Robotics
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Lecture - 20
Robot Kinematics (Contd.)

So, we have seen how to solve the forward kinematics problem and inverse kinematics problem for serial manipulator. To gain confidence, like how to assign the coordinate system, how to carry out the forward kinematics, how to carry out the inverse kinematics; in fact, we need more some example, more practice and that is why actually I have taken, I am just going to take two more examples just to explain it again like how to assign this particular the coordinate system because this is very important to assign the coordinate system correctly.

Now, if you can correctly assign the coordinate system, then we will be able to find out the kinematics equation correctly and once the kinematics equations are made correct, then only we can find out the correct dynamic equation and control is based on the dynamic equation. So, the accuracy lies with actually, how to assign the coordinate system correctly and that is why let us spend some more time on how to assign the coordinate system. And let us try to spend some more time and let us try to solve some very simple example.

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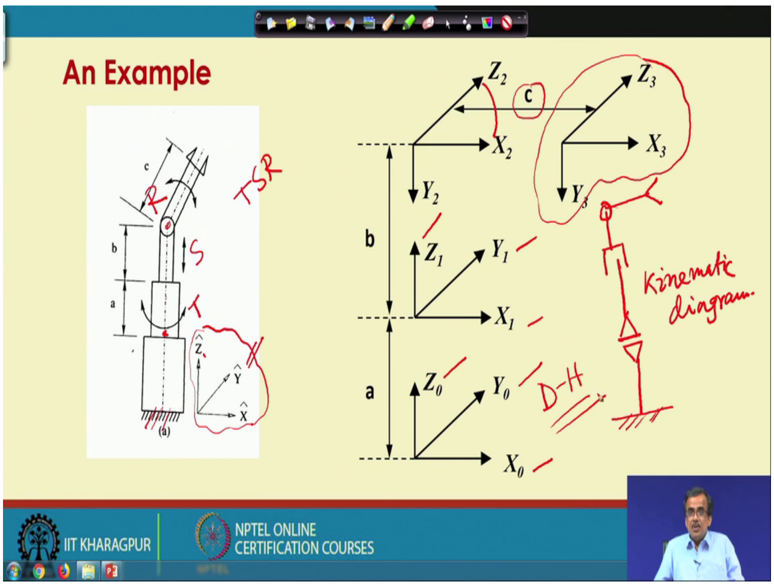

Now, this is one example; example of one manipulator, the serial manipulator having only 3 degrees of freedom; for example, say this is the fixed base. So, here, I have got one twisting joint say T and I have got one sliding joint that is S and I have got a revolute joint, that is R. So, this is nothing but a T, S, R manipulator and it is this kinematic diagram very easily we can draw the kinematic diagram. For example, say this is the fixed base. So, I have got the twisting joint here then I have got one sliding joint.

Now, this particular sliding joint is denoted by this particular symbol, then we have got one revolute joint and after that there is one end effector. So, this is nothing but the kinematic diagram, kinematic diagram for this particular the manipulator ok. Now, let us try to assign the coordinate system once again. Now, to assign the coordinate system as I told, the first thing, we will have to do is we will have to concentrate on the reference coordinate system. So, this is Z, X and Z cross X is Y and the first joint, so this particular joint is the twisting joint; that means, this will be rotated and Z will be along this particular direction. So, you select this particular as Z naught and this particular X naught, you try to take in the same direction of the reference. So, this is your X naught and Y is nothing but Z cross X. So, this is your Y naught ok.

Now, you see here, we have got one sliding joint. So, this particular with the help of the sliding joint, so it can move up and down and this is a linear joint and this particular linear joint, it will move along this. So, my Z 1 will remain same as in the direction of Z naught. So, this is my Z 1 direction and here, so, what you can do is, so they are on the same line. So, Z 1 and Z 2, they are on the same line and along this I take this particular X 1.

Now, I take along this as X 1; so that I can show this particular dimension because they are on the same line and if I do not show X 1 here, so I will not be able to capture this particular the dimension. So, let me take X 1 here and if it is X 1, then Y is Z cross X. Then comes your this revolute joint. Now, for this revolute joint, so Z will be different from this particular Z. So, let me take Z to along this particular direction; that means this is the Z 2 direction. So, this is the Z 2 direction ok.

Now, here, so, this particular Z and that particular Z, they are intersecting. So, to select this particular X, so I have got in fact, both the options but I selected this particular as X 2; so that I can capture this particular dimension because the this particular coordinate system I am just going to copy it here and show that the mutual perpendicular distance to the two Z, that will be your the length of the link. That means we will be able to capture this particular the C. That is why, instead of in this particular direction, so I have selected X 2 along this.

Now, if this is Z 2, this is X 2, then Z cross X, this will be Y 2 and as I discussed earlier, the same thing will be copied here and this is actually the hypothetical joint. So, Z 3 will be this, X 3 will be this, Y 3 will be this. So, this is the way actually we will have to assign so this particular the coordinate system. Now, once you assign this particular coordinate system, according to the D-H parameter setting rule, now what you can do is.

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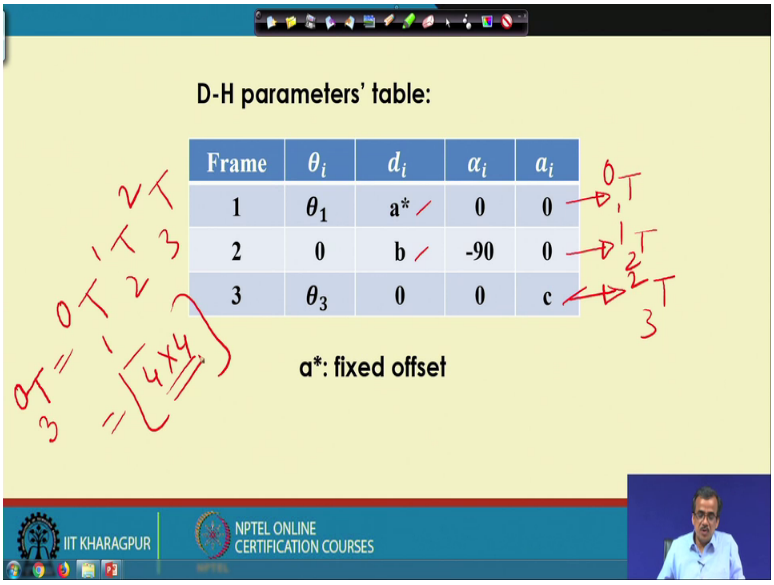

We will have to prepare the D-H parameter table and to prepare the D-H parameter table, so what we can do is. So, once again the D-H parameter table frame 1, 2, 3, the same sequence like theta, theta i, d i, alpha i, a i. The first one, the first joint is actually a twisting joint. So, definitely the variable will be your theta 1, then comes here d.

Now, we see the d is the distance between two X. So, this is X naught, this is X 1. So, this is the distance between two X and it is measured along Z ok. So, this particular a is nothing but an offset, but this offset is a fixed offset and that is why, this is shown as a star. So, this particular a star is the fixed offset. This is not the variable offset ok. The next is your alpha. So, alpha is the angle between two Z. So, the two Zs are lying on the same line. So, the angle between them is 0, then comes your a i; a i is the distance between two Z measured along X. The two Zs are on the same line. So, here, so this particular alpha is 0.

Now, next is your 2; that is 2 with respect to 1; so, 2 with respect to 1 ok. Now here, so this particular joint; for example, this particular joint is the sliding joint. So, the variable will be the offset. So, offset will be the variable and here the theta, that that is your angle that is your joint angle that will become equal to 0 because this is a sliding joint; not a rotary one. So, here, so this variable offset that is nothing but b and that is the distance between X 1 and X 2 measured along Z 1. So, this will be the variable offset and theta is equals to 0. Now, if you see this particular table, so, in place of theta i, I put 0; in place of d i, I put b.

Now, this particular b is actually the variable offset. Then comes your alpha. So here, so this is Z 1 and this is Z 2 ok. Now, if I draw the Z 2 here, if I draw the Z 2 here, say this is Z 2, if I draw then Z 1 to Z 2, so Z 1 to Z 2. So, this is clockwise. So, this will be your minus 90 and a i is nothing but the distance between two Z but they are intersecting. So, the distance between two Z is 0. The next is your 3 with respect to 2. Now here, so, this particular joint is actually a revolute joint. So, definitely the variable will be theta 3. Now, then comes here b; b is the distance between two X. The 2 X are lying on the same line. They are collinear. So, the distance will be 0, then comes your angle alpha and alpha is the angle between 2 Z, they are parallel. So, we will be getting 0 here, then comes a i.

Now, this a i is actually, the distance a i is actually the distance between 2 Z measured along this particular the X and here we will be getting c. And we can see that all the dimensions like a, b and c, we are able to capture a, b and c ok. And now, once you have got this particular thing, the way I discussed very easily you can find out the forward kinematics. And once you have got the forward kinematics equation for the known matrix, for the known position and orientation, you can carry out the inverse kinematics.

Now, as I told it is now, it is very easy like how to find out the forward kinematics solution like your T 3 with respect to 0. So, this is nothing but T 1 with respect to 0, then comes T 2 with respect to 1, T 3 with respect to 2. Now, T 1 with respect to 0. So, this will give actually T 1 with respect to 0, this will give T 2 with respect to 0, this will give your T 3 with respect to 0 with respect to 2. And if you multiply, then we will be getting this particular final the 4 cross 4 matrix which will carry position and orientation information.

So, this is the forward kinematics and by following the method, actually you can carry out the investment of kinematics the way I discussed in the last two problems. You can follow the similar type of approach to carry out the inverse kinematics. So, this is one problem. Now, just to gain more confidence like; let us try to look into another similar type of problem but slightly different.

Now here, I am just going to consider another serial manipulator having 3 degrees of freedom. So, this is nothing but the 3 degrees of freedom 3 degrees of freedom, serial manipulator.

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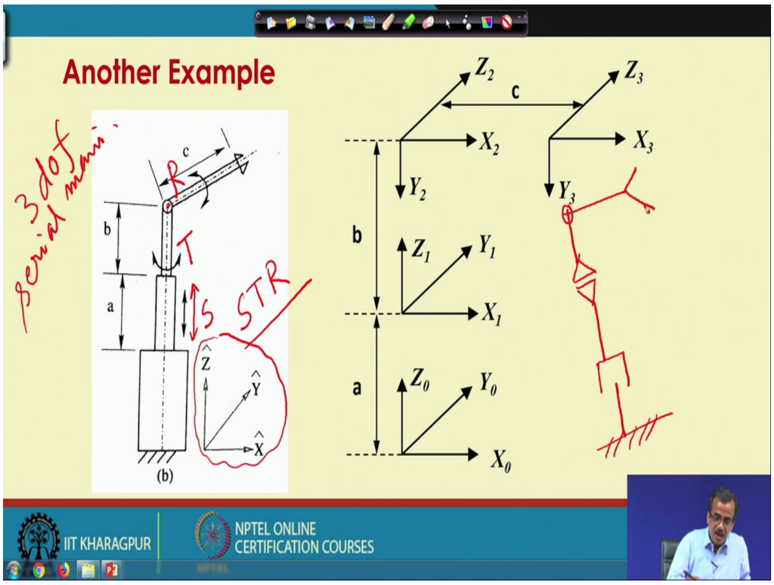

Now here, we have got the sliding joint first. So, this is S, then we have got the twisting joint that is T and here we have got the revolute joint that is R and this is nothing but actually S, T, R manipulator. S, T, R manipulator and following the same method first you look into that particular the reference coordinate system and before that we can also find out very easily the kinematic diagram; for example, say first we have got the sliding joint. So, this is actually the sliding joint, then we have got the twisting joint and after that, we have got the revolute joint and then we have got the end effector. So, this is nothing but the kinematic diagram.

And once you have got this, now you are in a position to in fact draw this particular the coordinate system at the different joints. Now this is the twisting joint, sorry this is a sliding joint. So, Z will be along this. So, I have taken Z naught along this particular direction; X naught is along my base X, Y naught is along my base Y. The next is your this particular twisting joint. Now for this twisting joint, once again, so Z 1 will be in the same line of Z naught and let me put this as X 1 and this as Y 1 so that I can capture; so this particular a value, this particular a dimension ok.

The next is your, next joint is the revolute joint. So, this is nothing but your Z 2 and once again the way I discuss to capture this particular c information. Now here, this particular Z and that particular Z, they are intersecting. So, I have got both the options as X 2 but I am going to take this as X 2, so that I can capture this particular the c. It is very simple very similar almost but slightly different. So, using this actually, now we can found the D-H parameters table. For the first one, the first joint is actually a sliding joint. So here, theta is equals to 0 and here the variable is nothing but is your a. So, a is nothing but the variable. This is the variable offset. So, a is the variable offset, alpha is nothing but 0, alpha is the angle between two Z and that is 0. Then comes a I, so a i is the distance between your two Z, they are on the same line. So, here a is equal to 0. So, we can find out this.

The next is your 2 that is 2 with respect to 1. Now, 2 with respect to 1; this is a twisting joint. So, theta 2 is the variable and then there is another offset. So, this is X 1, this is X 2, the distance between X 1 and X 2 measured along Z 1. So, this particular b is the offset but this is actually the fixed offset that is why we have used b star. Then comes your alpha; alpha is nothing but the angle between two Z. So, what we do is, here we draw this particular Z 2, the way I did earlier. Then, Z 1 to Z 2 is in clockwise sense ok. So, very easily actually, we can find out. So, this is minus 90 and a i is the distance between two Z they are intersecting. So, this is equal to 0.

Similarly, you can find out this particular thing also because this joint is the rotary joint that is a revolute joint. So, theta 3 will be the variable and d i, the distance between two x, they are on the same line. So, this is 0 and alpha, they are parallel. So, alpha is 0. Then comes a, that is the distance between two Z and that is nothing but is your c and your, so this is the c. So, you can prepare this particular the D-H parameter table and as I told once you have got this particular D-H parameter table correctly, you can carry out the forward kinematics and if you know the final position and orientation, so very easily you can find out the inverse kinematic solution but here if I solve the inverse kinematic solution, we will have to solve for theta 2, theta 3 and this particular the variable a ok.

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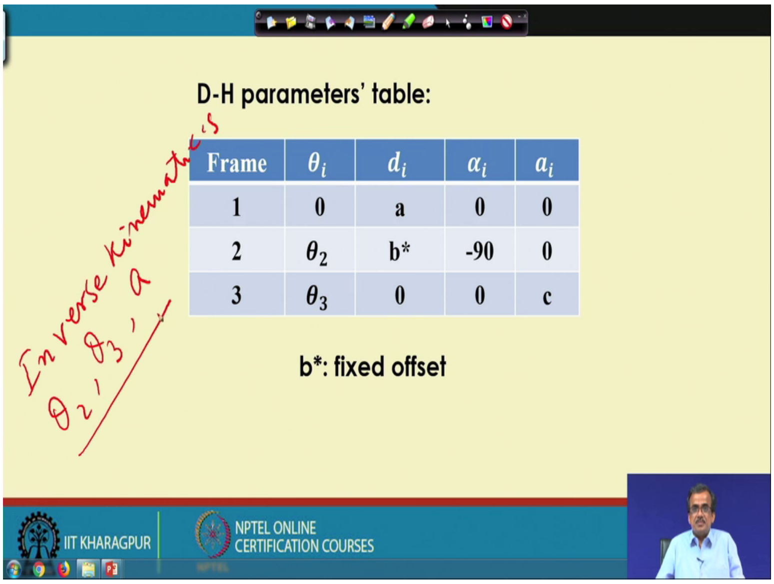

So, if I want to solve the inverse kinematics. So, in inverse kinematics here, inverse kinematics, the variables which we are going to find out are your theta 2, theta 3 and a. So, these are actually the variables whose values are to be determined ok. So, this is the way actually we can carry out your the forward and inverse kinematics.

Now, once you have done this kinematics, we know the movement of the different links, the movement of the different joints, but till now we did not consider the reason behind this particular movement that there could be a force, there could be movement, there could we talk. So, those things we did not consider in the kinematics and those things we are going to consider in the dynamics. But, before I go for this particular dynamics, we will have to discuss another thing another very fundamental thing that is called the trajectory planning that I am going to discuss next.

Thank you.