Robotics
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Lecture - 22
Trajectory Planning (Contd.)

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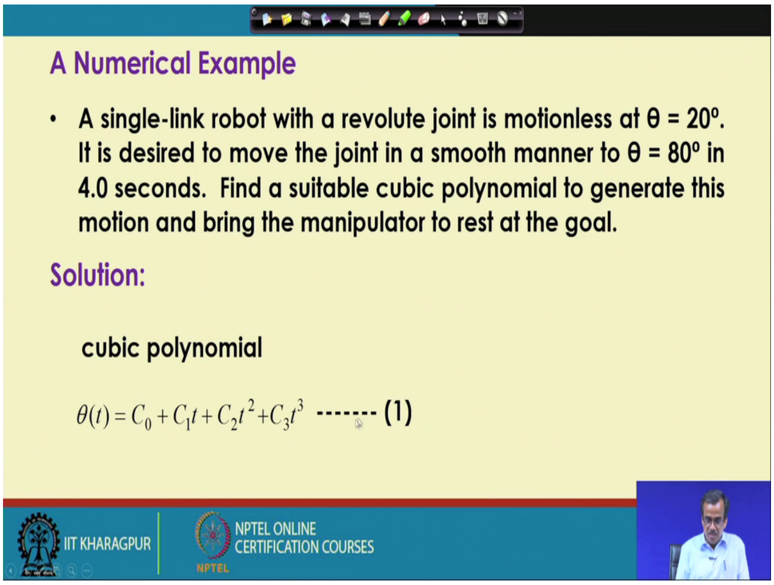

Now I am going to discuss one numerical example based on this trajectory planning. The statement of the problem is as follows. A single link robot with the revolute joint is motionless at theta equals to 20 degree. It is desired to move the joint in a smooth manner to theta equals to 80 degrees in 4 seconds. Find a suitable cubic polynomial to generate this motion and bring the manipulator to rest at the goal.

So, this is a very simple problem. And we know the displacement initial displacement initial velocity is equals to 0. We know the final displacement and the final velocity is once again equal to 0. And we know that the total time, that is nothing but the 4 second. So, very easily you can fit one cubic polynomial, and this particular problem actually relates to case 1, so very easily, in fact, we can put we can just fit one cubic polynomial. So, theta t is nothing but C naught plus C 1 t plus C 2 t square plus C 3 t cube.

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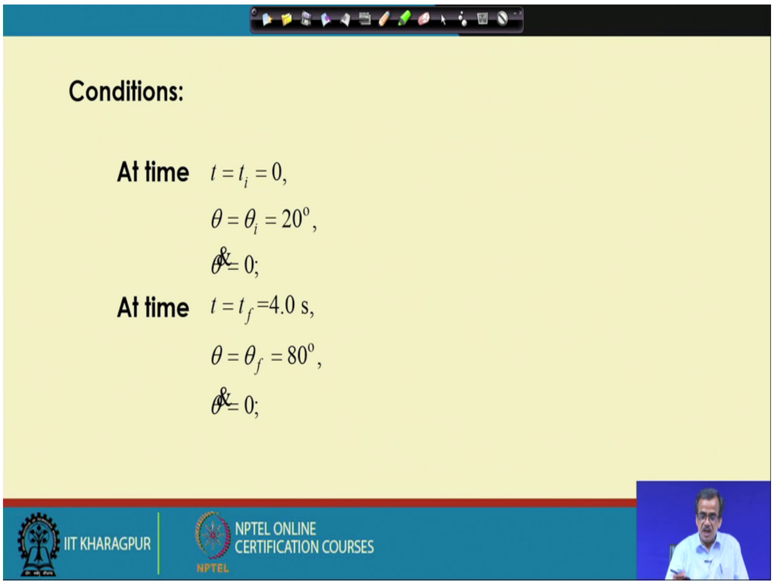

And the conditions are as follows. For example, say at time t equals to t i equals to 0, theta equals to theta i is equals to 20 degree, and this is nothing but theta dot theta dot is equals to 0. And similarly, at time t equals to t f that is equals to 4 second, theta equals to theta f equals to 80 degree. And this particular theta dot is equals to 0, so these are the conditions. And based on this particular condition, so we can find out what should be the values for this particular the coefficient.

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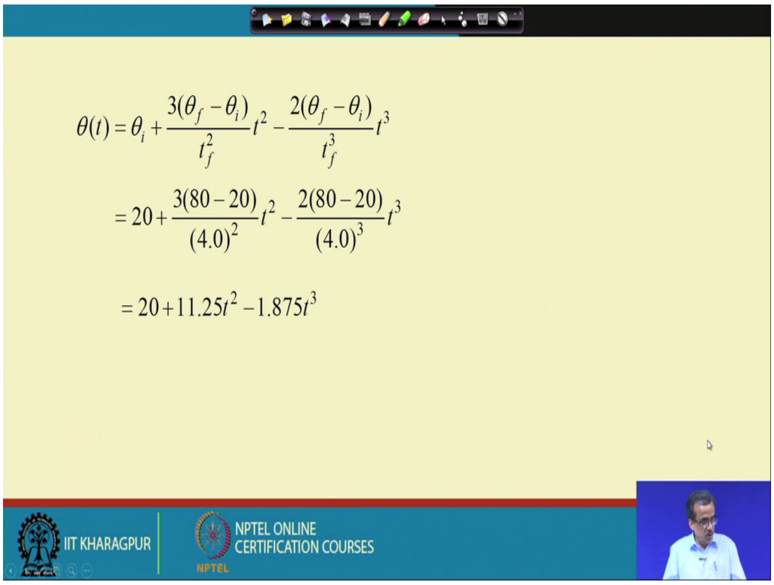

And we can find out the final expression of this particular the cubic polynomial. So, the final expression is coming as theta t, is nothing but the initial theta ok, and this is actually the known condition. Now, what you can do is, so we can actually fit we can just put the numerical values. And if I put the numerical values, then I will be getting the final expression.

And the final expression will be getting as something like this. So, theta t is nothing but 20 plus 11.25 t square minus 1.875 t cube, so this type of expression you will be getting. Now, very easily you can plot this particular theta as a function of time. So, this is how to find out the cubic polynomial.

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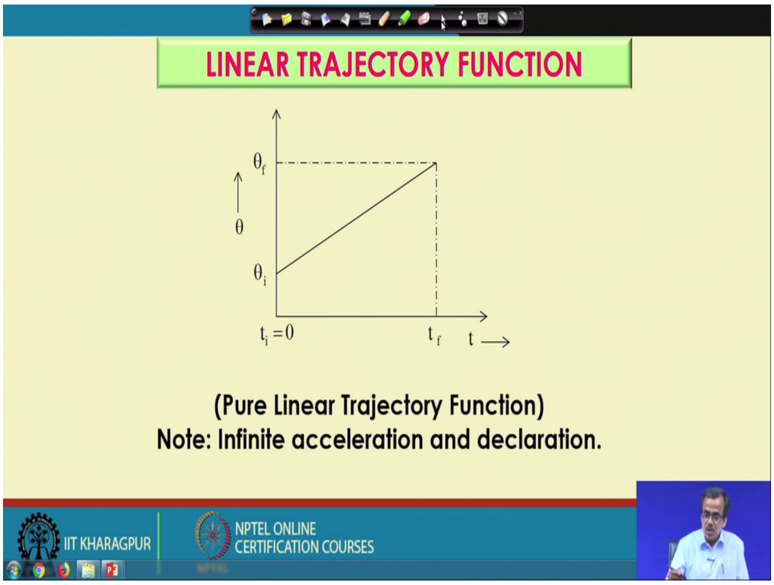

Now, if you see the literature on trajectory planning there is another form of the function, which is also in use that is called the linear trajectory function. Now, here the variation of theta as a function of time as a linear function of time; so, theta is going to vary linearly with this particular the trajectory function.

But, here we should remember although the theta is varying in the linear fashion like this. So, with time the movement of the end-effector cannot be linear, because if you see the expression for the position, that particular position we have got the cos theta, sin theta term. And this particular cos theta, sin theta terms are non-linear. So, the variation of the end-effector with time will become non-linear, of course, the theta is varying linearly.

Now, here as I told that this particular linear trajectory function is also in use. But, we have got a problem in this type of pure linear trajectory function. Now, the problem is as follows for this pure linear trajectory function. So, at the beginning there will be infinite acceleration; and at the end there will be infinite deceleration.

Now, this particular infinite acceleration, and infinite deceleration is not desirable. Now, what will happen, there will be some set of jerky movement initially, and at the end. And due to this jerky movement actually, there will be mechanical vibration, and there could be the failure of the robotic joint also. So, just to overcome this particular problem, actually what we do is, so we try to fit some set of cubic polynomial or not cubic polynomial.

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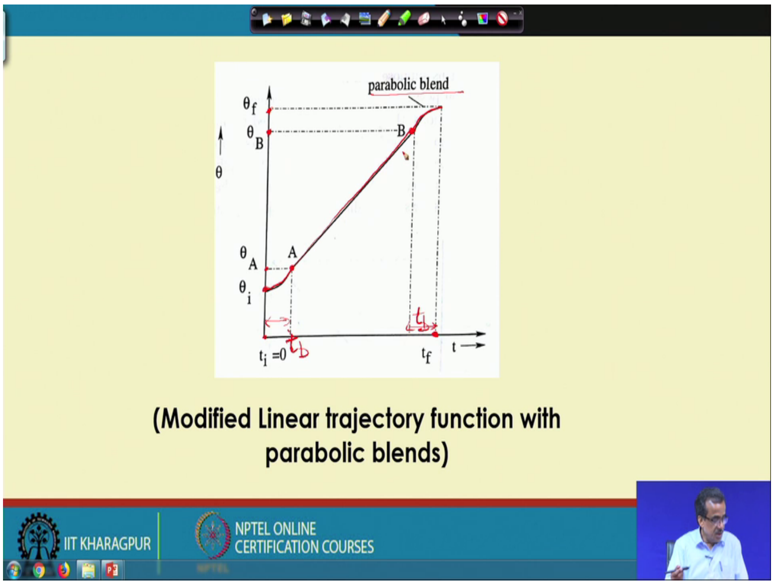

That is your the parabolic blend at the two ends of this particular the linear joint. Now, this is actually the modified linear trajectory function. So, here actually what we can do is, so from this particular point. So, this is theta as a function of time. So, from point A to point B, so this is nothing but a pure linear function. But, at the beginning we put one parabolic blend that is the second order curve, and at the end we put another parabolic blend that is the second order come, is not cubic polynomial this is actually a second order come parabolic blend.

Now, this parabolic blend is put at the beginning at the end, just to avoid that particular infinite acceleration here, and infinite deceleration here; just to avoid that jerky movement at the beginning, and at the end. Now, this particular parabolic blend, actually we put during some duration, and supposing that, that particular duration is denoted by t b. So, this particular time is denoted by say t b.

Similarly, here also we have got the duration, and that is your t b ok. And t f is nothing but the finishing time. And t i is the starting time. So, we are going to start from the 0, in terms of time. And here, at time t equals to t i, the angular displacement is theta i. And at time t equals to t b, the angular displacement is actually theta A.

Similarly, at time t equals to t f, the angular displacement is nothing but theta f. And at time corresponding to this particular point actually the angular displacement is theta B. So, we use this parabolic blend at the two ends as I told, just to avoid that the infinite acceleration and deceleration. And this is the modified linear trajectory function, which is generally used, and we do not use the pure linear trajectory function the reason behind that I have already explained.

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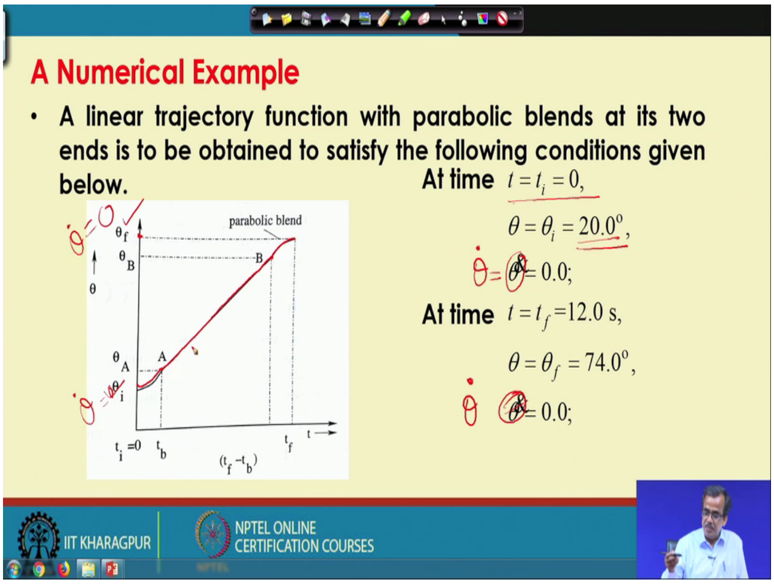

Now, actually what I am going to do is, I am just going to solve one numerical example, based on this type of the linear trajectory function with parabolic blend. Now, here the statement of the problem is as follows. Actually, so this is so from here from A to B once again, so this is the pure linear trajectory function. And we have got the parabolic blend here we have got the parabolic blend here.

Now, here actually what we are going to do. And let us see the condition at time t equals t i equals to 0, so theta equals to theta i theta i that is equals to 20 degree. Then comes your theta dot this is actually theta dot is equals to 0.0. So, here the angular velocity is equals to 0. Then at time t equals to t f t f equals to 12 second, theta is equals to your theta f. So this is given 74 degree, and theta dot that is your, so this is theta dot theta dot equals to 0.0, that means, at the end the angular velocity is equals to 0. So, this is the problem.

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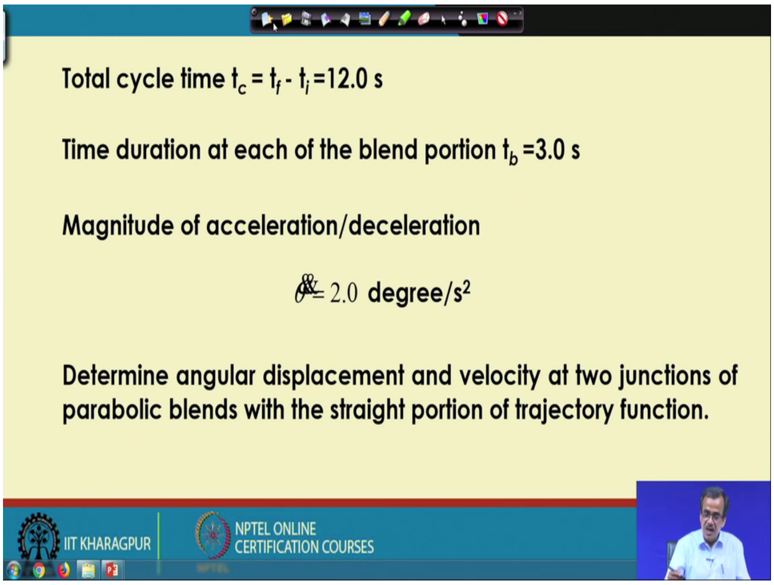

Now, let us see let us see how to how to determine the different parameters for this type of trajectory planning. Now, here actually the total cycle time that is nothing but t c is t f finishing time minus t i is the initial time, and that is nothing but 12.0 second. Then time duration at each of the blend portion that is t b is denote is nothing but 3 second.

And the magnitude of acceleration and deceleration that is nothing but is your theta double dot that is the angular acceleration, and that is equals to 2.0 degree per second square. So, these are all known conditions. Now, what I will have to do is, so I will have to find out the angular displacement. So, determine angular displacement and velocity at the two junctions like A and B of the parabolic blend with straight portion of the trajectory function.

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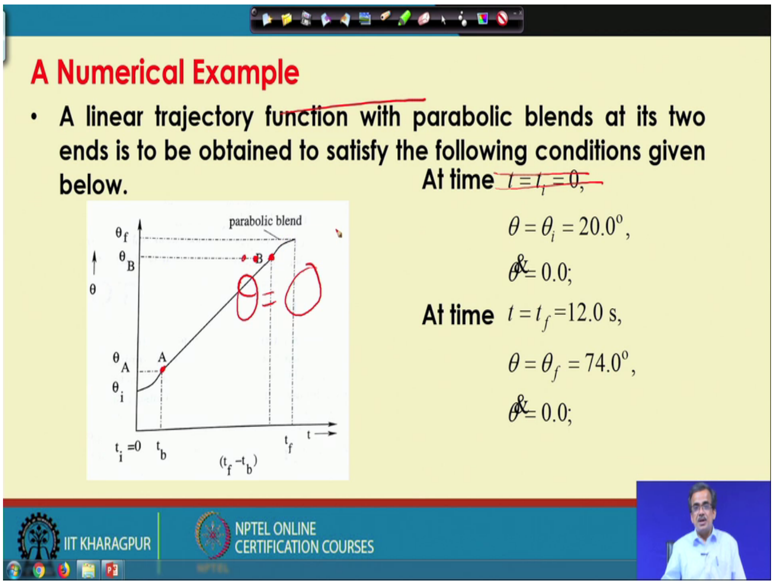

So, if you see that earlier that earlier picture. So, our aim is actually to determine the displacement and velocity at the two points that is actually at this point A and point B. So, I will have to find out the displacement as well as the velocity.

Now, here one thing is ensured, like if I want to keep this particular the continuous trajectory, so here the velocity at the end of the blend portion should be equal to the velocity at the beginning of the linear portion. Similarly, if I want to maintain continuity at point B, the velocity at the end of the linear portion should be equal to the velocity at the beginning of this particular the blend portion. So, those conditions are to be fulfilled.

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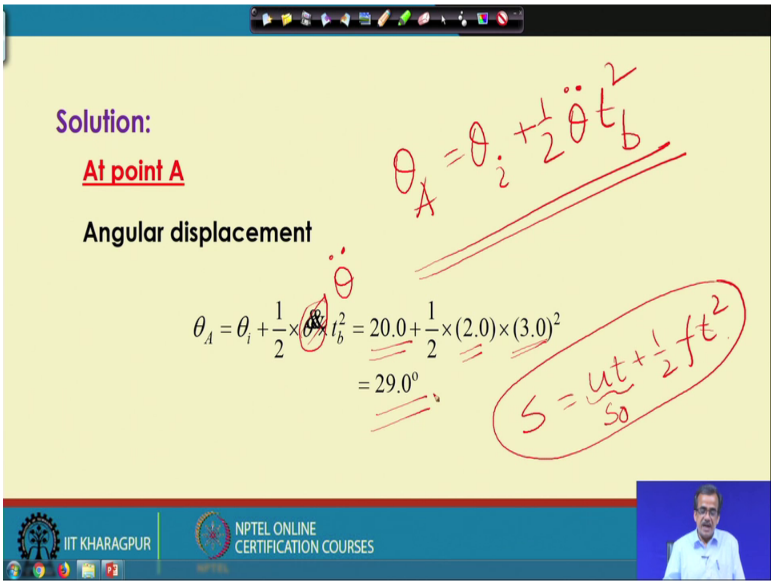

Now, let us see how to find out, and it can be determined very easily. Now, at point A, that is the junction point between the first parabolic blend, and the linear portion. The angular displacement can be determined as follows. Like your this angular displacement is nothing but theta A is nothing but the initial displacement theta i plus half theta double dot that is the angular acceleration multiplied by t b square, that means, here there is one typographical error. So, this will be theta double dot ok.

This is similar to the equation like if you remember in your school level we have studied, that is displacement S is nothing but u t plus half f t square, so this particular very well known formula. The displacement is the initial displacement plus half f t square; f is the acceleration t is the time u t is nothing but S naught, there is the initial displacement.

So, exactly in the same way, so we are writing theta A is the angular displacement is nothing but the initial displacement that is theta i plus half theta double dot is nothing but f angular acceleration, and time is t b square. So, using this particular formula, so very easily you can find out what should be the angular, the displacement at point A, that is theta A is nothing but the initial displacement is 20 half and theta double dot is 2 and t b is nothing but is equal to 3.0. So, you will be getting 29.0 degree. So, this is the way actually we can find out the angular displacement at point A.

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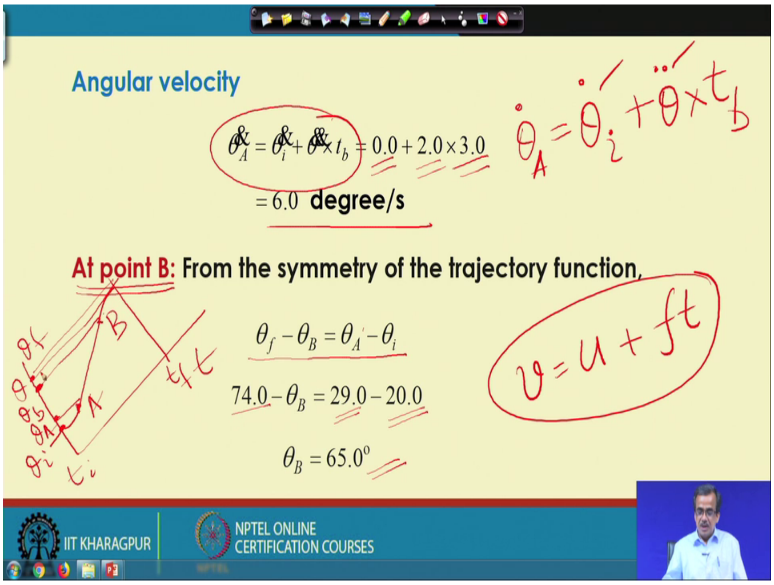

Now, let us see how to find out the velocity that is the angular velocity. And once again I am sorry we have got the typographical error here. So, let me just write down. So, this particular part that is your theta A dot, that means, I will have to find out the angular velocity at A, that is nothing but the angular velocity at initial, that is theta i dot plus your theta double dot is the angular acceleration multiplied by is your t b. And this is similar to the well known formula, that is v is equals to v is equals to u plus f t, so that particular formula ok.

Now, here, so this angular displacement sorry angular velocity is theta A dot is nothing but the initial velocity is theta i dot plus the angular acceleration multiplied by the time that is t b. And if we just put the numerical values theta i dot is equals to 0, theta double dot is equals to 2, t b is equals to 3.0, so this angular velocity will be 6.0 degree per second. So, this is the way actually we can find out the angular displacement, and angular velocity at the point A.

Now, I am just going to concentrate on point B. Now, if you remember the point B is the junction point between the linear the pure linear trajectory, and the second parabolic blend or the last parabolic blend. So, at point B actually I am trying to find out what should be the angular displacement and velocity.

Now, if you see this particular the plot if you remember, so this theta f minus theta B. So, this particular expression theta f minus theta B is nothing but theta A minus theta i. Now, roughly actually if I just plot it here, for example, say if I plot one rough sketch. So, for example, it will be something like this. So, here we have got the parabolic blend, and here also we have got the parabolic blend. Supposing that, so this particular time, so this is t f and this is t i ok.

Now, here actually this theta f is the finishing, and the blend portion starts here. So, this is point B, this is your point A. So, this corresponds to theta b this is theta b, and this is your theta f. So, theta f minus theta B is nothing but theta A minus theta i. So, here we have got actually theta A corresponding to these, and initial theta is nothing but theta i. So, theta f minus theta B is nothing but theta A minus theta i. And here we insert all the numerical values known numerical values like theta f is 74 degrees theta B will I have to find out theta A is 29 and theta i is 20. And if I calculate you will be getting theta B equals to 65. So, theta B can be determined as 65 degree.

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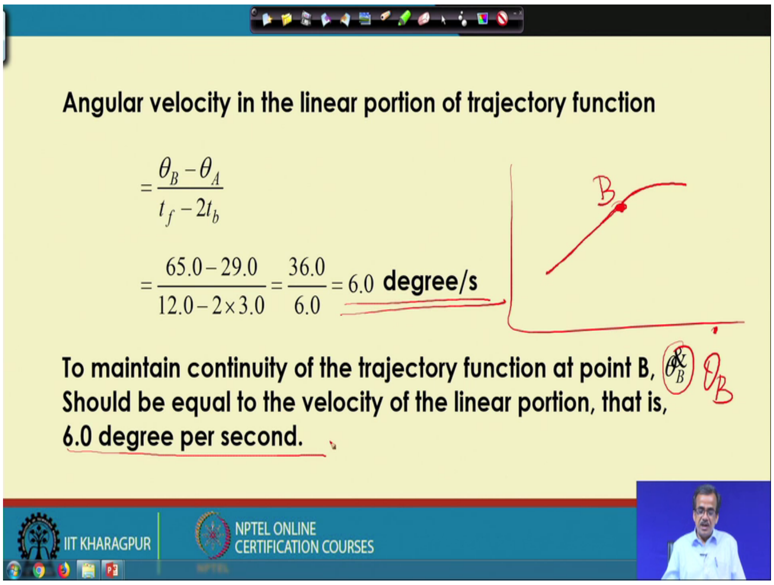

Now, I will try to find out what should be the angular the velocity at point B. Now, to determine the angular velocity of this particular point B; so what I am going to do is, I am trying to find out the angular the velocity in the linear portion. Now, how to find out the velocity at the linear portion? To find out the velocity at the linear portion, let us try to concentrate on theta B minus theta A.

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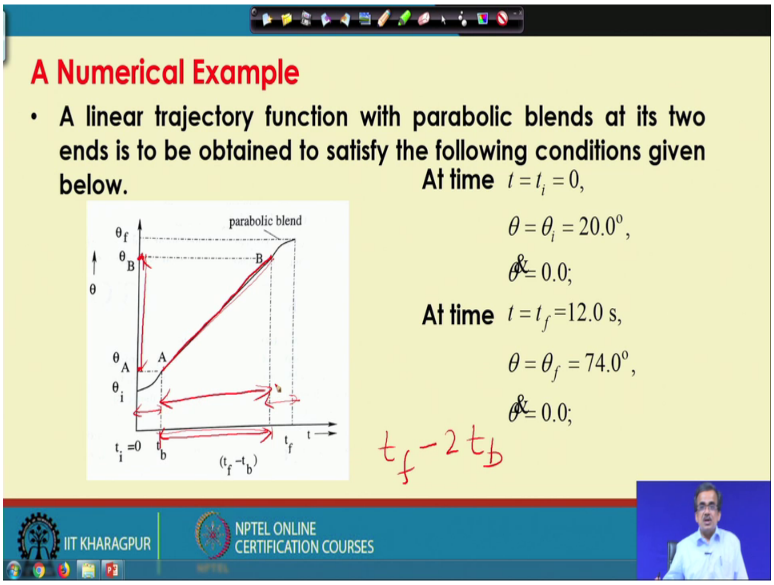

So, if you see this particular figure. So, if you see this particular figure theta B minus theta A that is your this thing theta B minus theta A. So, this is actually the change in angular displacement, and it has happened actually during this particular time. So, this is so this during this particular time, so this particular variation has come. So, if I see this particular thing, so I can find out if this is the change in displacement, and this is the change in time. And what is this change in time, it is very simple. So, this particular part is t b. So, this and this is also t b.

So, the time is actually t f t f is the total time minus 2 into t b is nothing but, so this particular the duration. So, I know the change in displacement, I know the change in time. So, very easily I can find out what should be this \ the linear velocity.

And we try to find out the linear velocity at this particular the portion. So, the at the linear portion, so we have got theta B minus theta A divided by t f minus t 2 t b, and that is nothing but the angular the velocity. And if we just insert the numerical values you will be getting your 6.0 degree per second, and that is nothing but the angular velocity at the end of the linear portion.

And as I men mentioned that at point B, so if this is the linear portion, so this is the point B. So, at point B, so what you will have to do is, the continuity has to be maintained So, the velocity at the end of this linear portion will become equal to the initial velocity of the blend portion, and that is why, the initial velocity of the blend portion that that particular theta dot B is nothing but is equal to 6.0 degrees per second.

So, here I have written here to maintained the continuity the trajectory function at the point B theta B dot this is theta B dot should be equal to the velocity of the linear portion, that is nothing but 6.0 degrees per second. So, this is the way actually we can fit the trajectory are depending on the requirement

Now, here I just want to mention that your. So, till now we have considered the polynomial function like cubic polynomial, and fifth order polynomial but here and the linear trajectory function with parabolic blend. But, here we can fit some other type of non-linear function. For example, some sort of log logarithmic function or some exponential function or sometimes a combination of say logarithmic and exponentieth exponential function, we can use just to find out the trajectory.

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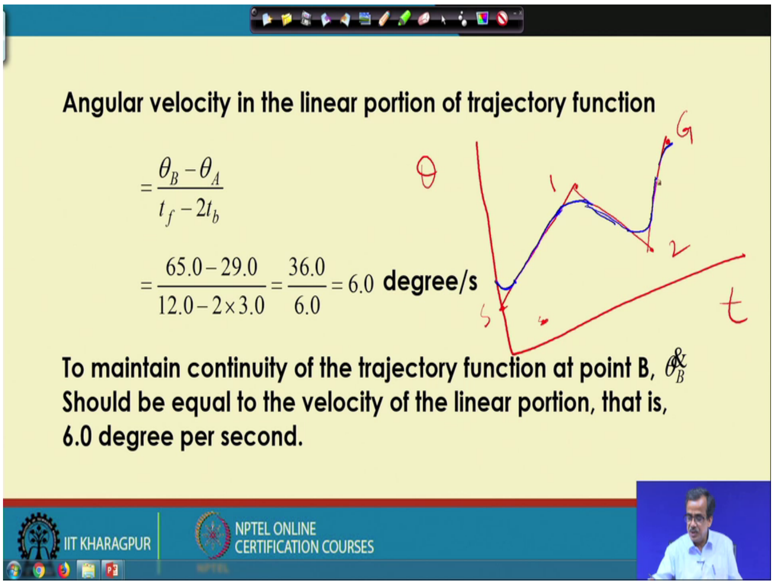

Now, another small thing actually I just want to discuss. Now, supposing that, say I have got one this type of distribution of theta. Let me just prepare one a rough sketch here; so theta as a function of time. Supposing that I have got so this is the initial position of the theta, and say the next point could be say here or let me consider the next point is say here, next point is here, the next point is here, next point is here. So, this could be the goal, and this could be the static ok.

Now, if I just fit one the linear curve. So, you forget about this particular point. So, this is starting point goal; the first intermediate, second intermediate. Now, if I just do it like this, so if I put one pure linear, so it will be something like this. If I put one pure linear, it will be something like this. And another pure linear, it is something like this. But, as I told that the pure linear is not possible, and that is why actually what I will have to do is, I will have to put some parabolic blend.

So, what I can do is, I can put one parabolic blend like this. For example, here I can put one parabolic blend up to this. Here, I can put one parabolic blend up to this so this type of parabolic blends ok. Then here also I can put one parabolic blend something like this, and at the end also I can put some sort of parabolic blend. So, the actual distribution of theta as a function of time will look like this. So, this is actually the actual distribution of theta, if I just use that particular the parabolic blend with linear trajectory function.

So, this is the way actually we try to find out a smooth curve, a smooth trajectory for this particular each of the robotic joint and the purpose of which I have already discussed.

Thank you.