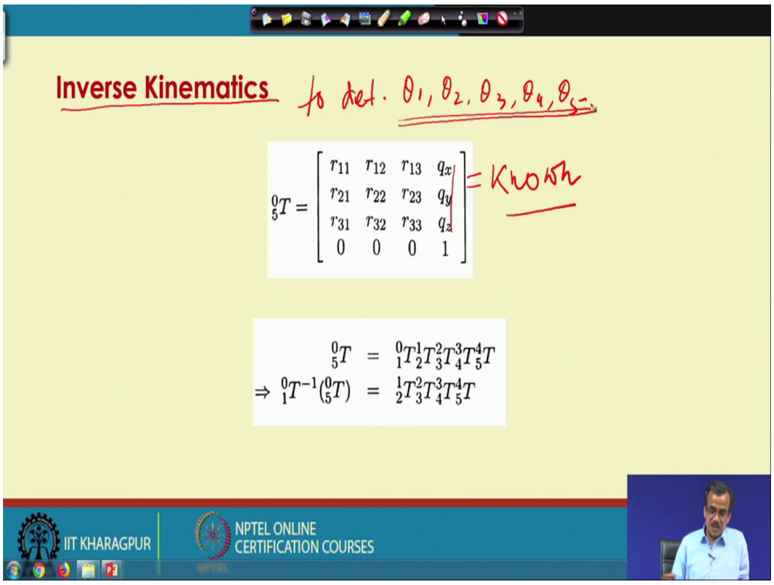
Robotics
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 19
Robot Kinematics (Contd.)

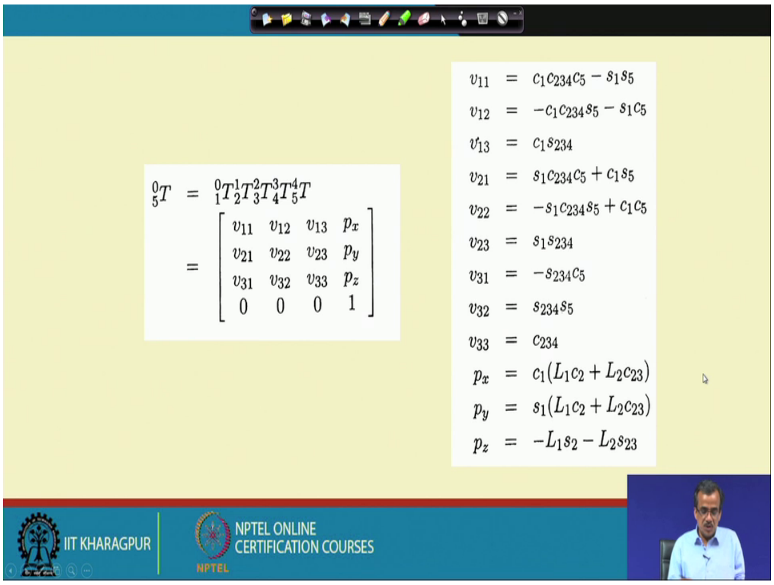
(Refer Slide Time: 00:17)



Now, let us start with the inverse kinematics of this particular the manipulator that is the mini mover. Once again the purpose is to determine the values of the joint angles like your theta 1, theta 2, theta 3, theta 4 and theta 5 provided the position and orientation of the end effector with respect to the base coordinate system those are known.

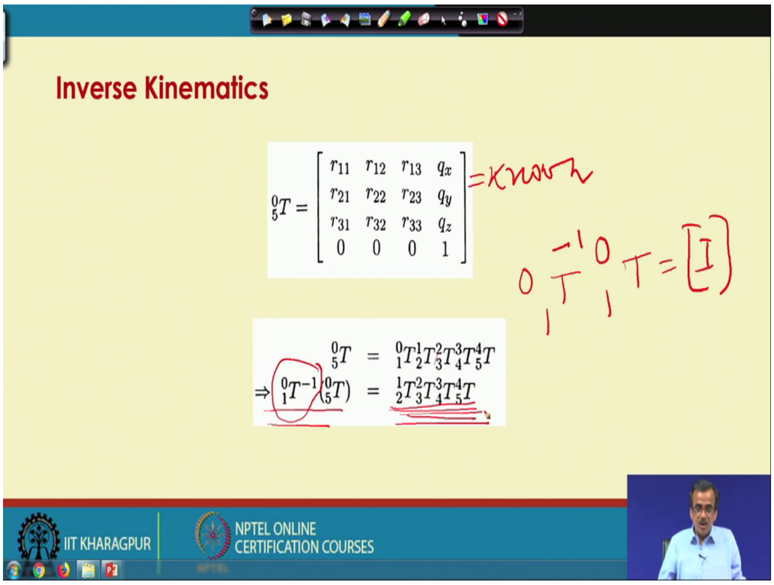
Now, here so this particular T 5 with respect to 0, so this is nothing but r 1 1, r 1 2, 1 3 so this is the. So, these three cross three the rotation of the orientation information. And q x, q y, q z is nothing but the position information. Now, supposing that this particular matrix is given. So, this is known ok. And our aim is to determine your so this particular theta 1, theta 2, theta 3, theta 4, theta 5. Now, here actually what you can do is so in the forward kinematics calculation like whatever we have already discussed.

(Refer Slide Time: 01:30)



For example, so this particular we have we have already got the final matrix and each of these element of this particular final matrix is having a very big expression. Now, here, so this particular calculated matrix that is T 5 with respect to 0.

(Refer Slide Time: 01:51)



Now, if I equate to the known matrix, so this particular known matrix, so I will be able to find out some equations. And we have got five unknowns. Now, for the five unknowns if you want to solve we need at least five equations, but here in fact we will have to use more than five equations otherwise we will not be getting all five joint angle values.

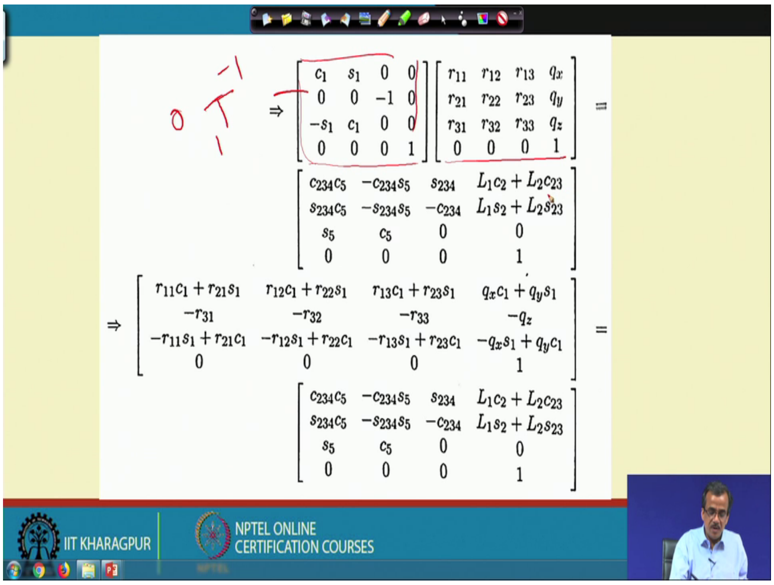
Now, if I just equate element wise these two matrices like the as I told the matrix which you have got through forward kinematics calculation and this particular known matrix. So, if I just equate element wise, we will be getting some equation, but it could be difficult to solve those equation to find out the values of the theta 1, theta 2 up to theta 5. And that is why actually we are going to use a particular tricks a particular method like to simplify so that we can get the equation in such a form that can be solved easily.

Now, here actually what we do is. So, from forward kinematics we know that T 5 with respect to 0 is nothing but T 1 with respect to 0 multiplied by T 2 with respect to 1, T 3 with respect to 2, T 4 with respect to 3, T 5 with respect to 4. Now, what I do is both the sides we multiply by T 1 with respect to 0 inverse. Now, if you remember we have already determined that particular matrix that is T 1 with respect to 0 inverse. So, both the sides both the left hand side and the right hand side so what we do is we try to we multiply by T 1 with respect to 0 inverse.

Now, if I multiply here like T 1 with respect to 0 inverse with your T 1 with respect to 0. So, I will be getting the identity matrix that is I ok. So, this particular identity matrix, I will be getting and that is why left hand on this particular side we will have T 2 with respect to 1, T 3 with respect 2, T 4 with respect to 3, T 5 with respect to 4, and we will be getting this particular expression.

Now, what I am going to do is I am just going to find out, so the final expression for this side and the final expression for this particular the side. Let us see how to find out those expression, and then we are going to equate just to find out like what should be the two sides of this particular the equation, and then element wise we are going to equate.

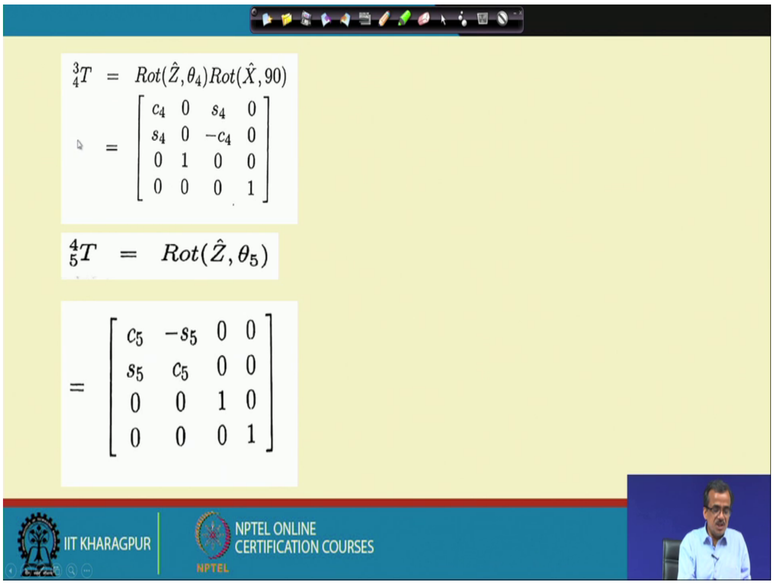
(Refer Slide Time: 04:49)



Now, left hand side if you see so what will can see this is nothing but is your T 1. So, this particular matrix is T 1 with respect to 0 inverse if you remember so this is actually the matrix. And this is the known matrix that are 1 1 1 2 1 3 q x. So, this is nothing but the known matrix.

So, these two matrices now we are going to multiply. And on the right hand side, so what we are going to write is your so on the right hand side if you see so we have got these T 2 with respect to 1, T 3 with respect to 2, and then comes your T 4 with respect to 3, T 5 with respect to 4. And each of these particular expression we have already seen while carrying out that forward kinematics calculation.

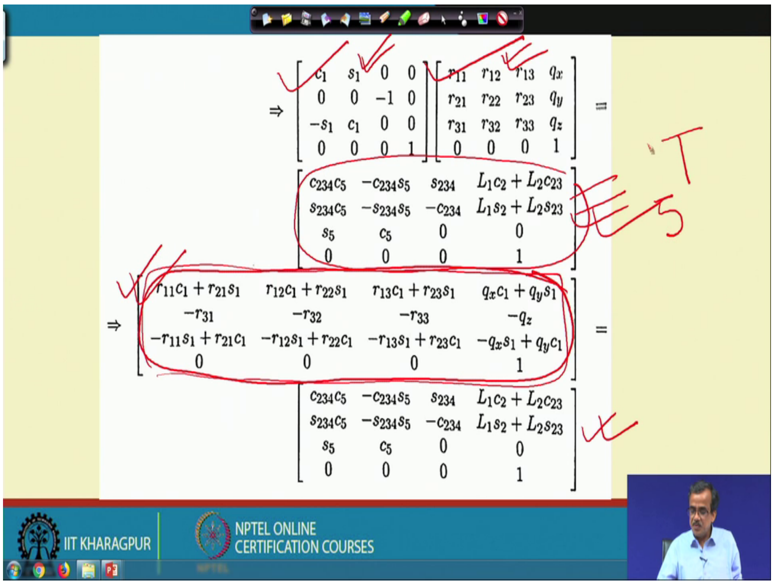
(Refer Slide Time: 05:48)



For example, say just anyone for example, if you want to have a look. So, this is my T 5 with respect to four or say this is T 4 with respect to three and so on. So, what you will have to do is here we have got four such matrices each having 4 cross 4 dimension and if you multiply then I will be getting finally, one 4 cross 4 matrix.

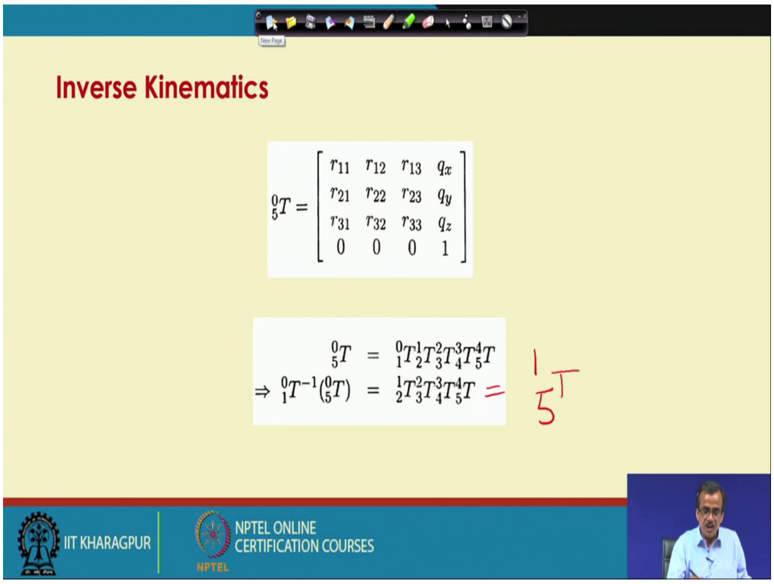
Now, if I just go, so you will be getting such a big expression and that is nothing but these 4 cross 4 matrix. So, these 4 cross 4 matrix will be getting. And on the other side, so we have got I am sorry I am sorry. So, these 4 cross 4 matrix will be getting ok. And if you multiply, so this one with this particular thing, so I will be getting so this particular the 4 cross 4 matrix. And on this side we have got the same thing I have written it here. So, once again let me repeat these particular matrix if you multiply with this, so I will be getting, so this particular matrix.

(Refer Slide Time: 06:58)



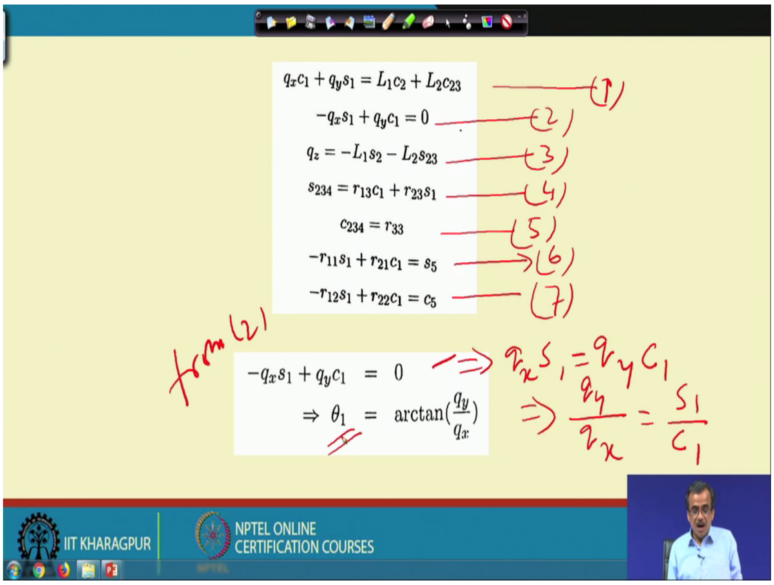
And this is nothing but is your T 5 with respect to your one that is your so this particular thing like your this is nothing but what so T 2 with respect to 1; T 3 with respect to 2.

(Refer Slide Time: 07:21)



So, this can be written as your T 5 with respect to 1. So, these T 5 with respect to 1. So, these T 5 with respect to 1, I am just going to write it here. So, this is nothing but your T 5 with respect to 1. So, this is T 5 with respect to 1. So, the this side I am getting these two, so this particular matrix I am getting; and this side I am getting this particular matrix. Now, element wise I am just going to equate ok. For example, first row first column, this particular element will be put equal to this if it is required if it is going to help us similarly. So, this particular position term. So, I can equate to this I can equate to this, then this I can equate to this and this I can equate to this ok. So, by equating these particular elements, we will be able to find out the different terms the different equations rather.

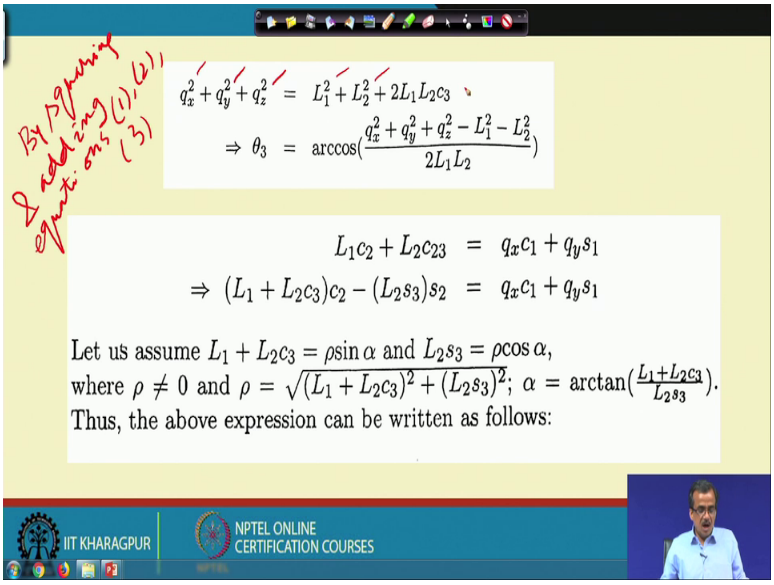
(Refer Slide Time: 08:31)



Now, if I just equate, so I will be getting a set of equations. For example, say if you see the position term, so q x cos theta 1 plus qy sin theta 1 is nothing but L 1 cos theta 2 plus L 2 cos of theta 2. And this is nothing but is your equation 1. Similarly, the second position term if you see that is your minus q x sin theta 1 plus q y cos theta 1 that is equals to 0. So, you will be getting equation 2. Then the third position term if we equate, so we will be getting equation 3.

And now that orientation term or the rotation term, we will have to equate. And if we equate then we can find out s 234 equals to this and c 234 is equals to this. So, what I can do is I am just going to find out s 234 and c 234. So, this is nothing but is your s 234 s 234, and this is minus c 234. So, this has to be equated to s 234 should be equal to this; and minus c 234 should be equal to minus r 32 ok. And then we can also find out another expression sin 5 sin theta 5 and cos theta 5. So, this particular sin theta 5 will be made equal to this ok; and this cos theta 5 will be made equal to this ok. And position terms we have already equated.

(Refer Slide Time: 10:16)



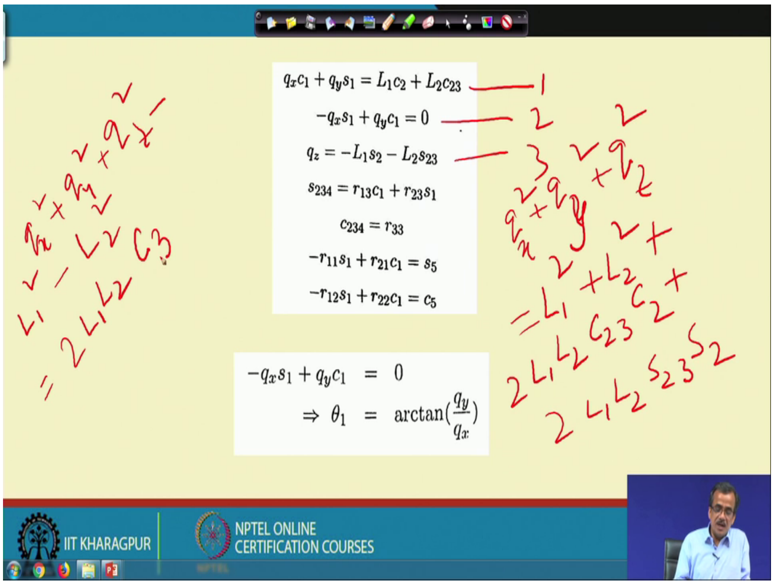
Now, if this is the situation, so I will be getting all such equations like this is your this is equation 1, this is equation 1; this is your equation 2; this is equation 3; equation 4; then we have got 5th equation; this is your 6th equation; and this is your 7th equation. Now, although we have got only 5 unknowns, we will have to take the help of these seven equations to find out the solution. So, although the minimum number of equations required is 5, with the help of 5 equations will not be able to find out all 5 theta values.

Now, let us see how to determine the values of the theta like theta 1, theta 2 up to theta 5 using this set of equations. Now, first let us concentrate on equation 2. So, from equation 2, so from equation 2, from equation 2, we can find out minus q x sin theta 1 plus q y cos theta 1 that is equals to 0. So, this can be written as your q x sin theta 1 is nothing but q y cos theta 1 then we can write down q y divided by q x is nothing but sin theta 1 by cos theta 1. So, this is nothing but tan theta 1. So, theta 1 is nothing but tan inverse q y by q x. So, out of these five theta values, so one theta value I am able to find out very easily and that is nothing but theta 1.

Now, let us see how to find out the other theta values. Now, to find out the other theta values actually what I will have to do is we will have to take the help of other equations. For example, say once again if you write down the equation number, so this is 1, this is 2, this is 3, this is 4, this is 5, this is 6 and this is 7. So, we will have to take the help of equations 1, 2 and 3. And let us see by taking the help of 1, 2 and 3, how can you find out the other the joint angle values.

So, what I do is your we use actually the same method like by squaring and adding by squaring and adding equations 1 2 and three we get q x square plus q y square plus q z square and that is nothing but L 1 square plus L 2 square plus 2 L 1 L 2 cos theta 3, how to get it let us try to see.

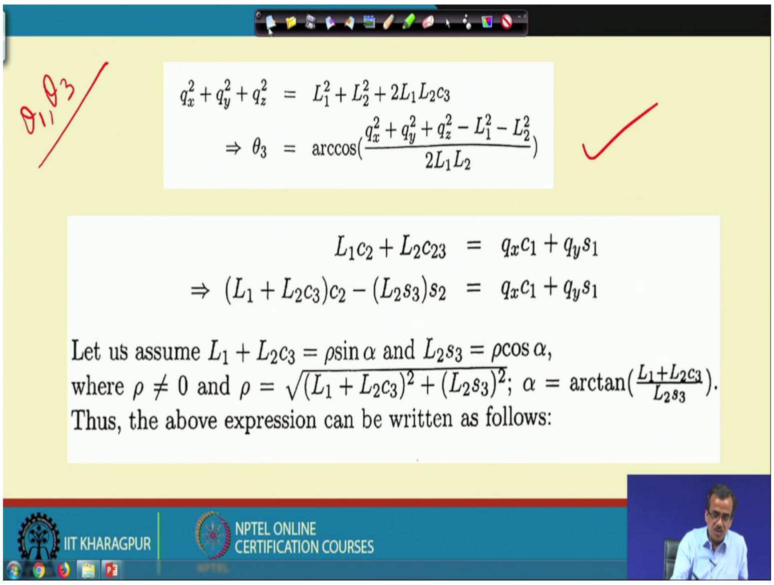
(Refer Slide Time: 13:42)



So, what I do is so this was my equation one this was my equation 1, this is 2, and this is 3. So, what I do is so by squaring and adding, so you will be getting q x square q x square, then I will be getting q y square, then I will be getting q z square and that is nothing but what that is nothing but is your L 1 square then comes your L 2 square plus 2 L 1 L 2 cos of theta 2 plus theta 3 cos theta 2. And then here I will be getting 2 L 1 L 2 sin of theta 2 theta 3 then comes your sin theta 2. So, this is what we are going to get

Now, this part actually can be written as your q x square plus q y square plus q z square minus L 1 square minus L 2 square and that is equal to 2 L 1 L 2 cos of theta 2 plus theta 3 cos theta 2 sin of theta 2 plus theta 3 sin theta 2. So, this can be written as cos of theta 2 plus theta 3 minus theta 2. So, this will become cos of theta 3 ok. And from here we can write down so that particular expression that is your that is theta 3 is nothing but cos inverse of this particular the expression cos inverse of this particular expression. So, theta 3 is known.

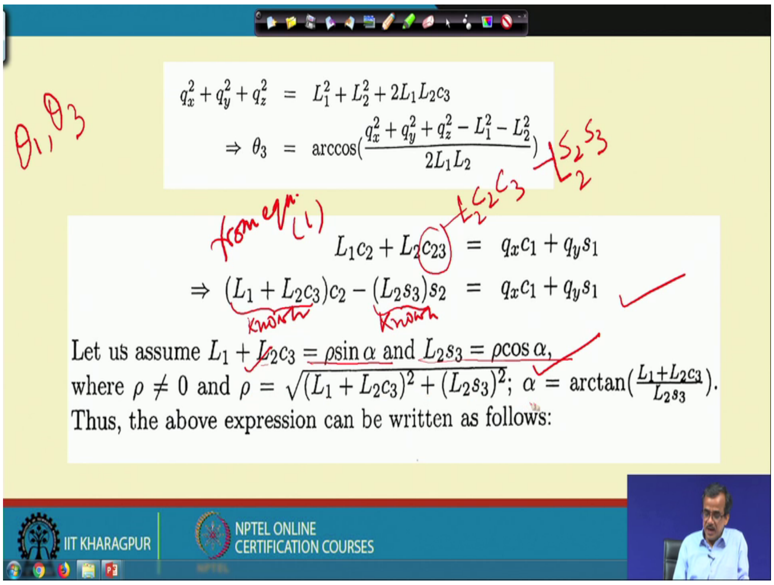
(Refer Slide Time: 15:42)



So, till now we have determined theta 1, theta 1 we have already got and we have got now theta 3; out of 5 values, 2 values we have already got. Now, we concentrate on the first equation that is your q x cos theta 1 plus q y sin theta 1 and that is nothing but L 1 cos theta 2 plus L 2 cos of theta 2 theta 3.

Now, this is that particular equation. So, this particular equation q x cos theta 1 plus q y sin theta 1 is nothing but L 1 cos theta 2 plus L 2 cos of theta 2 theta 3. Now, let us try to concentrate here now here. So, this is from equation 1. So, from equation 1, so this can be written as so L 2 cos of theta 2 plus theta 3 is nothing but L 2 cos of theta 2 cos of theta 3 minus L 2 sin of theta 2 sin of theta 3.

(Refer Slide Time: 16:54)

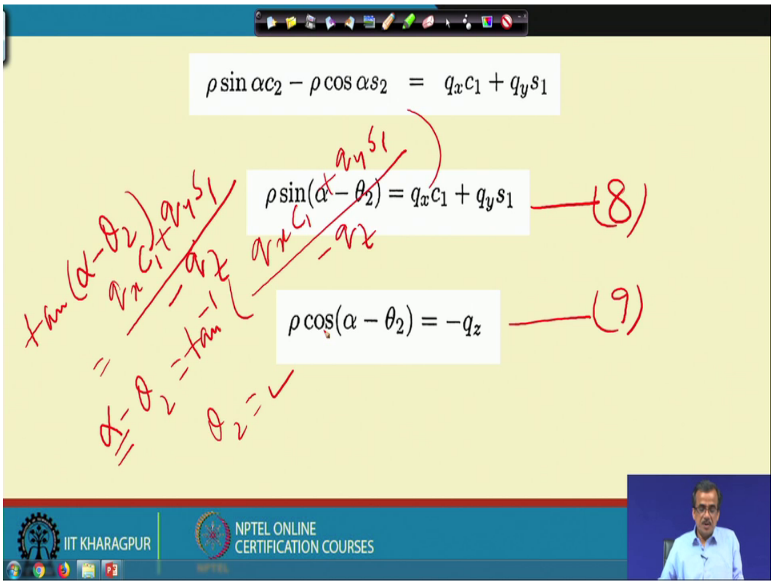


So, this particular thing can be written as cos of theta 2 cos of theta 3 minus sin of theta 2 sin of theta 3. So, if I just write here, so I can write I can take this particular c 2 common and I can within bracket I can write down L 1 plus L 2 cos theta 3 minus L 2 s 3 within a bracket then s 2 because this is nothing but what this is nothing but L 2 cos of theta 2 theta 3 minus L 2 sin of theta 2 theta 3. So, what you can do is, so I can take c 2 common here. So, this become L 1 plus L 2 c 3 minus s 2 I can take common this will become your L 2 s 3. And this is nothing but q x cos theta 1 plus q y sin theta 1 ok.

Now, if you see so till now as I mentioned that we have calculated theta 1, we have calculated theta 3; Now, here if you see so this L 1 plus L 2 cos theta 3, so this is the known quantity because I know that L 1 L 2 I know theta 3. So, this is the known quantity; similarly, this L 2 sin theta 3, so this is also the known quantity. And here so we can assume that the known quantity L 1 plus L 2 cos theta 3 is nothing but rho sin alpha; and L 2 sin theta 3 is nothing but rho cos alpha.

And here rho is nothing but square root of L 1 plus L 2 cos theta T square plus L 2 sin theta 3 square and rho is not equals to 0, and moreover alpha can be determined as tan inverse L 1 plus L 2 cos theta 3 divided by is your this thing L 2 s 3. So, I can find out rho; I can find out this particular alpha. And if I know this rho and alpha ok, so I can write down I can find out this expression as follows.

(Refer Slide Time: 19:22)

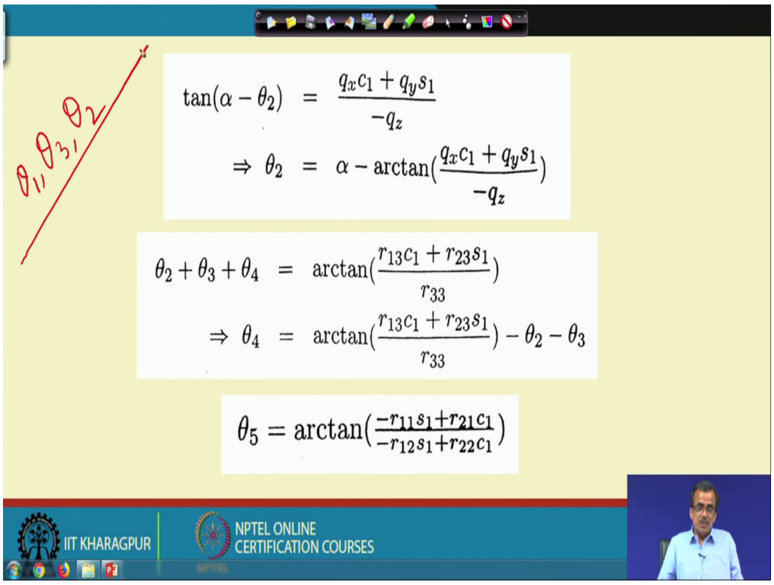


So, this can be written as rho sin alpha cos theta 2 minus rho cos alpha sin theta 2 that is nothing but q x cos theta 1 plus q y sin theta 1. So, this is actually one equations will be getting. So, previously we got seven equation and these can be written as rho sin alpha minus theta 2 that is nothing but q x cos theta 1 plus q y sin theta 1. So, this is actually nothing but equation number this is equation number this is equation number 8. So, this is your equation number 8.

Now, by following the same principle actually what you can do is so we can concentrate on so this particular equation that is your equation 3. So, we can concentrate on this particular equation, and we can follow the same method. And if you follow the same method, then I can find out so this particular expression that is rho cos alpha minus theta 2 is nothing but minus q z. So, this is actually this is actually equation 8; this is equation 8, and this is your equation 9 ok.

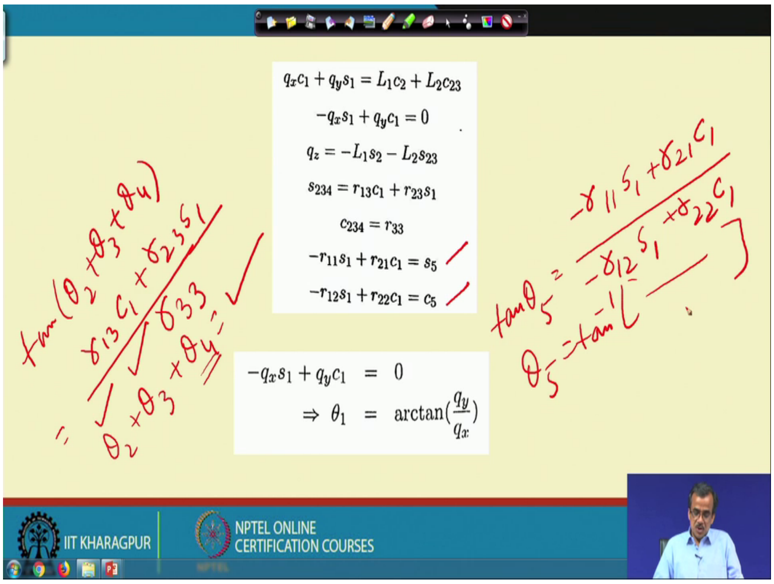
Now, by solving this equation 8 and 9, we can find out actually tan of alpha minus theta 2 is nothing but q x cos theta 1 plus qy sin theta 1. And this is nothing but is your minus q z. So, this is nothing but tal of tan of alpha minus theta 2. And alpha minus theta 2 is nothing but tan inverse this particular the expression that is q x cos theta 1 plus q y sin theta 1 minus this q z. So, this is the expression of this particular alpha minus theta 2. And alpha we have already determined so we can find out this particular the theta 2. So, theta 2 is also known now.

(Refer Slide Time: 21:55)



So, once you have determined this particular theta 2, now so till now actually we have got theta 1 then we determine theta 3, then we determined actually theta 2 so, out of these 5 values, like 3 values are known. Now, if we just go back to the set of equation, we have got some equations like your like this sin of theta 2 plus theta 3 plus theta 4 cos of theta 2 plus theta 3 plus theta 4.

(Refer Slide Time: 22:43)



So, I can find out very easily that is your the tan of theta 2 plus theta 3 plus theta 4 is nothing but is your r 1 3 c 1 plus r 2 3 s 1 divided by is your r 3 3. So, this particular thing we can find out. So, this is known quantity, so I can find out what is theta 2 plus theta 3 plus theta 4. Now, this is known and theta 2 I have already calculated; theta 3 I have already calculated. So, I can find out what is theta 4.

And once you have got this particular theta 4 then using this these two equation very easily we can find out theta 5 that is tan theta 5 is nothing but is your this particular expression r 1 1 s 1 plus r 2 1 c 1 then comes minus r 1 2 s 1 plus r 2 2 c 1. So, from here you can find out tan inverse of that tan inverse of this particular expression if you write down so you will be getting this particular theta 5. So, this is the way actually we can find out all the values of the joint angles that is your theta 1, theta 2, theta 3, theta 4 and theta 5. And we can solve this particular inverse kinematics in this particular the way.

Now, remember one thing here we may not get and we will not be getting the unique solution for this particular inverse kinematics. Like let me take a very simple example supposing that I have got a manipulator having 5 degrees of freedom or say 6 degrees of freedom. And if I give the task to the manipulator that the that the tip of the manipulators who touch this particular point.

Now, this point can be touched with the different configuration of this particular the different joints of the manipulator and that is why to touch the same point actually there will be your the different combination of the theta values with the help of which actually I can reach the same point. And there will be multiple sets of solutions for this particular joint angles with the help of which so the tip of the manipulator or the end effector will be able to reach a particular point in 3D space.

Thank you.