but minus l by 2 and the z coordinate is 0. So, this is actually the coordinate of the mass center and that is why so minus m l by 2 can be written. So, in place of minus l by 2 I am writing y i bar and then integration z dm is equals to 0.

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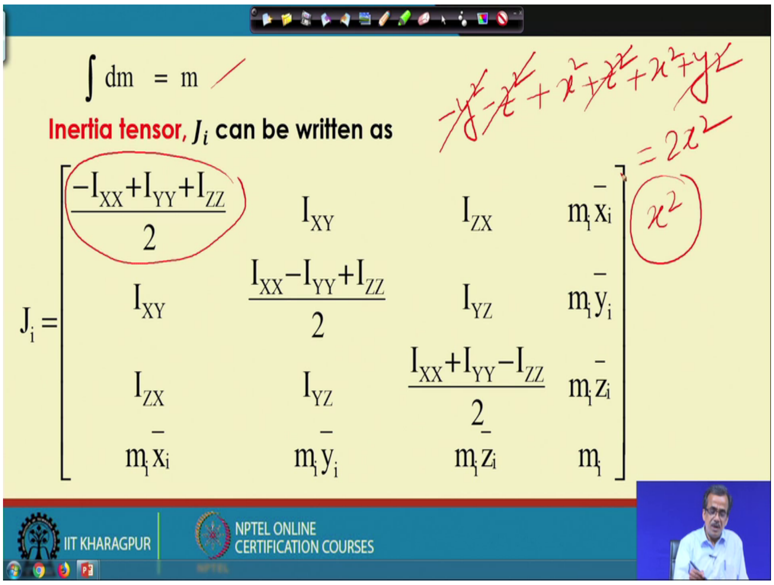

So, if you carry out this integration now we can also find out integration dm equals to m, now I am just going to concentrate on this particular very well known inertia tensor, which is available in all the textbook ok. But how to derive this particular inertia tensor the general expression, so let us try to find out let us try to concentrate on this.

Now, while determining this I xx if you remember we consider y square plus z square and there is a minus. So, I can write down minus y square minus z square. While determining I yy I consider x square plus z square; while determining I zz I consider x square plus y square. So, minus y square plus y square minus z square plus z square gets cancelled so I am getting 2 x square divided by 2, so I will be getting x square.

Now if you see the previous expression of the inertia tensor, so this particular expression of the inertia tensor the first term is your x i square. So, to get this x i square actually I am using so and to get the other terms, so we are using this particular the final expression for this inertia tensor. So, this particular inertia tensor can be derived starting from the first principle and now if I just put all the expression the values of I xx I yy I zx and all such things.

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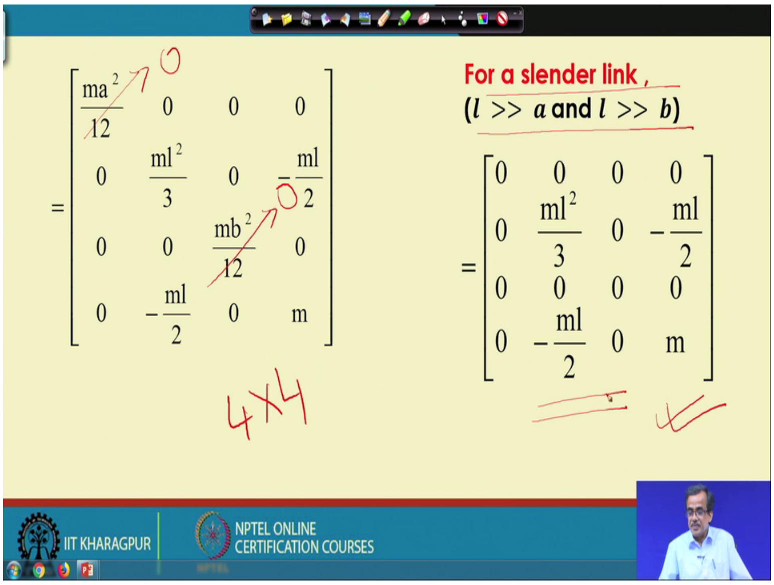

So, I will be getting this particular the matrix, this is nothing but 4 cross 4 inertia tensor for this particular the rectangular link ok. Now there is a concept of slender link for which l is very large compared to a the length of the link is very large. That means if I consider the slender link. So, m a square by 12 will tend to 0, m b square by 12 will tend to 0 and I will be getting this particular matrix as a inertia tensor matrix for a slender link, so this is the inertia tensor.

Thank you.