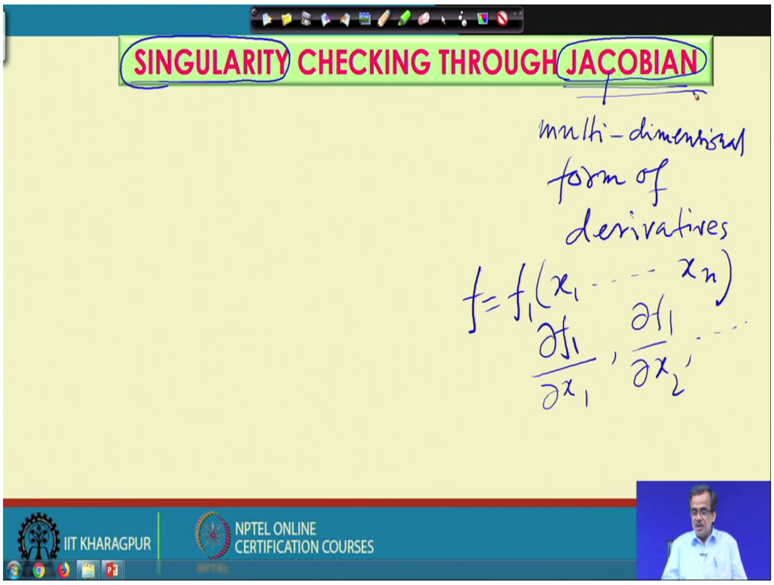
Robotics
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 23
Singularity Checking

(Refer Slide Time: 00:22)

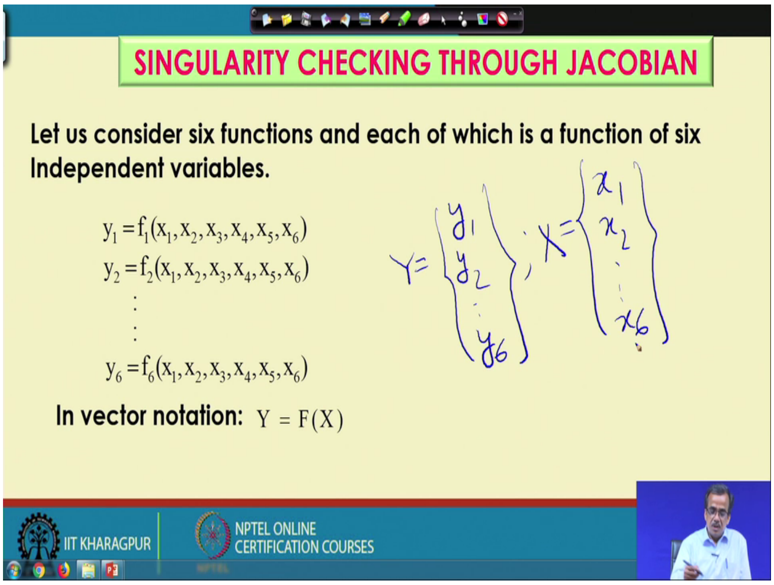


Now, I am going to start with another subtopic that is how to determine the singularity condition of a particular the manipulator. Now, remember one thing singularity condition is a condition, during which the manipulator will lose either one or more degrees of freedom. Let me take one example. Supposing that I have got a manipulator having 6 degrees of freedom and out of 6 joints, so might be say 1 or say 2 joints got locked. And during due to this the locking condition, the manipulator is going to lose its 1 or 2 degrees of freedom depending on the situation. This is what we mean by the singularity condition of a particular the manipulator.

Now, let us see how to check the singularity condition using the concept of the Jacobian. Now, what is Jacobian, you might have studied in the partial differential equation. Now, this particular Jacobian is nothing but the multi-dimensional, multi-dimensional form of derivatives. Supposing that, I have got a function, which is a function of many variables. For example, say f is a function of say f 1 is a function of say n variables.

So, if I want to find out the derivative, so we will have to take the help of partial derivative. So, we will have to find out, so partial derivative of f 1 with respect to x 1 partial derivative of f 1 with respect to x 2 and so on. And by Jacobian, we mean the multi-dimensional form of derivative. Now, this particular mathematical concept of Jacobian can be used in robotics, just to check the singularity. Moreover, we can also use just to control the manipulator.

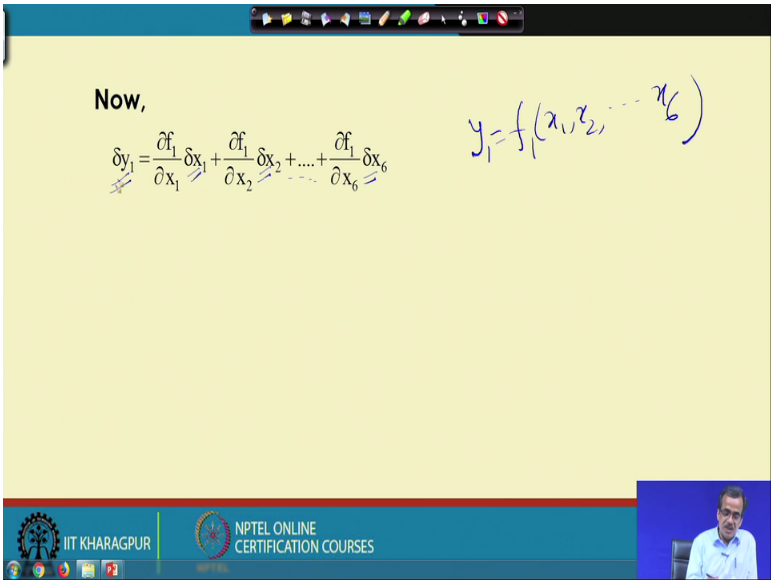
(Refer Slide Time: 02:47)



Now, let us let us try to define the Jacobian first. And let us see, how to use the concept of Jacobian to check the singularity condition of a manipulator. Now, here I am just going to take the help of six function say y 1, y 2 up to y 6. Now, each of these six function is a function of six independent variables. For example, x 1, x 2, up to x 6, so these are all independent variables and this y 1 is a dependent one. So, the y 1 is a function of, so x 1, x 2, up to x 6.

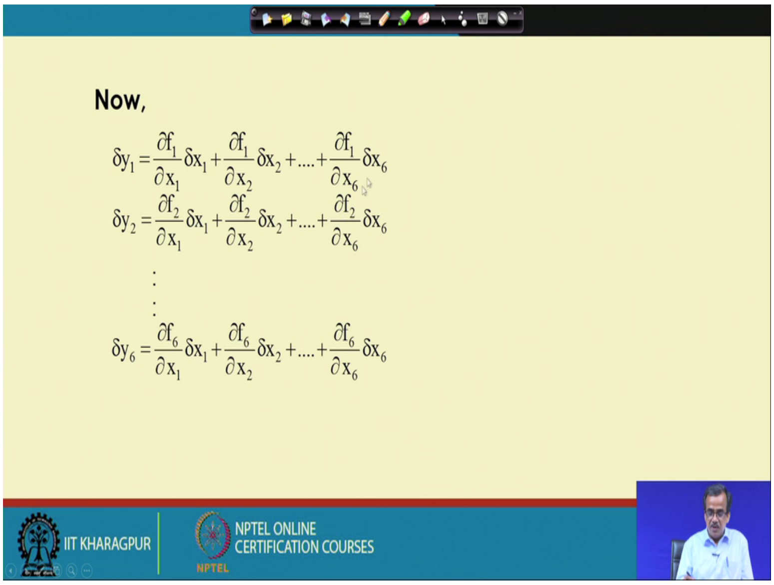
Similarly, y 2 is a another function, f 2 of the same independent variable. Similarly, this y 6 is a another function of the same independent variable x 1, x 2, up to x 6. So, I am considering six function and each function is a function of 6 independent variable. Now, here if you see, so this particular thing in the vector form can be written like this Y capital Y is nothing but capital F capital X. So, this Y is a collection of all small y values. Similarly, X is a collection of all small x values. For example, so I can write down, so this particular Y is nothing but a collection of small y that is y 1, y 2 up to say y 6. Similarly, this X is nothing but a collection of all small x values, so x 6.

(Refer Slide Time: 04:42)



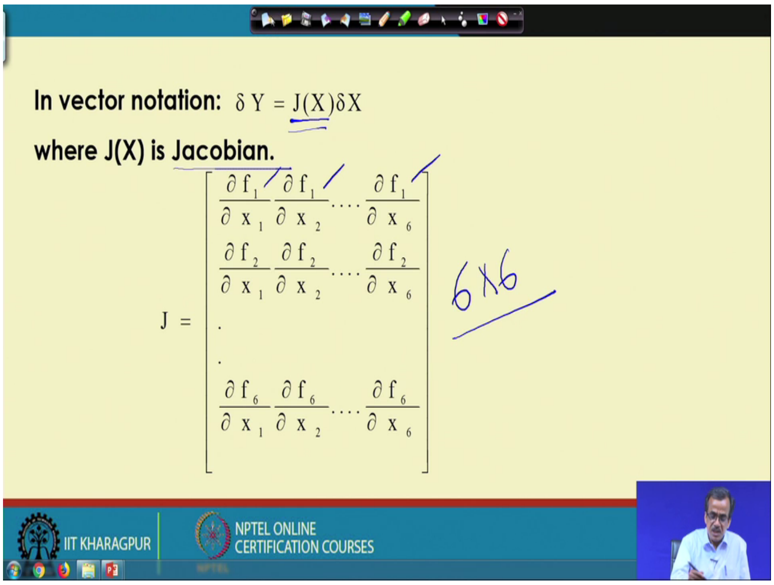
Now, here now let us see how to proceed further. So, what you can do is, we will try to find out how to find out a change in y 1, a small change delta change in y 1. So, if we just write down. So, y 1 is a function of your x 1, x 2 up to x 6. So, I am just going to find out a small change in y 1 that is delta y 1 is nothing but the partial derivative of f 1 with respect to x 1 multiplied by a small change in delta x 1. Similarly, the partial derivative of f 1 with respect to x 2 multiplied by a small change in x 2 that is delta x 2 and so on. And the last term that is your partial derivative of f 1 with respect to x 6 multiplied by delta x 6. So, this is the way actually we can represent the small change in your y 1.

(Refer Slide Time: 05:52)



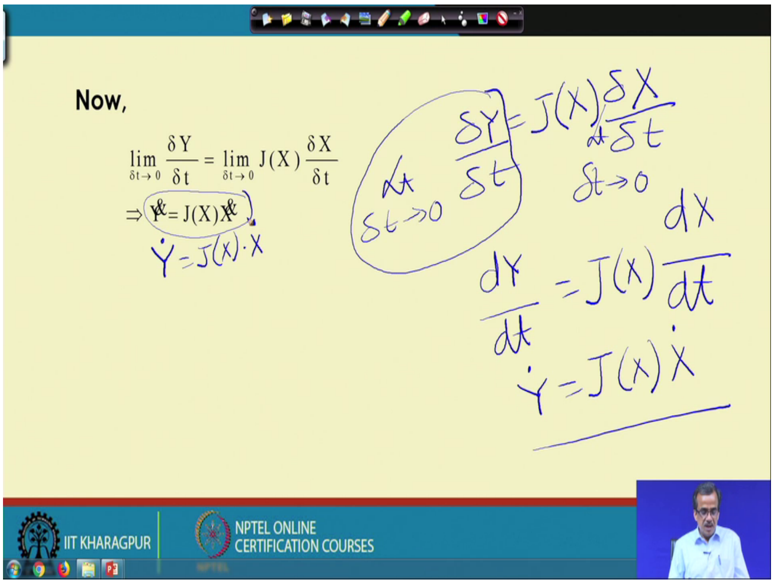
Following the same principle, so I can also write down, what is the small change in your y 2. Similarly, we can also find out, what is the small change in y 6 by following the same method.

(Refer Slide Time: 06:06)



And actually in vector notation this can be written something like this, the change in y 1 that is delta capital Y is nothing but J X multiplied by the delta X that is small change in X. Now here, so his particular J X is actually nothing but the Jacobian. And this J matrix that is the Jacobian matrix is nothing but the partial derivative of f 1 with respect to x 1, partial derivative of f 1 with respect to x 2, partial derivative of f 1 with respect to x 6. So, so this particular the J matrix is having the dimension here 6 cross 6. So, this is the way actually we can find out the Jacobian matrix.

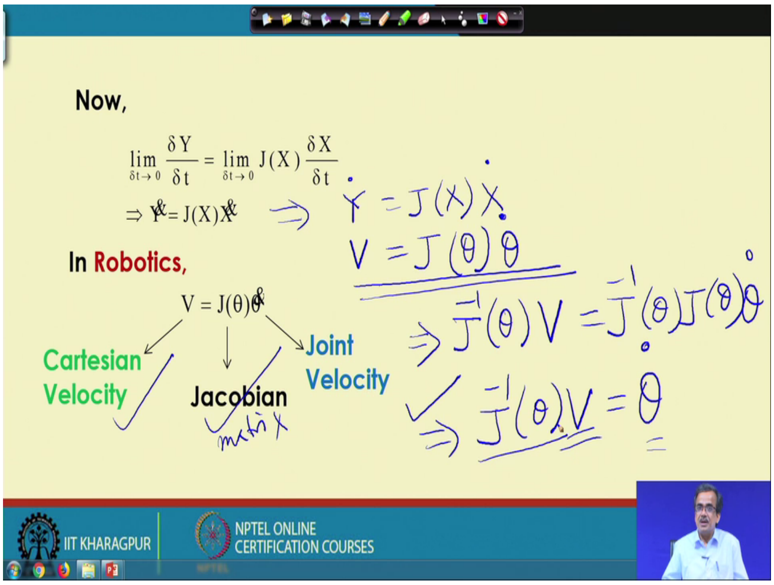
(Refer Slide Time: 07:08)



Now, let us see how to use this particular Jacobian matrix further in robotics. Now, we have got this particular thing like the change in Y that is delta Y is nothing but the Jacobian J X multiplied by delta X. Now, both the sides you divided by a small time that is delta t, and small time delta t, and put limit delta t tends to 0, and here also you put limit delta t tends to 0.

Now, if I put limit delta t tends to 0, so by definition, so this is nothing but the definition of derivative that is d Y d t, and this is nothing but J X multiplied by d X d t, so this is by definition the derivative. And this can be written as Y dot is nothing but J x multiplied by X dot, so this is the expression. And here, once again there is some typographical error, this is in fact, Y dot is equals to J X into x dot.

(Refer Slide Time: 08:36)



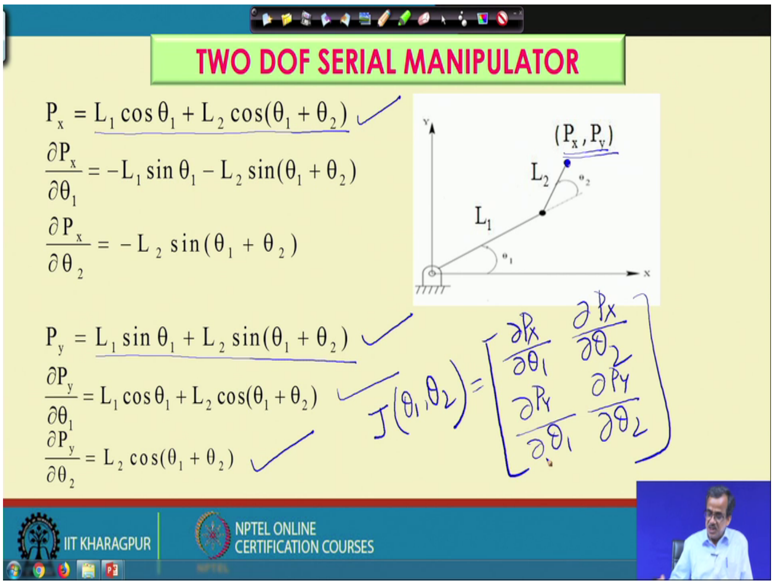
Now, here in robotics actually we are going to use this particular expression like. So, this is nothing but this is nothing but Y dot is equals to J X into X dot. So, in robotics actually this Y dot we can replace by V, and this J X we can replace by J theta and this is nothing but theta dot.

Now, why theta, because theta is independent X was also independent; so this joint angles, supposing that I have got a robot having six joints, all six are rotary joint, so joint angles are the variable. And this particular theta are determined with the help of say six motors. And actually this theta are nothing but the independent things. So, this can be written as V equals to J theta dot.

Now, V is nothing but the Cartesian velocity. And J theta is nothing but the Jacobian matrix. And theta dot is nothing but the joint velocity. So, this is the way in robotics, we can connect the Cartesian velocity with the joint velocity with the help of Jacobian matrix. Now, here we have written V equals to J theta dot. Now, what you can do is, so we can multiply both the sides by J inverse theta multiplied by V and this is nothing but J inverse theta multiplied by J theta multiplied by theta dot. So, this becomes your J inverse theta V is nothing but is your theta dot ok.

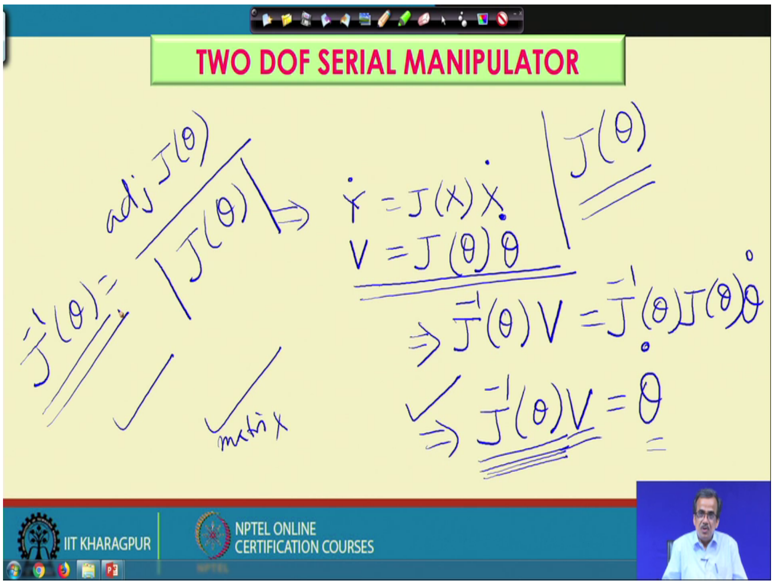
So, this particular relationship we can find out very easily, that means, your we can find out we can relate the Cartesian velocity with the joint velocity provided we know, the your provided we know, the inverse of this particular the matrix. Provided we know the inverse of this particular matrix that is your J inverse theta.

(Refer Slide Time: 11:16)



Now, let us let us try to concentrate on, so this particular the J inverse theta thing. Now, let me try to take the help of one manipulator. But before that let me tell one more thing, this V is nothing but the V is nothing but is your J theta dot, and J inverse theta is multiplied by V is equals to theta dot.

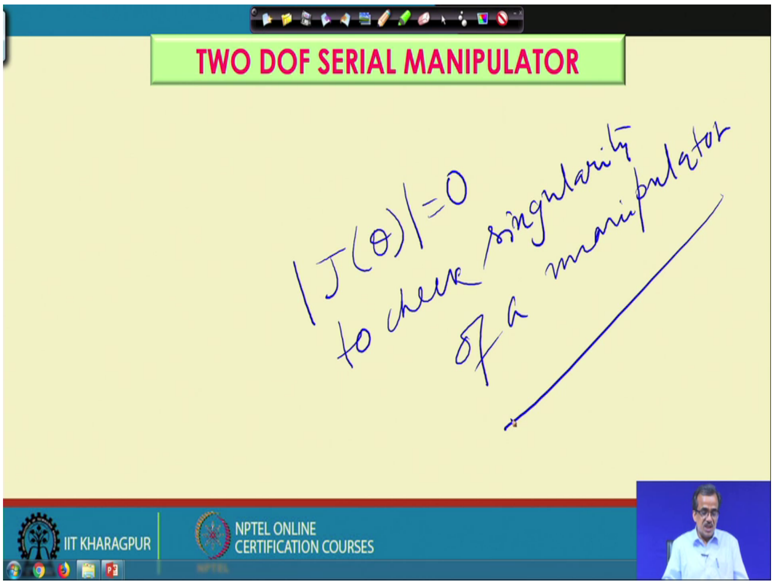
(Refer Slide Time: 11:51)



And as I told that this particular theta J inverse theta, that means, this particular J theta that is your Jacobian matrix, we should be able to find out the inverse of that. And if you want to find out the inverse of this particular Jacobian matrix, because inverse is nothing but your J inverse theta is nothing but adjoint of J theta divided by is your determinant of J theta.

So, this particular J inverse theta to exist, that means, if you want to find out the inverse of J theta that is the Jacobian matrix, it has to be invertible, that means, it has to be non-singular. And that is why, so this particular J theta determinant of J theta has to be non 0. So, to exist this particular inverse of the J theta, so the determinant of J theta has to be non 0. Now, I can state in the different way that if I want to check the singularity condition of the manipulator, so what I will have to do is, I will have to put, so the determinant of this particular J theta is equal to 0.

(Refer Slide Time: 13:12)



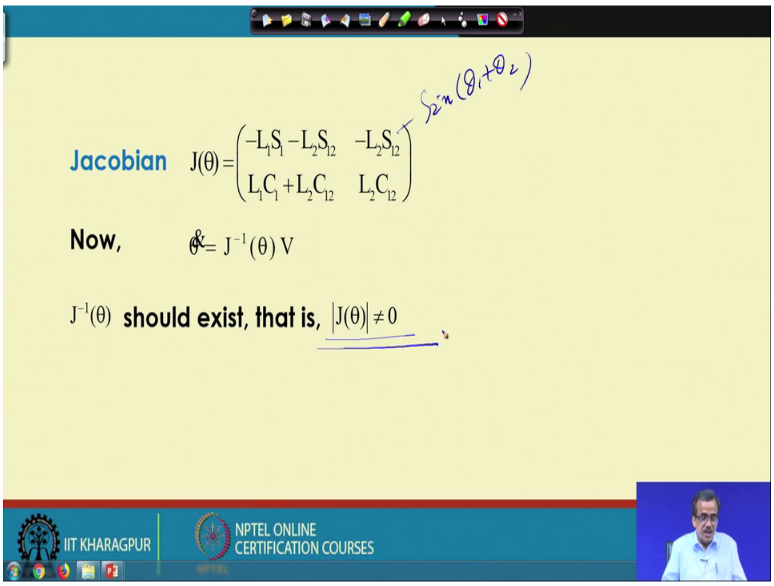
Now, here actually what we can do is, so the determinant of this particular J theta, we can put equals to 0, just to find out to check the singularity condition to check singularity condition of a manipulator. So, this is the condition, which I am going to use. Now, let us let us take one example. And let us check how to determine the singularity of a particular the manipulator.

Now, here I am just going to take the example of a two degrees of freedom serial manipulator ok. Now, once again this is the well known two degrees of freedom serial manipulator having the length of the link L 1 and L 2. And this is the position of the end-effector, whose coordinate is nothing but P x and P y. And all of us we know that very easily you can find out the expression for P x, P x is nothing but L 1 cos theta 1 plus L 2 cos of theta 1 plus theta 2.

Similarly, this P y is nothing but P y the general expression is L 1 sin theta 1 plus L 2 sin of theta 1 plus theta 2. So, what I am going to do here is, I am trying to find out the partial derivative of these P x with respect to theta 1. So, so partial derivative with respect to theta 1, so I will be getting minus L 1 sin theta 1 minus L 2 sin of theta 1 plus theta 2. Similarly, the partial derivative with respect to theta 2, so it will have no contribution, and here I will be getting minus L 2 sin of theta 1 plus theta 2. Then comes your P y is nothing but your L 1 sin theta 1 plus L 2 sin of theta 1 plus theta 2, this is a general expression. I can find out the partial derivative of P y with respect to theta 1 that is L 1 cos theta 1 plus L 2 cos of theta 1 plus theta 2.

Then partial derivative of P y with respect to theta 2 is nothing but L 2 cos of theta 1 plus theta 2. Now, how to write down its Jacobian? Now, the Jacobian matrix J is a function of that that variables theta 1 theta 2 is nothing but the partial derivative of P x with respect to theta 1, then partial derivative of P x with respect to theta 2, then comes partial derivative of P y with respect to theta 1 partial derivative of P y with respect to your theta 2 ok.

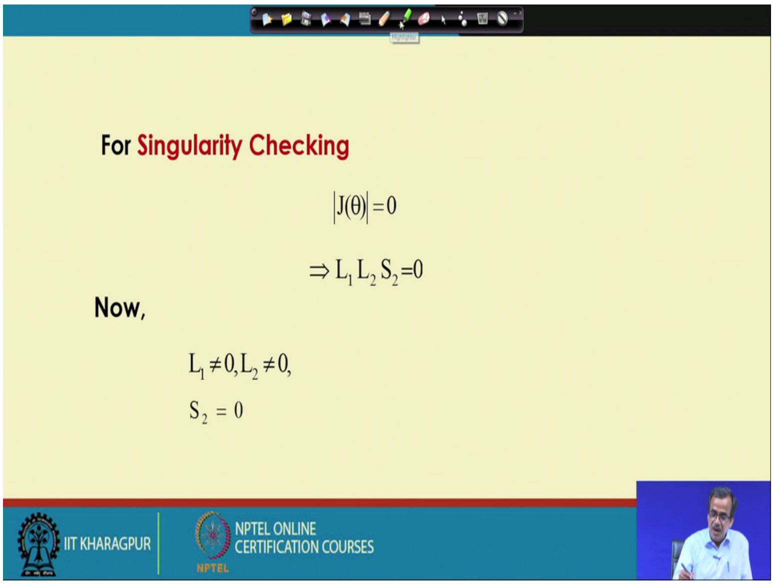
(Refer Slide Time: 16:29)



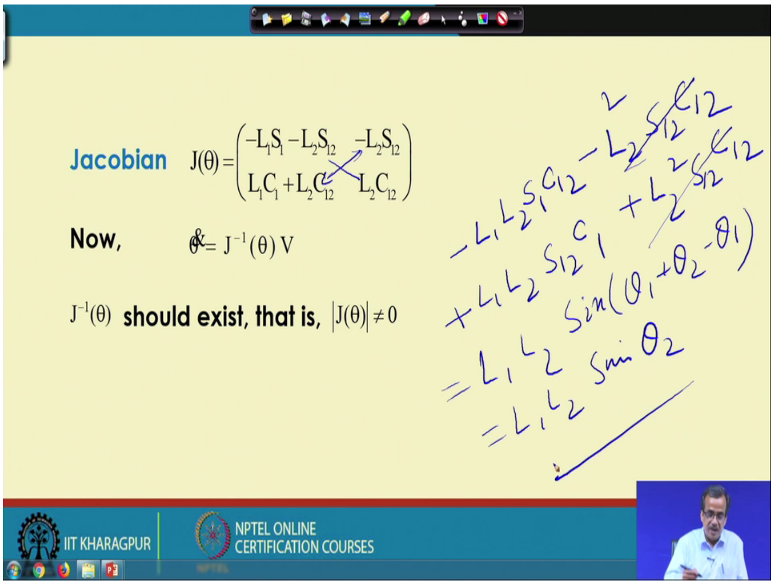
Now, we can insert those values for the partial derivative. And if we insert the your the expression for the partial derivative, we will be getting the Jacobian matrix like this. So, this is the Jacobian matrix like your minus L 1 sin theta 1 minus L 2 sin of theta 1 plus theta 2 minus L 2 sin of actually theta 1 plus theta 2; similarly, L 1 cos theta 1 plus L 2 cos of theta 1 plus theta 2 and L 2 cos of theta 1 and theta 2.

Now, this thing I have already discussed that for J inverse theta to exist, so this particular condition has to be fulfilled, that means, the determinant of J theta has to be non has to be non 0. So, if I want to check the singularity condition, so what I am going to do is, I am just going to put the determinant of the J theta is equals to 0.

(Refer Slide Time: 17:34)



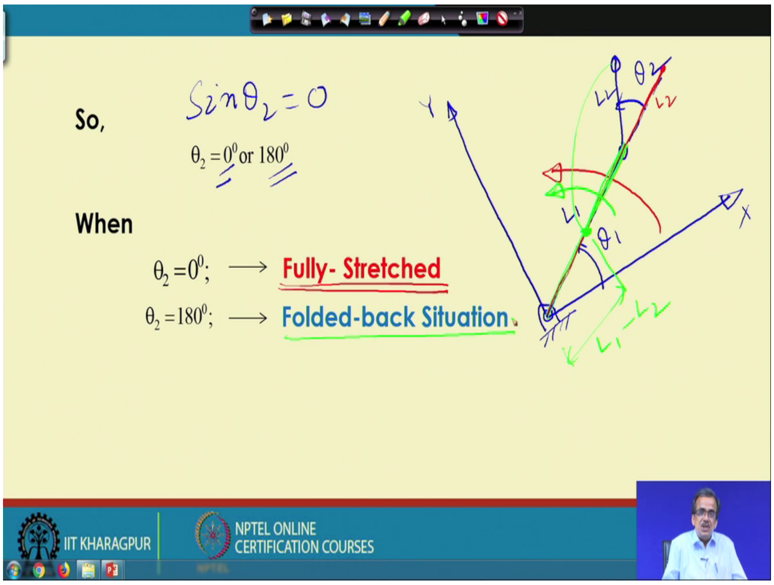
(Refer Slide Time: 17:49)



Now, if I put the determinant of J theta is equals to 0, then what you can get is your, so very easily you can find out the determinant, determinant of this particular is very simple, so what we do is, so we multiply, so this one that is minus L 1 L 2 sin of theta 1 cos of theta 1 plus theta 2 minus L 2 square sin of theta 1 plus theta 2 cos of theta 1 plus theta 2 minus this minus the this multiplied by this, that means, your plus L 1 L 2 sin of theta 1 plus theta 2 cos theta 1 plus your L 2 square sin of theta 1 plus theta 2 cos of theta 1 plus theta 2. So, this is minus L 2 square sin of theta 1 plus theta 2 cos of theta 1 plus theta 2 and this is plus.

So, this minus and plus gets cancelled. And now we are we are having L 1 L 2. Now sin of theta 1 plus theta 2 cos theta 1 minus your cos of theta 1 plus theta 2 sin theta 1; so sin of sin of theta 1 plus theta 2 minus theta 1. So, I will be getting your L 1 L 2 sin of theta 2. So, I will be getting this particular as the determinant. Now, if I put equals to 0, if I put, so this particular determinant equals to 0. So, L 1 L 2 is equals to 0. But, L 1 L 2 are nothing but the length of the links, so that cannot be 0. So, the only possibility is your S 2 equals to 0 that is sin theta 2 is equals to 0.

(Refer Slide Time: 19:38)



Now, if I put sin theta 2 equals to 0, in fact, there are two solutions for sin theta 2 equals to 0 sin theta 2 is equals to 0. There are two solutions for theta 2; one is theta 2 equals to 0; another is theta 2 equals to 180 degree. Now, let us see what happens for this particular the two degrees of freedom serial manipulator. So, this is your say I am just drawing here roughly that two degrees of freedom serial manipulator. So, this is the Cartesian X the Y coordinate. And this is your L 1, and this is say L 2, L 1, L 2. So, this particular angle is your theta 1, this particular angle is your theta 2.

Now, let us see what happens, if I consider theta 2 equals to 0, now if I consider that theta 2 equals to 0, if I consider that theta 2 equals to 0, so what will happen is your, so this theta 2 if I put equals to 0, so this point will come here. So, the link will become this is up to this it is L 1, and this will become L 2, so the total length of this particular link will be L 1 plus L 2. And as if at this particular point, it is locked ok. Now, it will behave just like a manipulator having only one degree of freedom, and the length of the link is like L 1 plus L 2, because theta 2 is locked to 0. Now, this will rotate, and there will be only one variable, one degree of freedom that is your theta 1. So, this is actually is known as the fully-stretched condition.

Now, then comes if I consider that your theta 2 is equals to 180 degree. So, this particular point is going to come here, so my theta 2 will be here. So, this is my L 1, this is L 2. So, I will have one link, whose length is nothing but L 1 minus L 2 provided L 1 is greater than L 2. Now, this is once again is going to have, so this is my L 1, this is L 2 the tip will be here, and it will have only one movement that is theta 1, and practically it will have only one degree of freedom. So, although it is having two degrees of freedom, so one degree of freedom is lost, and this is known as the folded-back situation of this particular the manipulator.

So both, at fully stretched condition of this particular two degrees of freedom serial manipulator and the folded-back situation of this particular manipulator; so it is going to lose one degree of freedom out of two. And this will behave as a manipulator having only one degree of freedom, and that is what you mean by the singularity condition of this particular the manipulator.

Thank you.