Robotics

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Lecture - 21
Trajectory Planning

Now, I am going to start with a new topic that is topic 3 that is Trajectory Planning. Now, before I start, let me tell you that this particular trajectory planning is not the same with the robot motion planning. The purpose of trajectory planning is to ensure the smooth variation in the robotic joint. On the other hand the purpose of robot motion planning is to make that particular robot intelligent.

Now here, I am just going to consent it on the principle of the trajectory planning only, like how to make that particular trajectory planning so that that operation or the variation in a particular robotic joint becomes smooth. Now, before I proceed further, let me tell you the reason behind going for this particular the trajectory planning. Now, till now, we have discussed the robot kinematics. The purpose of kinematics is to study the motion of the different robotic joint without considering the effect of or without considering the reason behind that particular the motion that is the force or the torque.

Supposing that we have got a linear joint, so we will have to find out the force if I have got the rotary joint, you will have to find out the torque, torque. Now, supposing that I have go to rotary joint, now in that particular rotary joint, so I will have to find out the joint torque and in a particular cycle time, the variation of joint torque has to be gradual and there should not be any such abrupt change in the value of that particular torque.

Now, if you see the expression of joint torque, which I am going to derive after some time, we will see that this particular joint torque has got a few components like the inertia component, coriolis and centrifugal component, gravity terms, friction terms and so on. And if you see the expression, we have got the terms like joint, the angle that is nothing but the angular displacement, angular velocity, angular acceleration. Now, as I told that will have to ensure the smooth variation of this particular joint torque with time; that means, the joint angle should vary in a very smooth way with time and to ensure that, the faster that time derivative of this angular displacement, that is angular velocity and the second order derivative that is angular acceleration are to be continuous.

Now, to ensure that actually, we will have to study this particular the trajectory planning before you start with the dynamics. And that is why actually, we should study this particular dynamics, this trajectory planning before the robot dynamics.

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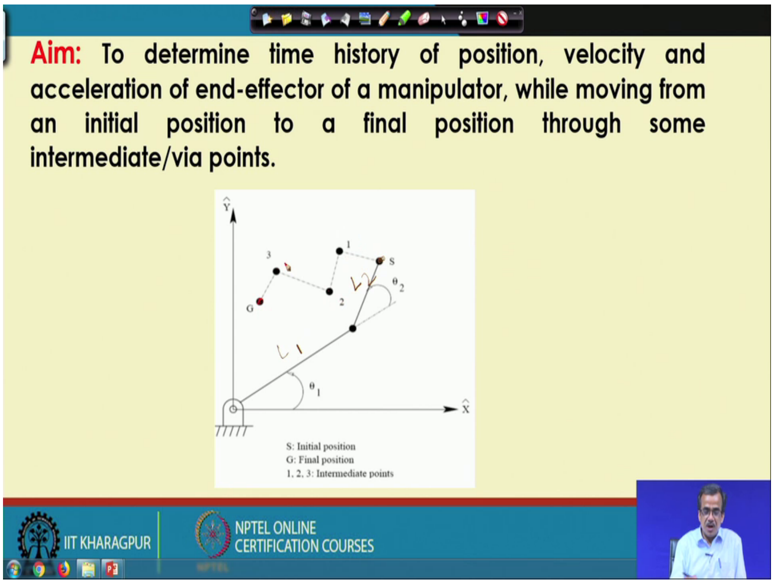

Now, supposing that we have got a serial manipulator like 2 degrees of freedom serial manipulator. So, let me concentrate on, so this is nothing but your link 1. So, this is the link 1, L 1 and the link 2, the length is L 2. Now here, so that the aim is, so this is the starting point of this particular end effector; the end effector starting from here will reach this particular, the final point that is the goal point through a number of intermediate points like 1, 2, 3 and so on.

Now, the purpose of this particular trajectory planning is to determine the time history of position, velocity and acceleration of end effector while moving from the initial position S to the final position G through a number of intermediate points or the higher points. Now, let us see how to ensure the smooth variation.

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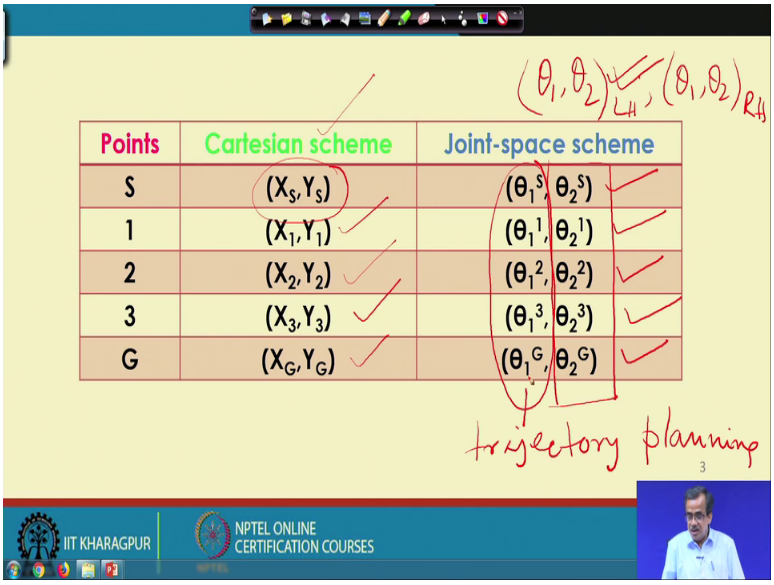

So, corresponding to this particular the point S, so in Cartesian coordinate system like we have got X S, Y S, this particular coordinate and corresponding to this the first intermediate point, we have got X 1, Y 1.

Similarly, second intermediate point X 2, Y 2, third one X 3, Y 3 and the final one that is X G, Y G. Now, we have already discussed inverse kinematics. Now supposing that in Cartesian coordinate system, we know the position of this particular the end effector and if we know the position of the end effector, we can solve the inverse kinematics and we can find out two sets of theta values like your theta 1, theta 2, the left hand solution and we have got the theta 1, theta 2, right hand solution.

Now, here out of this left and right hand solution, so let me consider any one, supposing that I am concentrating on the left hand solution. So, corresponding to these X S, Y S, so I will be getting theta 1, theta 2 S. Similarly, corresponding to X 1, Y 1 in Cartesian, in joint space scheme, I will be getting theta 1, theta 2 1. Corresponding to X 2, Y 2, I will be getting theta 1, theta 2 2, then corresponding to X 3, Y 3, I will be getting X 1 sorry, theta 1, theta 2, 3. Then corresponding to X G, Y G, I have got theta 1, theta 2 G.

Now, as I know that at each of the robotic joint we have got, the motor, the dc motor and this particular angular displacement will have to generate with the help of that particular the motor. Now that is why for the controlling of the motor, so there is no way out but will have to go for the joint space scheme. Now, if I want to ensure the smooth variation of theta during a particular cycle time, so what I will have to do is, I will have to feed a smooth curve passing through your theta 1 S, theta 1 1, theta 1 2, theta 1 3, theta 1 G.

Similarly, I will have to actually find out another smooth curve for the variation, the smooth variation of theta 2. This is what you mean by the trajectory planning. So, in trajectory planning actually what I do is we try to fit a smooth curve for this particular the joint variable. Now, if it is a rotary joint, then of course it will be the joint angle.

So, here let us see how to fit a smooth curve so that I can ensure the smooth variation of theta 1 and theta 2 separately. So, this is actually the thing which I am going to discuss in details.

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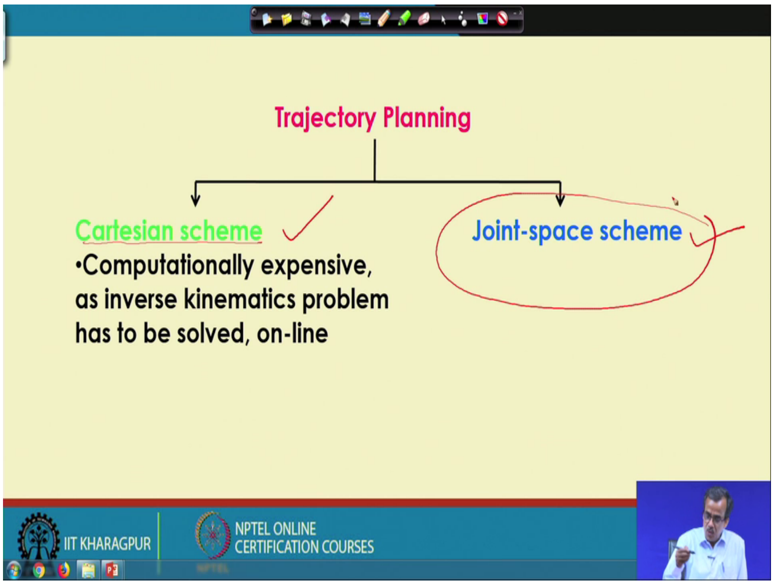

Now, if you see the trajectory planning, the trajectory planning can be done either in Cartesian coordinate system or in joint space scheme. Now, as the motor is controlled in joint space scheme, so if I want to follow the trajectory planning in Cartesian coordinate system. So we will have to carry out the inverse kinematics and we have seen how much complex does this particular inverse kinematics is and it is bit difficult to solve this inverse kinematics problem online. And that is why, the trajectory planning in Cartesian scheme is not so much important but you will have to concentrate more on trajectory planning in joint space scheme because the motor is controlled in the joint space.

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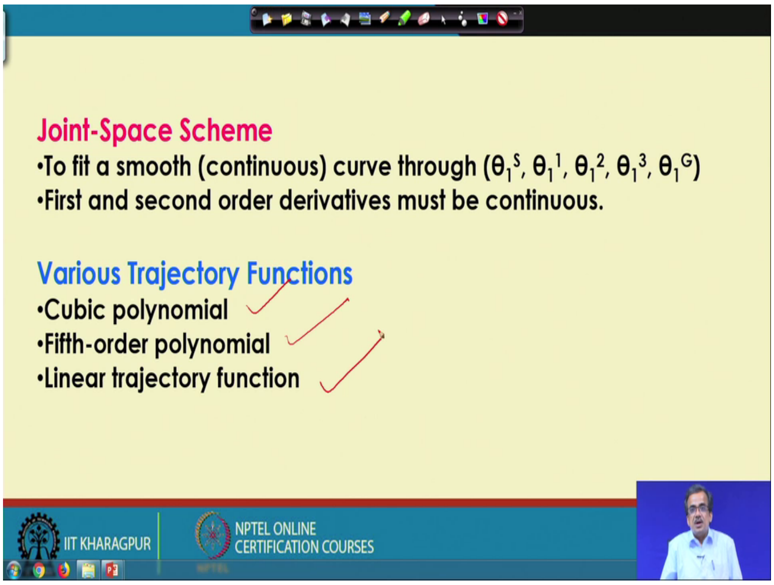

Now, let us see how to make this particular planning for this particular the variation of joint angle in joint space scheme. Now, as I told the purpose of this trajectory planning in joint space scheme is to fit a smooth curve passing through theta 1 S, theta 1 1, theta 1 2, theta 1 3, theta 1 G and to ensure actually, the first order derivative and the second order derivative must be continuous; that means, the first order derivative that is the angular velocity. So, we will have to find out this particular theta dot and this theta double dot is the actually the angular acceleration and they are to be continuous.

Now here, actually what we do is we try to find out like how to how to fit different types of trajectory function. For example, say we are going to concentrate on the cubic polynomial, then comes the fifth order polynomial and the linear trajectory function. Let us see how to fit the cubic polynomial first, then I will move to the fifth order polynomial and we will also see how to fit one linear trajectory to ensure the smooth variation of this particular the joint angle.

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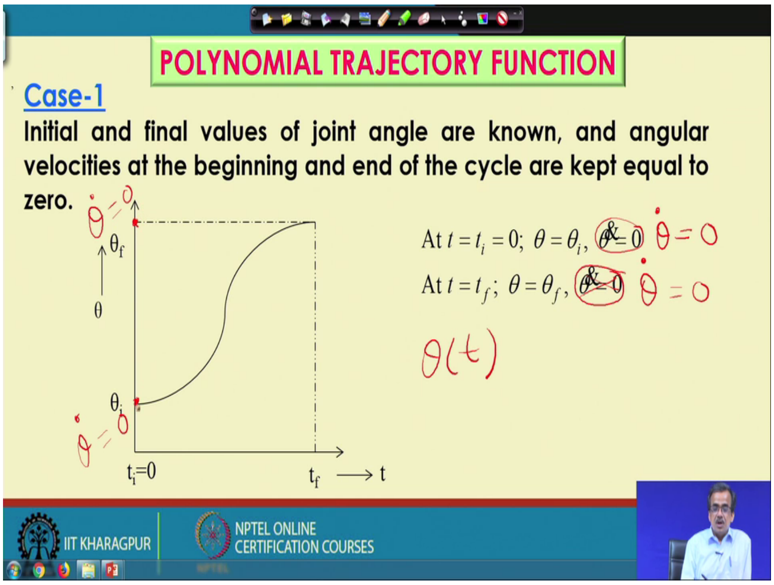

So, let me concentrate it on the polynomial trajectory function first. Now, case 1; now this is a very typical problem where, so, this is the variation of theta that is the joint angle with time. Now what you do is we will have to find out a smooth curve of theta as a function of time. So, theta should be, theta as a function of time and we will have to find out a smooth curve. Now here, the conditions are as follows; a time t equals to t i equals to 0; that means, initially, so theta equals to theta i. So, theta is equals to theta i and theta dot that is actually the angular velocity that is equals to 0. So, here, there is some typographical error. So, truly speaking, it should be theta dot is equal to 0.

So, angular velocity here will be equal to 0, that is a time t equals 2 t i, your theta dot that is the angular velocity that is equals to 0. Similarly, a time t equals t f, t f indicates the finishing time. So, theta is nothing but theta f. So, this is actually the theta f and the angular velocity, that will be your theta dot and that is equals to 0. So, here once again, the theta dot is equals to 0.

So, let me repeat a time t equals to t i initially, theta equals to theta i, angular displacement and theta dot, that is the angular velocity that is equals to 0; a time t equals to t f, theta equals to theta f, the angular displacement and velocity that is theta dot is equals to 0. So, there are four known conditions. So, for these four known condition actually, I can fit one cubic polynomial in the form of, so this. So, let us consider the cubic polynomial like this.

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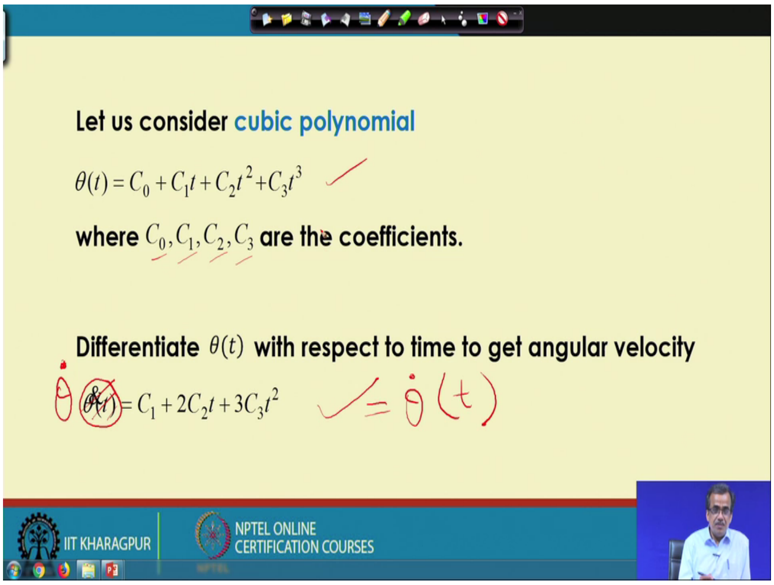

Now, this particular cubic polynomial theta t is nothing but C naught plus C 1 t plus C 2 t square plus C 3 t cube. Here, C naught, C 1, C 2, C 3 are the coefficients. Now, we have got four4 known conditions and those four known conditions will have to utilize just to find out the expressions or the values for this coefficient C naught, C 1, C 2 and C 3.

Now here, actually, what we do is, we differentiate theta t with respect to time that is, we try to find out actually what is theta dot. Now this particular theta dot is nothing but is nothing but is your d theta d t. So, this is theta t. So, if you find out the derivative. So, this here, this continuation will be 0. So, we will be getting C 1 plus 2 C 2 t plus 3 C 3 t square. So, this is nothing but theta dot or the angular, the velocity and truly speaking, so this is also a function of time. So, theta dot t.

So here, we can find out this angular displacement and angular velocity and now I am just going to put all four conditions which I discussed little bit early. So, so what I am going to do is, so I am just going to put those four condition in these two equations.

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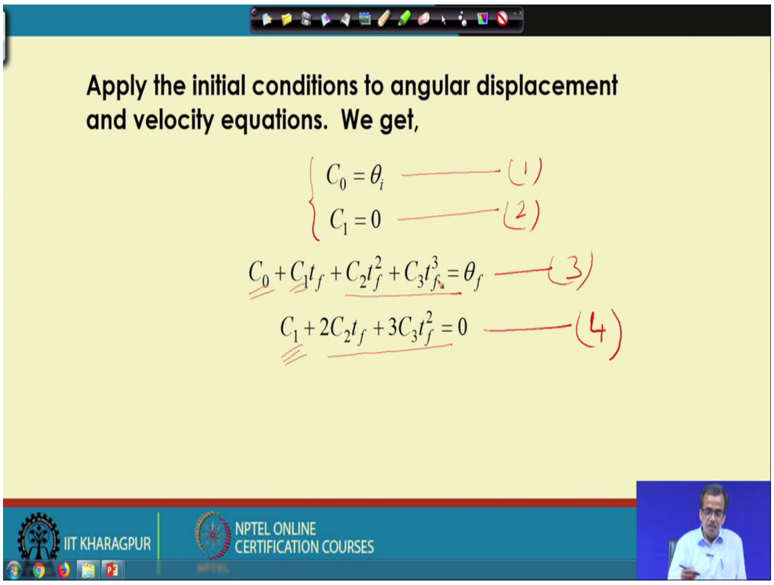

Now, if I just put those equations, so we will be getting this particular the equations; for example, the first equation will be C naught equals to theta i, then the second equation is C 1 is equals to 0, then C naught plus C 1 t f plus C 2 t f square plus C 3 t f cube equals to theta f that will be equation 3. Then comes your C 1 plus 2 C 2 t f plus 3 C 3 t f square is equals to 0. So, this is equation 4.

So, there are four equations and there are four unknowns and out of these four unknowns, two have already determined, that is C naught and C 1. Now, if I substitute the values of C naught and C 1 here similarly, the value of C 1 here. So, I will be getting one equation in terms of C 2, C 3, I will be getting another equation in terms of C 2, C 3.

So, there are two equations and there are two unknowns C 2 and C 3 and those things can be very easily solved.

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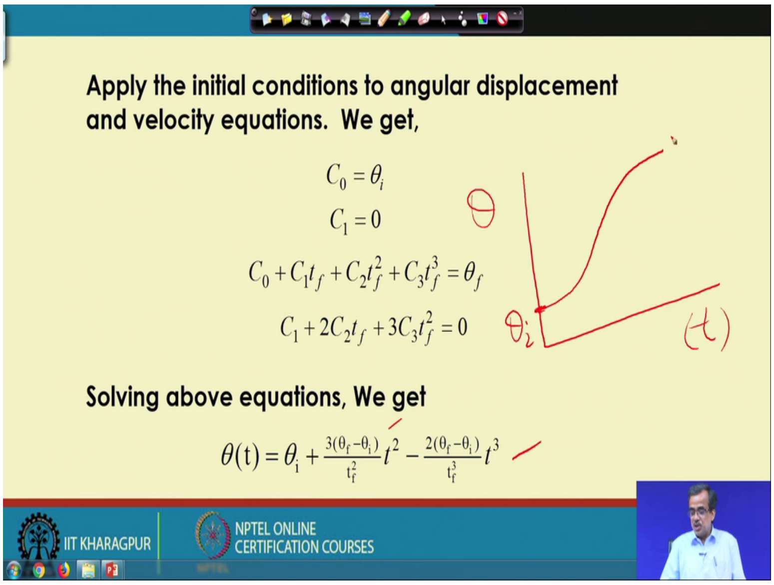

And if I solve it, then I will be getting actually your the final expression and this particular final expression is nothing but theta t that is your theta i plus 3 into theta f minus theta i divided by t f square into t square minus 2 into theta f minus theta i divided by t f cube into t cube. So, this particular theta is actually a function of time.

Now, if I just plot it, so for example, now we can plot theta as a function of time and I can plot this particular curve and this is up to t cube. So, this will be a some sort of cubic polynomial and very easily, I can find out the sweet able plot. Now here, approximately I am just putting just writing the plot but that may not be the correct one; for example, at time t equals to 0. So, these will be theta i and there is a possibility that it will be something like this. So, this type of plot will be getting. So, this is the approximate plot for this particular the function. Now this could be your theta i, the starting is theta i a time t equals to 0 theta is nothing but theta i.

So, I can find out approximately this type of the smooth curve further, the theta i. So, this is the way actually I can use cubic polynomial to find out what should be

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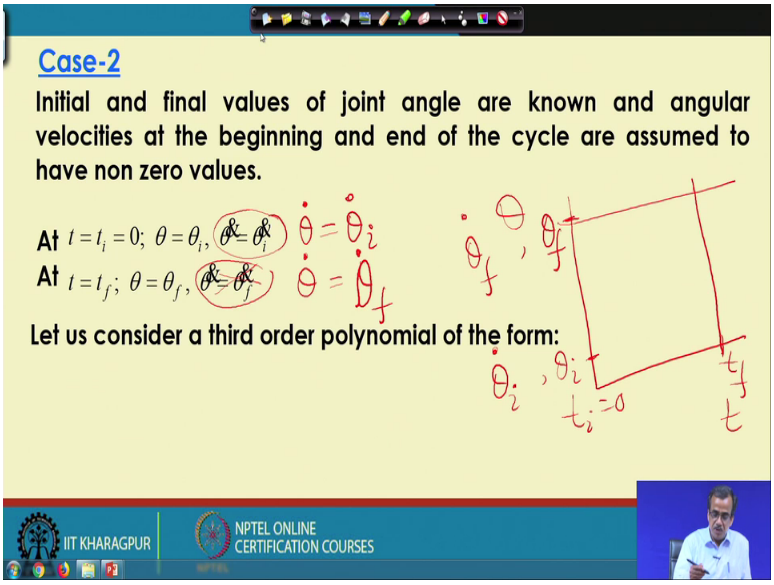

The expression for theta as a function of time so that the variation of theta becomes more; now, I am just going to concentrate on case 2. Now here, actually what I do is, this is slightly different from case 1. So, in case 1, at time t equals to t 1, the velocity was equal to 0 but here it will be non 0.

So, at time t equals to t i, that is t i equals to 0, so if I just plot it here, so at time t equals to t i. So, this is time and this is theta. So, time t i is equals to 0 and t is nothing but t f. So, at time t equals to t i equals to 0, so theta is equals to theta i and here there is typographical error. So, this particular theta dot that is the angular velocity is nothing about theta i dot it is non zero. So here, the angular velocity is nothing but theta i dot.

Similarly, at time t equals to t f, theta equals to theta f, so might be at time t equals to t f theta equals to theta f. So, I have got theta f here and once again there is typographical error ; so, this particular theta dot that is the angular velocity that will be equal to theta that will be equal to theta f dot. So, this is non zero. So, here the angular velocity will be theta f dot. So, using these four conditions, so once again, we can go for some set of cubic polynomial. So, there are four known condition. So, and I can fit one cubic polynomial.

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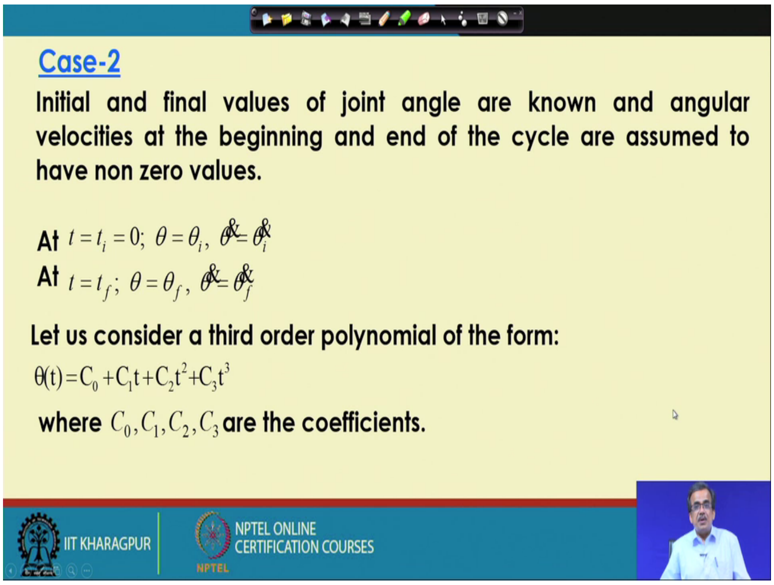

So, let us try to fit one cubic polynomial. Now, if I just fit try to fit the cubic polynomial, so this is nothing but the expression for the cubic polynomial. Theta t is nothing but C naught plus C 1 t plus C 2 t square plus C 3 t cube. Once again, there are four coefficient C naught, C 1, C 2 and C 3 and we will have to find out the values for this C naught, C 1, C 2 and C 3.

Now, by following the same method, actually what you can do is, there are four conditions and we can put the conditions on the equation and we can try to find out like, what should we the equations.

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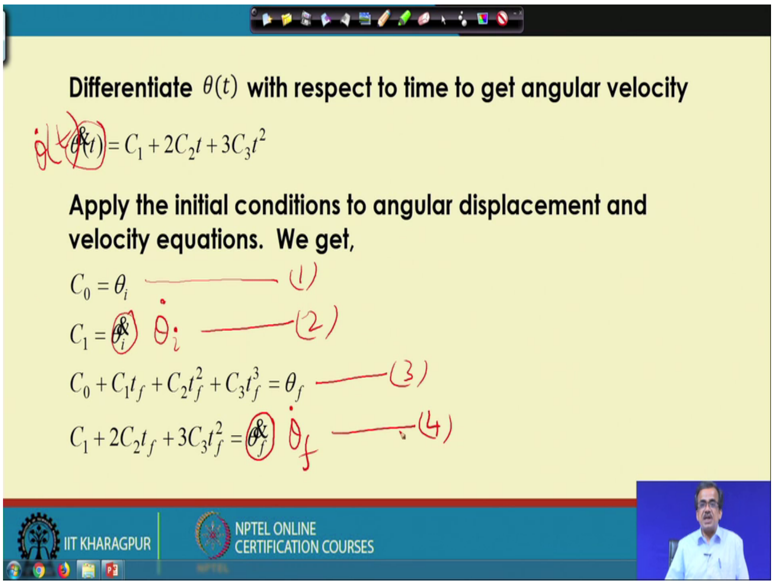

For example, say what you can do is; so, once again we can find out, we can differentiate this particular theta with respect to t and this you will be your theta dot as a function of time. So, that is nothing but C 1 plus 2 C 2 t plus 3 C 3 t square and we apply the four initial condition just to find out the equations. The equations are as follows; C naught equals to theta i. So, this is equation one then C 1 is nothing but is actually theta i dot, that is equation 2. Then, C naught plus C 1 t f C 2 t f square plus C 3 t f cube equals to theta f, that is your equation 3; then comes your C 1 plus 2 C 2 t f plus 3 C 3 t f square and this will be nothing but your theta f dot.

So, this is your equation 4. Now, there are four equations and there are four unknowns. So, these four equations can be solved just to find out the values or the four coefficients. Now, if we just solve those equations.

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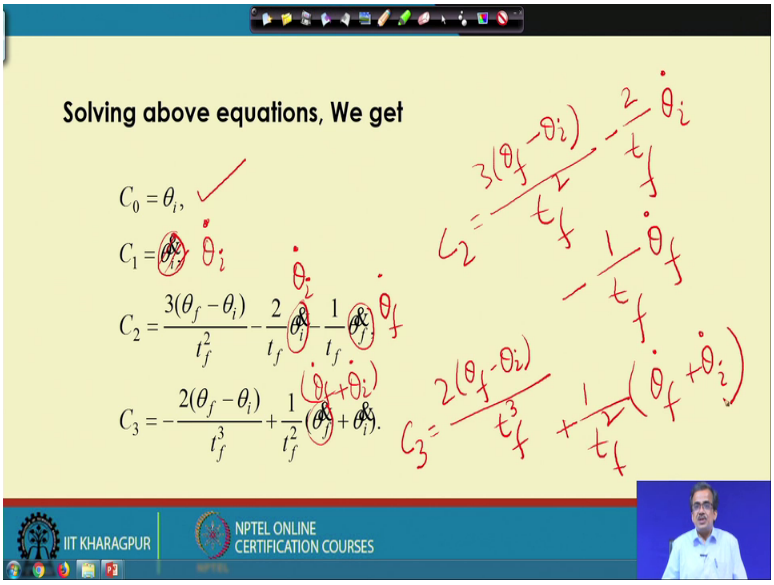

So, we will be getting the terms like this, the coefficients like this. For example, the C naught will become equals to your theta i, then C 1 will become equals to theta i dot. So, I am sorry for this type of graphical error, then C 2 is nothing but 3 into the theta f minus theta i divided by t f square minus 2 divided by t f and this will be your theta i dot minus 1 by t f and this will be nothing but is your theta f dot. Now, if I just a write down, let me write down once again; C 2 will be nothing but 3 minus theta f minus theta i divided by t f square minus 2 divided by t f theta i dot minus 1 divided by t f theta f dot. So, this is the final expression for C 2.

Now, similarly, the C 3 will become equal to 2 into theta f minus theta i divided by t f cube plus 1 divided by t f square plus this should be your theta f dot plus this will be your theta i dot. Now, if I write down once again. So, this C 3 will become equal to your 2 multiplied by theta f minus theta i divided by t f cube plus 1 divided by t f square into theta f dot plus your theta i dot. So, this is the expression which will be getting for C 3.

Now, I will request all of you just to carry out this particular derivation and check whether you are getting the same expression or not. So, this is how to, so find out the cubic polynomial.

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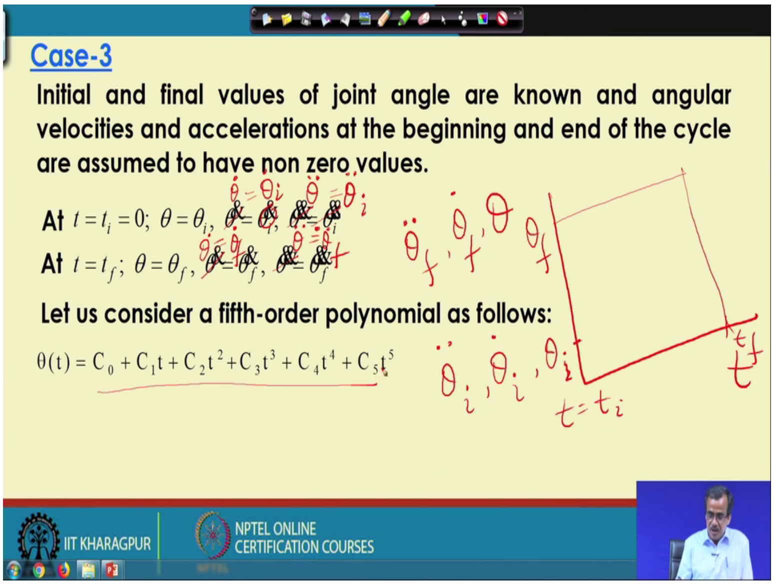

Now, I am just going for case 3. Now the case 3 is more complicated. More complicated in the sense, like we have got more known conditions.

Now, once again the problem is as follows, say I will have to feed one smooth curve for this particular theta. So, theta as a function of time, so at time t equals to t i that is the initial at time t equals 2 t f. So, at time t equals to t i, so I have got the initial displacement as theta i. Then, comes the initial velocity that is nothing but theta i dot and initial acceleration that is nothing but theta i double dot; that means, this will be your, this will be theta dot is equals to theta i dot. Then comes this will be theta double dot that is the angular acceleration is nothing but theta i double dot.

Similarly, at time t equals to t f, so we have got some other conditions like theta equals to theta f. So, at time t equals 2 t f, theta equals to theta f. Then, comes your theta dot theta dot is nothing but theta f dot. Then comes your theta double dot is nothing but theta f double dot that is the angular acceleration. So, if I just make the correction, so this particular thing theta dot will become equal to theta f dot and your theta double dot is nothing but is your theta f double dot ok.

So, at time t equals 2 t i, we have got three conditions like angular displacement, velocity and angular acceleration. At time t equals to t f, we have got three conditions like your theta equals to theta f, angular velocity is theta f dot and angular acceleration is theta f double dot. So, there are six conditions. So, we can fit actually the fifth order polynomial like theta t is nothing but C naught plus C 1 t plus C 2 t square plus C 3 t cube plus C 4 t raised to the power 4 plus C 5 t raised to the power 5.

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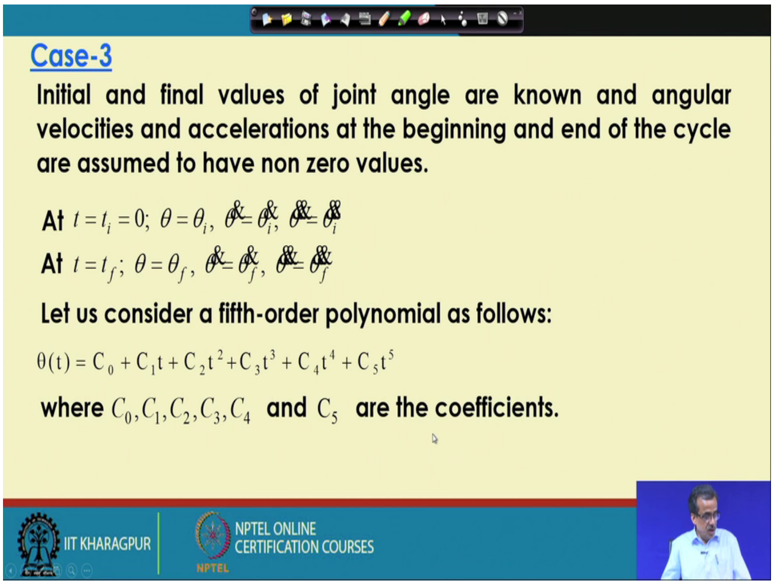

Now, here actually, what I can do is, so we can put those conditions and just to find out all such coefficient.

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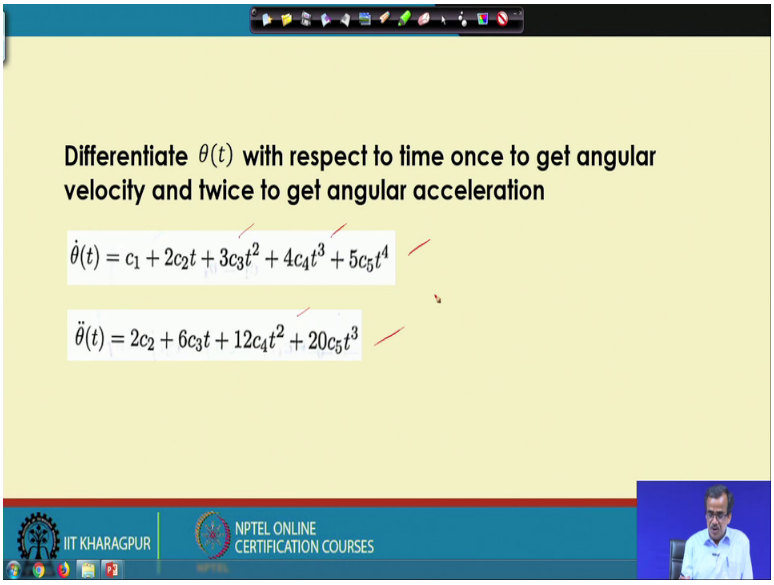

So, there are six coefficients and there are six known condition. So, I will be getting actually the equations. Now here, so theta t we have seen, now if I find out the derivative, the first order derivative, so this is nothing but the first order derivative theta dot t is C 1 plus 2 C 2 t plus 3 C 3 t square plus 4 C 4 t cube plus 5 C 5 t raised to the power 4, then theta double dot t that is the angular acceleration is 2 C 2 plus 6 e 3 t plus 12 C 4 t square plus 25 C 5 t cube.

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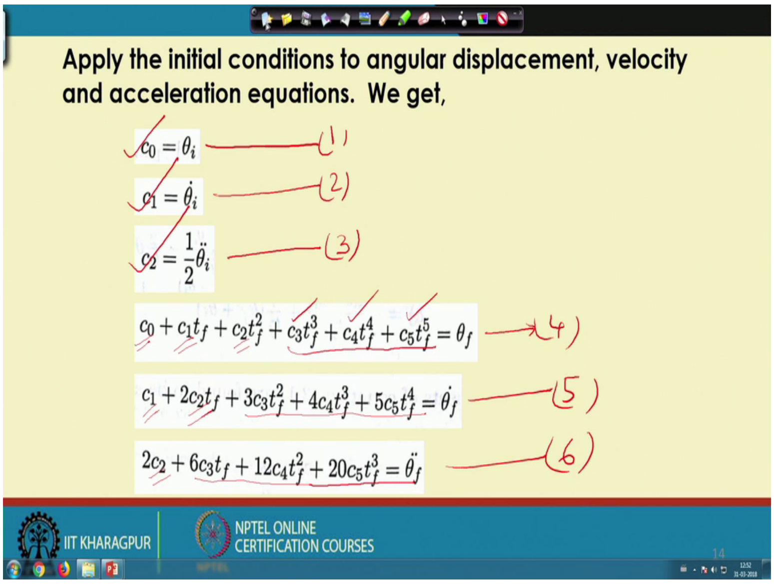

Now here, if I just put all such conditions now, so I will be getting the equations like this and I will be getting these six equations; for example, say C naught equals to theta i. So, this is equation 1. C 1 equals to theta i dot is equation 2 and C 2 is equals to half theta i double dot is equation 3. Similarly, I will be getting equation 4 here, then comes equation 5 here and equation 6 here. Now, out of these 6 unknowns, 3 we have already got like C naught, C 1 and C 2. Now, if I put in this particular equation like C naught, C 1 and C 2, similarly if I put C 1 and C 2 here, so I will be getting another equation in terms of C 3, C 4, C 5 and here also I will be getting C 3, C 4, C 5.

Now, the same thing you do here, you put this expression of C 2. So, you will be getting another equation in terms of C 3, C 4 and C 5. So, there are three unknowns C 3, C 4 and C 5, C 3 C 4 and C 5 and there will be three derived equation. So, there are three unknowns and three equations. So, those can be solved and if you solve it then finally, we will be getting the expression like this.

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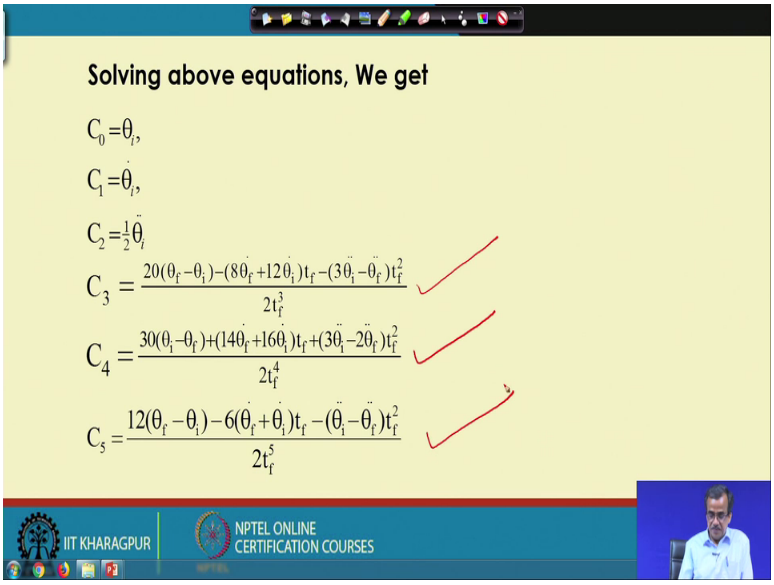

So, your C naught equals to theta i C 1 is theta i dot C 2 is actually your theta half theta i double dot, then comes your C 3 will be getting this particular big expression involving your theta f, theta i, theta f dot, theta i dot, theta i double dot, theta f double dot ok. So, I will be getting this big expression for C 3. Similarly, I will be getting another big expression for C 4 and another big expression for C 5.

So, this is the way actually we can utilize the known condition just to find out the smooth variation of this particular theta, that is theta as a function of time and this is actually the purpose of trajectory planning.

Thank you.