Robotics
Prof. Dilip Kumar Pratihar
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 18
Robot Kinematics (Contd.)

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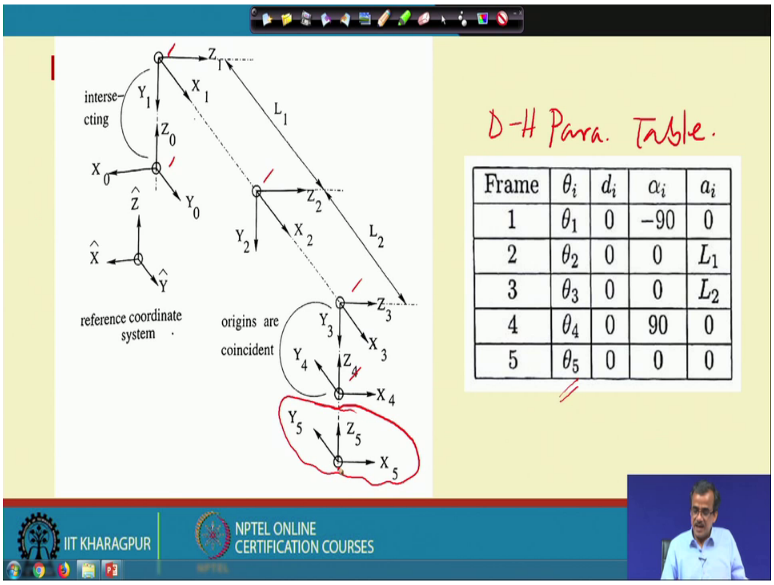

Now this is the DH parameter table which we got for this mini mover; so DH parameters table. Now as I mentioned earlier. So, we have got 5 joints here the first joint that is denoted by this, second joint is denoted by this, third joint fourth joint and fifth joint and this particular joint is an extra whatever coordinate system I am showing it here. So, this is hypothetical and what we do is, exactly whatever we have like your Z 4 X 4 and Y 4, the same thing I have just copied it here and this is hypothetical and this is required just to define this particular the joint angle.

In fact, this particular coordinate system has got no physical existence this is an imaginary coordinate system it is attached at the end otherwise. So, first second third fourth fifth joint. So, all 5 joints I have assign the coordinate system. So, this is the purpose of attaching this hypothetical extra coordinate system at the end.

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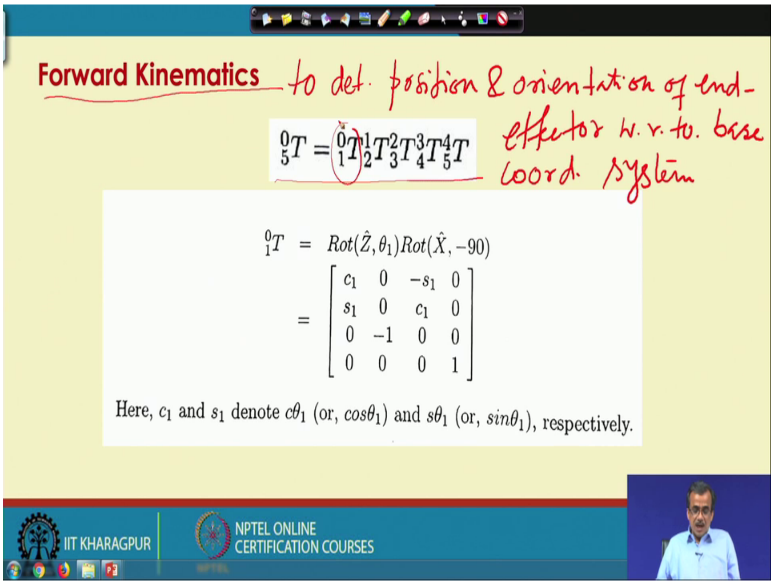

Now, I am just going to start with the forward kinematics of this particular the manipulator. Once again for the forward kinematics the purpose is the same to determine the position and orientation of the end effector with respect to the base coordinate system, provided the length of the links and the joint angles are given or they are known.

So, let me write here, the purpose is once again to determine the position position and orientation position and orientation of end effector; end effector with respect to with respect to the base coordinate system base coordinate system, provided the length of the links and the joint angles are known. So, this is the purpose of this particular the forward kinematics.

Now, let us see how to carry out this particular the forward kinematics, it is exactly the same way the way we did for the 2 degrees of freedom serial manipulator. So, our aim is to determine T 5 with respect to 0, that is transformation matrix of 5 with respect to 0 that is a end effector with respect to the base coordinate frame and this is nothing but T 1 with respect to 0 multiplied by T 2 with respect to 1 multiplied by T 3 with respect to 2 multiplied by T 4 with respect to 3 multiplied by T 5 with respect to 4.

Now, we will try to find out each of these particular the transformation matrix. Now what you will have to do is. So, you will have to go back to this particular the DH parameter table. Now if we just go back to the DH parameter table. So, I am just going to concentrate here, now here if you see we have got a 1 rotation here and another rotation here. Now this particular rotation is rotation about Z by an angle theta 1 and this is the rotation about X by an angle minus 90. And as I told that you follow this particular sequence like your screw Z and screw X that particular rule and if I just follow that. So, very easily I can I can I can write down that, T 1 with respect to 0 is nothing but rotation about Z by theta 1 then rotation about X by minus 90. And we know this expression this the rotation matrix in 4 cross 4 that is nothing but is your the rotation about Z rotation about Z by an angle theta 1.

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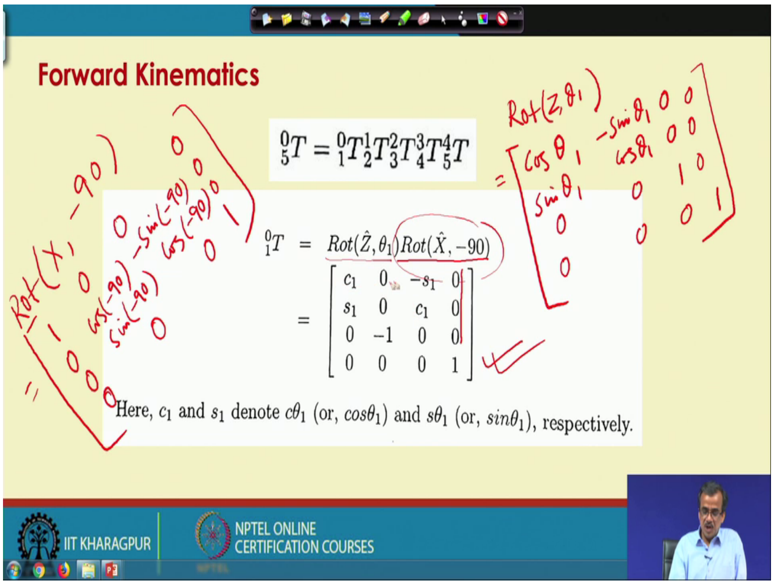

So, in terms of 4 cross 4. So, we can write down that is nothing but cos theta 1 minus sin theta 1 0 then comes your sin theta 1 cos theta 1 0, then comes 0 0 1 0 0 0 1 and this is here there is no position term. So, these things are all 0. So, this is nothing but rotation about Z by an angle theta 1 similarly this rotation about X by an angle minus 90. So, we can write down. So, rotation about X by minus 90 is nothing but. So, rotation about X is 1 0 0 then comes 0 cos of minus 90 then comes minus sin of minus 90 then comes your a 0 sin of minus 90 and cos of your minus 90, then comes 0 0 0 0 1 and the position terms are all 0.

So, this is nothing but that 4 cross 4 matrix now these 4 cross 4 matrix we will have to multiply. Now if I multiply this particular matrix with this. So, I will be getting this particular the 4 cross 4 matrix that is nothing but cos theta 1 denoted by c 1 0 minus sin theta 1, that is denoted by minus s 1 0 sin theta 1 0, c 1 0 0 minus 1 0 0 0 0 0 1. So, this particular final matrix, you will be getting corresponding to this particular 4 sorry T 1 with respect to0.

Now here if you see: I have multiplied 2 rot matrices rotation matrices and finally, I am getting 1 pure rotation matrix and here the position terms are all zeros. That means, this is the pure rotation matrix and if I want to find out the inverse of this particular matrix very easily I can find out because inverse of this particular matrix is nothing but the transpose of this matrix. Now if I try to find out the transpose of this particular the matrix and that is nothing but is your T 1 with respect to 0 inverse.

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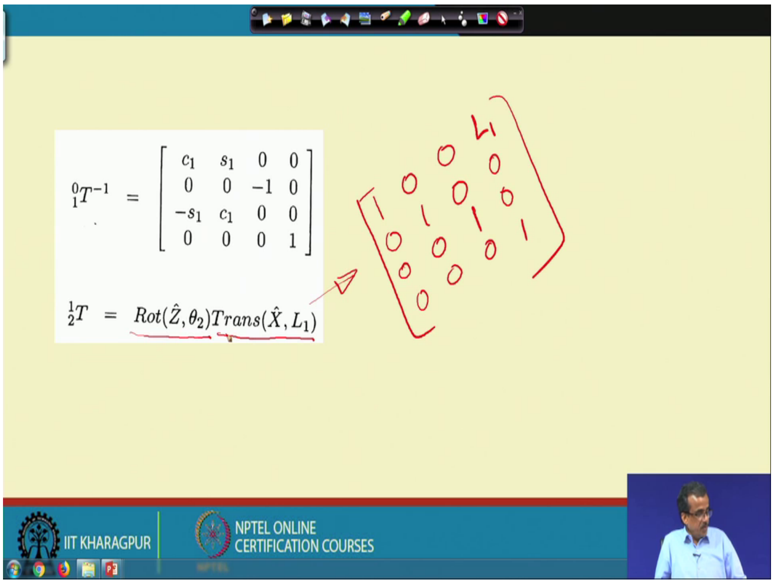

So, whatever was the row, so that became the column. So, first row became first column second row became second column and so on. So, we will be getting the inverse of this particular T 1 with respect to 0. So, this is nothing but the inverse. So, in future we may need this particular the inverse, we will see that how to utilize this particular the inverse now the next is your T 2 with respect to 1. So, what I will have to do is once again I will have to go back to I will have to go back to the DH parameter table, and I will have to concentrate on this particular like 2 with respect to 1. So, I have got theta 2 here and I have got L 1 here.

So, this theta 2 is nothing but the rotation about Z and this particular L 1 is nothing but the translation along X ok. So, this thing I am just going to write it here just to find out T 2 with respect to 1. So, T 2 with respect to 1 is nothing but rotation about Z by theta 2 then translation along X by L 1 and once again I know the expression and for this particular the translation along X by L 1, the 4 cross 4 matrix is very simple and this will be your the rotation terms will be just identity matrix.

So, this is the rotation term then 0 0 0 1 and this is translation along X is L 1 0 0 and we know the expression that is rotation about Z by theta 2 and those 2 4 cross 4 matrices if I just multiply, then we will be getting your like this particular expression that is T 2 with respect to 1 so, this is the expression which will be getting.

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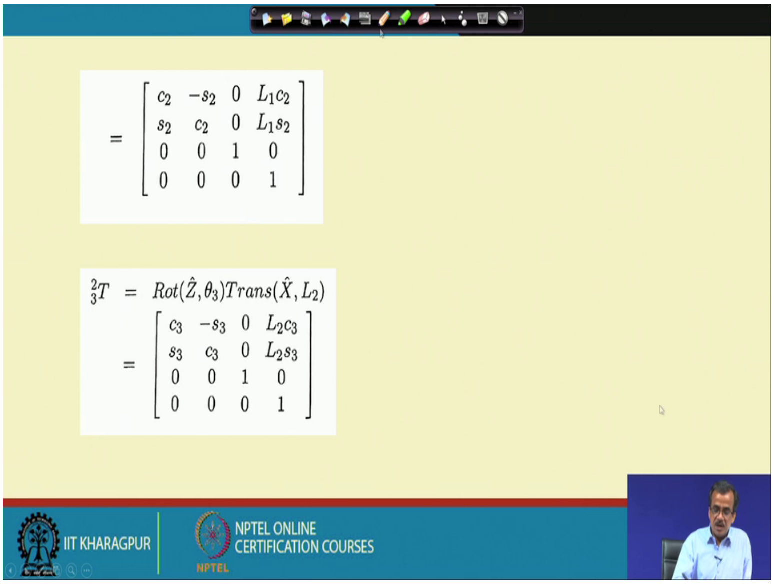

Following the same procedure; so, we can also find out what is this T 3 with respect to 2 and if you see that DH parameter table. So, in the DH parameter table let me once again go back. So, T I am just going to find out T 3 with respect to 2. So, I will have to concentrate here. So, I have got theta 3 I have got L 2. And now corresponding to this what we can do is very easily we can find out the T 3 with respect to 2 is nothing but rotation about Z by theta 3 translation along X by L 2 and I will be getting these 2 4 cross 4 matrices.

And if you multiply you will be getting this particular the matrix that is T 3 with respect to 2.

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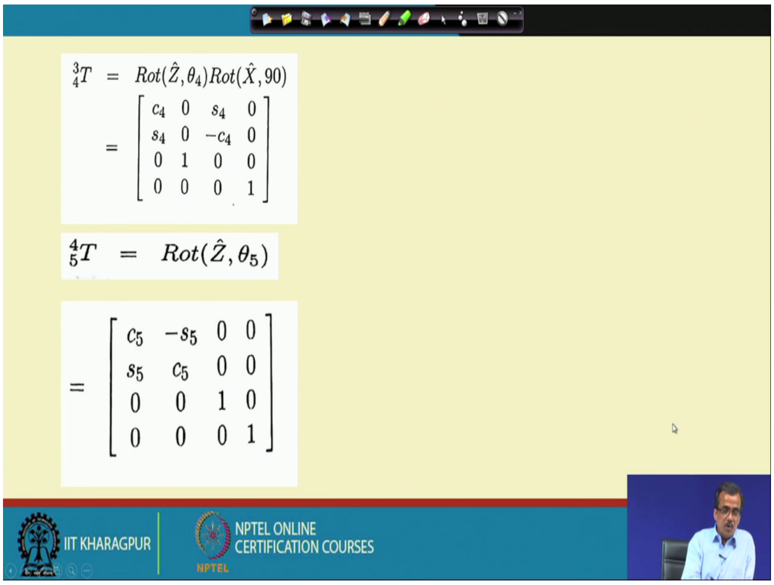

Now, is your T 4 with respect to 3, from the DH parameter table we can find out this is nothing but is your rotation about Z by theta 4 rotation about X by 90. So, we know the expression and if we multiply then we will be able to find out. So, this particular T 3 a T 4 with respect to 3; and by following the same method we can also find out T 5 with respect to 4 and this is nothing but rotation about Z by theta 5. So, this is the expression for is your T 5 with respect to 4.

Now, here once you have got this particular all the expression the individual expressions for transformation matrix now, you can multiply.

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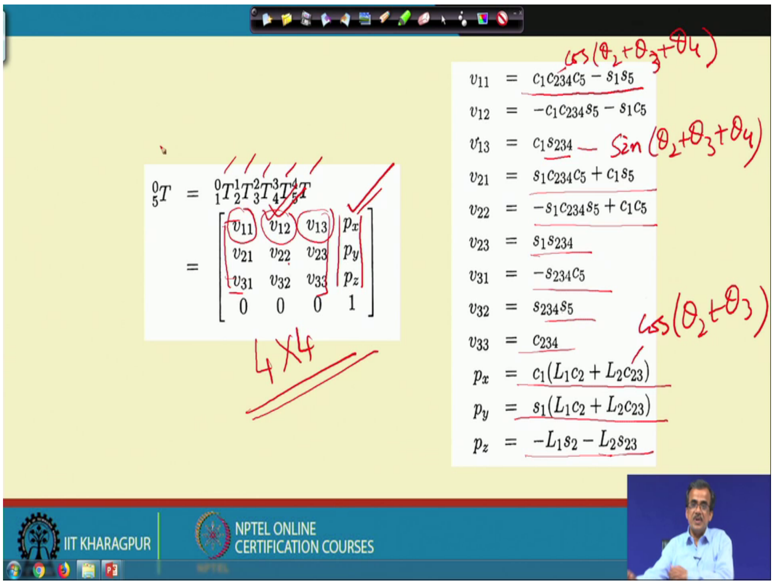

And if we multiply that is T 5 with respect to 0 if you want to find out that is nothing but is your T 1 with respect to 0 then T 2 with respect to 1, T 3 with respect to 2, T 4 with respect to 3, T 5 with respect to 4 and this particular the 5 transformation matrices if you multiply then ultimately I will be getting one 4 cross 4 matrix like this.

Now, this particular matrix actually is going to carry information of the position and orientation of the end effector with respect to the base coordinate pair now here. So, this particular px py and pz. So, this is going to carry the position information and your orientation information that is denoted by a actually your. So, this particular the 3 cross 3 matrix. So, this is these are actually the elements of the rotation terms. Now if you see the expression for each of these particular terms. So, we will be getting a very complex expression.

For example say if you see; so this particular v 1 1 that is the first term the first element of the rotation matrix. So, you will be getting the expression like c 1 c 2 3 4 c 5 minus s 1 s 5. Now by this c 2 3 4 we mean cos of theta 2 plus theta 3 theta 3 plus theta 4 then c 5 is cos theta 5, c 1 is cos theta 1, s 1 is sin theta 1 and s 5 is sin theta 5.

Similarly, if you see this particular v 1 2; so here once again c 2 3 4 will come then v 1 3 we have got c 1 s 234 and s 234 is nothing but sin of theta 2 plus theta 3 plus theta 4, then v 2 1 is this particular expression, then v 2 2 is this particular expression v 2 3 is this v 3 1 a s 2 3 4 that is sin of theta 2 plus theta 3 plus theta 4, 3 2 is this and 3 3 is this.

Now, if you see the position term px, px is nothing but cos theta 1 multiplied by L 1 cos theta 2 plus L 2 cos of theta 2, cos of theta 2 plus theta 3. So, this is actually the expression for this particular the px and similarly we can find out. So, this particular py is sin theta 1 multiplied by L 1 cos theta 2 plus L 2 cos of theta 2 plus theta 3. Then pz is nothing but minus L 1 sin theta 2 minus L 2 sin of theta 2 theta 3. So, such a big expression you will be getting for this particular the final matrix.

Now, you see. So, this particular manipulator is having only 5 degrees of freedom, now if I consider a manipulator ideal spatial manipulator like say puma, which is having 6 degrees of freedom. So, this particular expression will be even more complex and another term will come that is theta 6 and more complex expression will be getting for each of these particular elements of this 4 cross 4 matrix which is nothing but the position and orientation information of the end effector with respect to the base coordinate frame.

And this is actually the purpose of the forward kinematics. So, once again let me repeat in forward kinematics, we try to represent the position and orientation of the end effector of the manipulator with respect to the base coordinate frame provided we know, the length of the links there is L 1, L 2 and all such things we know and all the joint angles like theta 1, theta 2 up to theta 5 are known, then only we can find out the position and orientation information of this particular the end effector with respect to base.

Now, here once again let me just try to take 1 physical example, supposing that say this is my serial manipulator. So, this is the end effector and supposing that this is the fixed. So, the fixed base; so I have got a number of joints number of links and this is actually the serial manipulator. And with the help of this manipulator supposing that I am just going to manipulate this particular object. So, it is something I want to grip it, it is something like this I want to grip it, it is something like this I want to grip it.

So, I am just going to grip this particular object. So, I should know the position and orientation and accordingly, I will have to orient this particular end effector I will have to position and orient. So, that I can grip this particular the object and I can do this particular the manipulation task.

So, in forward kinematics we try to find out the position and orientation of this particular end effector with respect to base, but remember we can also think in the reverse direction like for example; say if I know this with respect to base, can I not find out the base with respect to this.

Now, let me take a very simple example, if I know this particular joint with respect to this I should also be able to find out, so this particular thing with respect to this and vice versa that is this with respect to this. So, we should know the information in both the direction. That means this particular transformation matrix has to be invertible.

Thank you.