Robotics
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Lecture - 17
Robot Kinematics (Contd.)

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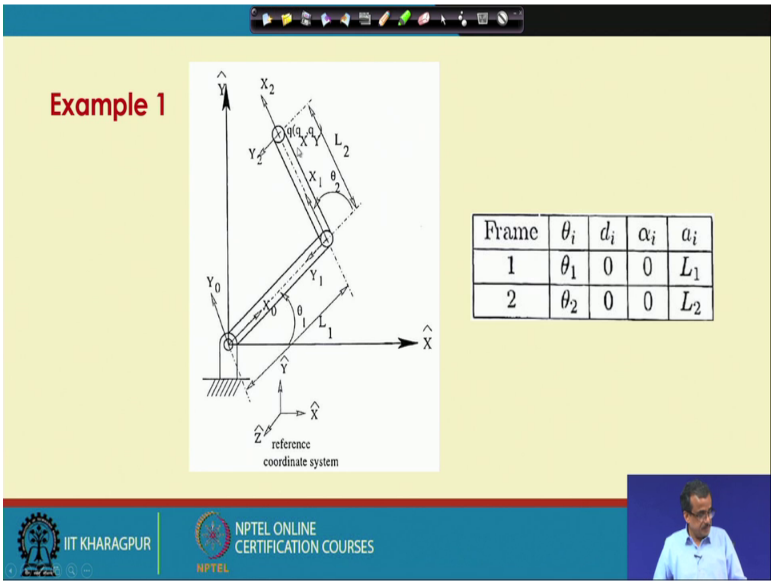

Frame	θ_i	d_i	α_i	a_i
1	θ_1	0	0	L_1
2	θ_2	0	0	L_2

Now, we are going to discuss how to carry out the analysis related to inverse kinematics of this particular 2 degrees of freedom serial manipulator.

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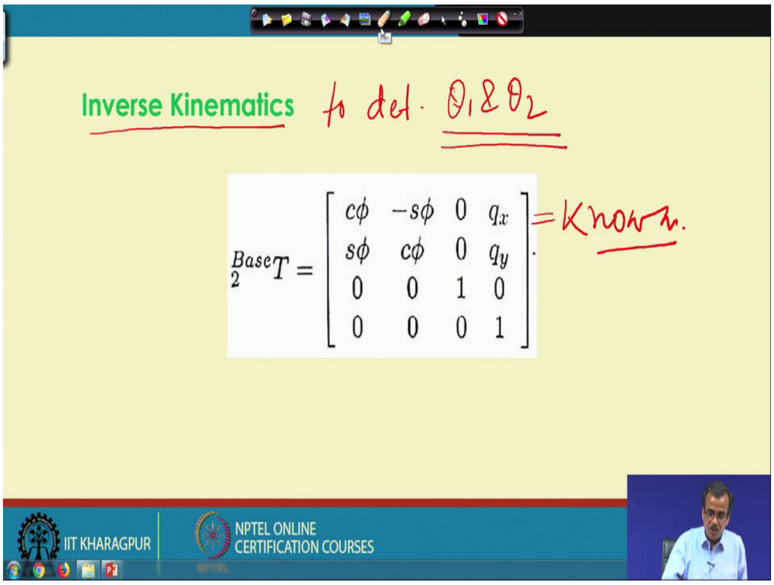

$$ {}^{Base}_{2}T = \begin{bmatrix} c\phi & -s\phi & 0 & q_x \\ s\phi & c\phi & 0 & q_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Now in inverse kinematics, we try to find out the joint angles; that means, the purpose is to determine the purpose is to determine the joint angles to determine the joint angles theta 1 and theta 2. Provided the position and orientation of the end effector with respect to the base coordinate frame and length of the links are known.

Now, this particular matrix that is t 2 with respect to base; so these carries information of the position and orientation of the end effector and this particular matrix is known to us. And our aim is to determine the values for this particular joint angle theta 1 and theta 2 that is the purpose of inverse kinematics.

Now, let us see how to carry out; so this particular inverse kinematics the problem, how to find out the solution.

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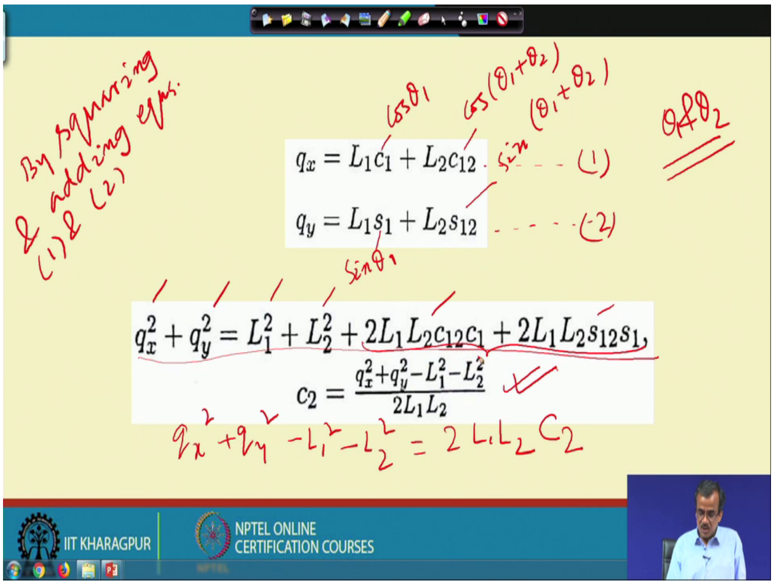

Now, now here if I just compare like this particular q x and q y that is the coordinate of this particular the position of the end effector with the position term that is nothing but. So, this is the q x and q y this particular position term that is q x and qy. So, this is known and if I see from the forward kinematics we have seen that this q x is nothing but L 1 cos theta 1 and L 2 cos half theta 1 plus theta 2.

So, this is nothing but cos theta 1 cos theta 1 and C 12 is nothing but cos of theta 1 plus theta 2. So, we have got 2 equations one is q x that is equals to L 1 cos theta 1 plus L 2 cos of theta 1 plus theta 2. So, let me consider this is equation 1.

Similarly, this q y the q y is known and from the forward kinematic that calculation we have seen that this particular q i is nothing but L 1 sine theta 1 plus L 2 sine of theta 1 plus theta 2. So, S 1 2 is nothing but sine of theta 1 plus theta 2 and S 1 is nothing but is your sine theta 1 now this particular equation is also known. So, there are 2 equations and there are 2 unknowns that is unknowns are theta 1 and theta 2. So, we can solve for these the 2 unknowns theta 1 and theta 2.

Now, let us see how to solve it now this is the very simple set of equation and as I told there are 2 unknowns 2 equation. So, what we can do is we can square and add equations one and 2. So, by squaring by squaring and adding equations 1 and 2 one and 2 we get that q x square plus q y square that is nothing but L 1 square plus L 2 square then comes 2 L 1, L 2 C 1 2 C 1. So, this particular term plus 2 L 1 L 2 S 1 2 S 1.

So, this particular expression will be getting now here we can write down. So, this part these part of this expression can be further simplified and we can write that q x square plus q y square minus L 1 squared minus L 2 square is nothing but 2 L 1 L 2, then cos of theta 1 plus theta 2 cos theta 1 plus sine of theta 1 plus theta 2 sine theta 1; so cos of theta 1 plus theta 2 minus theta 1. So, this will be getting cos of theta 2.

Now, from here it will be getting this particular the expression, that is C 2 is nothing but q x square plus q y square minus L 1 square minus L 2 square divided by 2 L 1 L 2.

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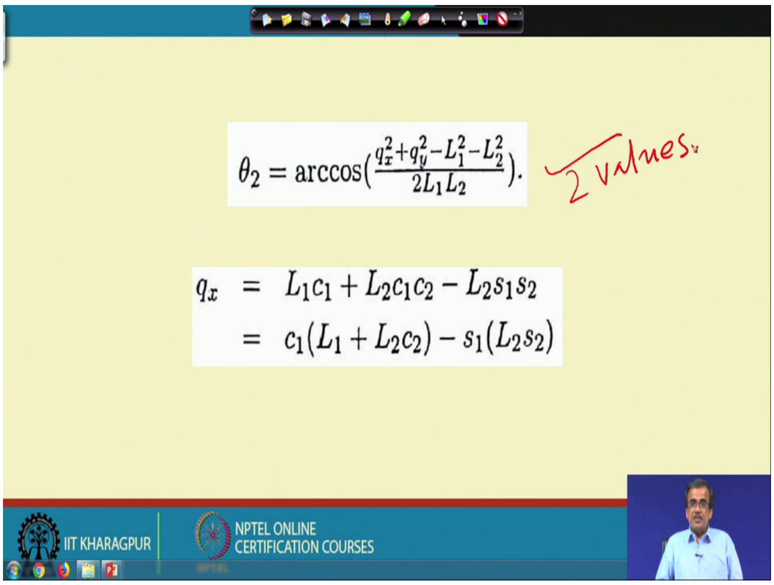

Now, once you have got this particular your cos theta 2. So, very easily you can find out what is theta 2. So, theta 2 is nothing but cos inverse q x square plus q y square minus L 1 square minus L 2 square, divided by 2 L 1 L 2 and this is cos inverse. So, we will be getting 2 values for this particular theta 2.

So, theta 2 is known now will have to determine what is theta 1. Now to determine this particular theta 1; so what we do is we try to concentrate on this equation that is equation one. So, from equation one actually we can find out we can further write this q x is nothing but L 1 cos theta 1, plus L 2 cos of theta 1 plus theta 2.

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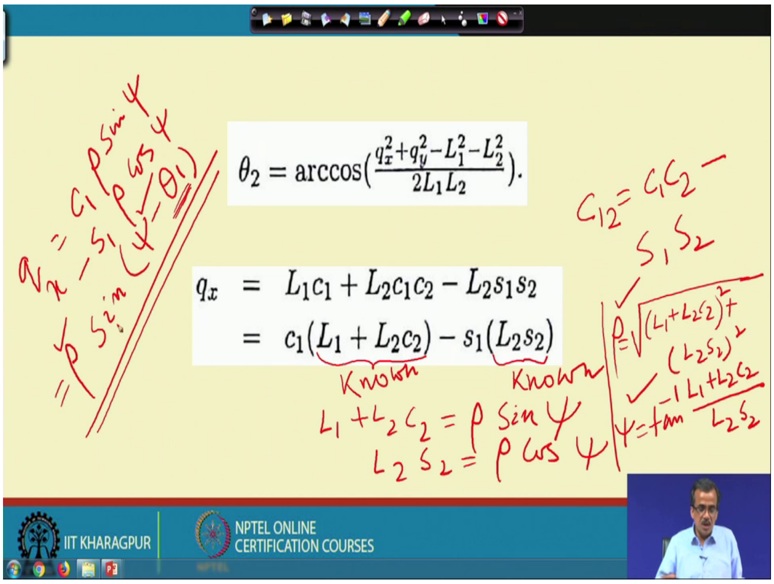

So, cos of theta 1 plus theta 2 can be written as cos theta 1 cos theta 2 minus sine theta 1 sine theta 2; so this will become q x becomes L 1 cos theta 1 plus L 2 cos of theta 1 cos of theta 2 minus L 2 sine of theta 1 sine of theta 2 because C 1 2 is nothing but cos theta 1 cos theta 2 minus sine theta 1 sine theta 2.

Now, here now this particular expression can be rearranged, we can take this cos theta 1 as common and within the first bracket we can write down L 1 plus L 2 C 2 and here I can take S 1 is common and within bracket I can write down L 2 S 2 now here if you see. So, this L 1 plus L 2 C 2, C 2 we have already determined theta 2 is known. So, cos theta 2 is known then L 1 and L 2 are the lengths of the links. So, this L 1 and L 2 are known. So, this particular expression is actually known. So, this is known and your cos theta 1 is unknown.

Similarly, L 2 is known sine theta 2 is known. So, this part is known. Now this known part actually we can assume that your this L 1 plus L 2 cos theta 2 is nothing but rho sine psi and we can take this L 2 cos theta sorry sine theta 2 is nothing but rho cos psi ok.

So, if I just assume like that, that this known part is nothing but your row signs psi and L 2 S 2 is nothing but is your rho cos psi. So, very easily we can write down that q x is nothing but q x is nothing but cos theta 1 multiplied by rho sine psi then comes your minus sign theta 1 and this is nothing but is your row cause psi and from here I can take rho constant and sine psi cos theta 1 minus cos psi sine theta 1 is nothing but sine of psi minus theta 1.

Now here, these part qs can be obtain as rho sine psi minus theta 1. Now from here actually we can also find out what is rho now rho is nothing but is your square root of your L 1 plus L 2 C 2 square plus this your L 2 S 2 square. So, square root of that. So, this is nothing but rho and we can also find out what is psi. Psi is nothing but is your tan inverse tan inverse is your L 1 plus L 2 cos theta 2 divided by L 2 sine theta 2 and that is nothing but is your psi.

So, we can find out rho we can find out psi. So, here rho is known here psi is known. So, only unknown is your theta 1. So, from here directly you can find out by sine inverse or what we can do is we take the help of we take the help of the another equation that is your that q y equation.

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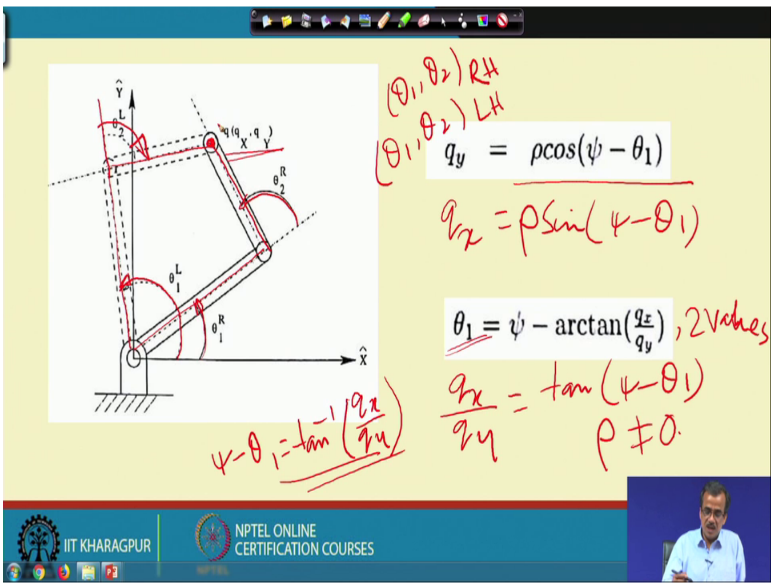

Now, this q y by following the same procedure, can be written as rho cos psi minus theta 1 and if I get q x equals to say rho sine psi minus theta 1 that we have already seen.

So, from here we can find out is your this q x by qi q x by qi is nothing but is your tan of psi minus theta 1 and rho is not equals to 0 no is not equals to 0. So, from here we can find out that psi minus theta 1 is nothing but tan inverse your q x by qy. So, tan inverse q x by qy. So, from here we can find out the theta 1 this particular theta 1 is nothing but psi minus tan inverse that is arc tan q x by qi. So, we can find out this particular the theta 1.

Now, once again will be getting 2 values for this particular theta 1; so for this problem of 2 degrees of freedom serial manipulator, there are 2 sets of theta values we are getting one is called theta 1 theta 2 right hand solution right hand solution, another is called L theta 1 theta 2 left hand solution.

Now, if I consider that I this particular manipulator will have to reach this particular point whose coordinate is q x qy, now this is one configuration with the help of which it can reach this particular point, and here this is your theta 1 theta 1 R right hand solution and this is theta 2 R that is the right hand solution.

Now, another solution could be another configuration could be this one. So, this is L 1 this is L 2 where now theta 1 is nothing but this. So, this is my theta 1 and theta 2 will be clockwise. So, theta 2 will be negative. So, we will be getting a 2 sets of theta 1 and theta 2 values; that means for this particular serial manipulator having 2 degrees of freedom, there are 2 solution for this particular the inverse kinematics.

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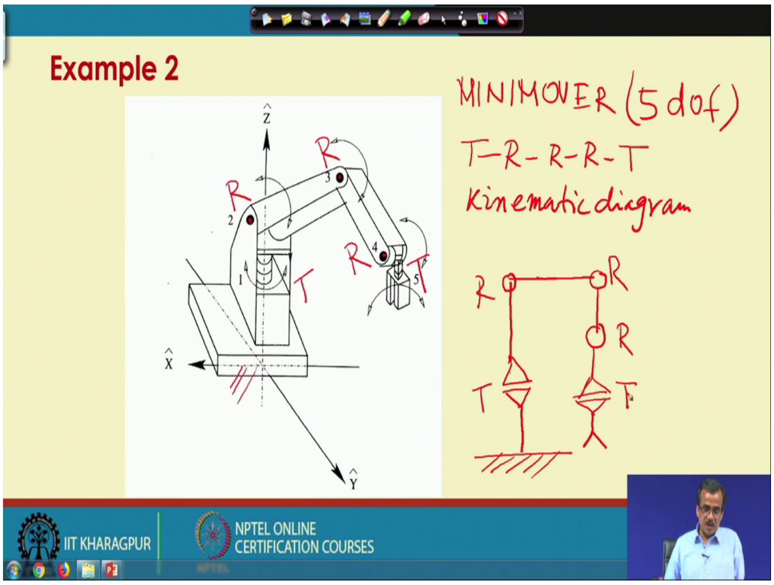

So, one is called the left hand solution another is called the right hand solution. So, this is the way actually we can carry out the inverse kinematics and forward kinematics for the serial manipulator. Now I am just going to take the example of a more complex a manipulator and this particular manipulator is having actually the 5 degrees of freedom and this is a spatial manipulator. So, spatial manipulator with 5 degrees of freedom. So, this is nothing but an under actuated manipulator as we discussed.

So, this is a manipulator the name of this particular manipulator is mini mover. So, this is the name of the manipulator is mini remover, and it is having 5 degrees of freedom. Now here if I just try to understand the nature of the joints, we can see that. So, this is nothing but the fixed base and with respect to the fixed base. So, here we have got one joint now this particular joint is actually the twisting joint, then we have got another joint here. So, this is nothing but a revolute joint the third joint is here this is once again a revolute joint, the fourth joint is here this is once again a revolute joint and we have got another joint here the twisting joint here and this is nothing but the fifth joint.

So, this particular robot as we discussed is known as is your T R R T manipulator. So, each of these particular joints rotary joint each having one degree of freedom and this is the serial manipulator does it is having 5 degrees of freedom. Now if you draw the kinematic diagram. So, let us try to draw the kinematic diagram of this particular manipulator; so the kinematic diagram. So, we start with the fixed base. So, this is the fixed base the first joint is the twisting joint. So, this is the symbol for the twisting joint. So, let us draw this twisting joint.

The second joint is the revolute joint. So, this is actually the revolute joint the third joint is once again a revolute joint, the fourth joint is once again a revolute joint and the fifth joint is nothing but a twisting joint. So, this is nothing but a twisting joint and this is the symbol for the gripper. So, we have got twisting joint, revolute joint, revolute joint, revolute joint and this is the twisting joint. So, this is nothing but the kinematic diagram of this particular the serial manipulator having 5 degrees of freedom.

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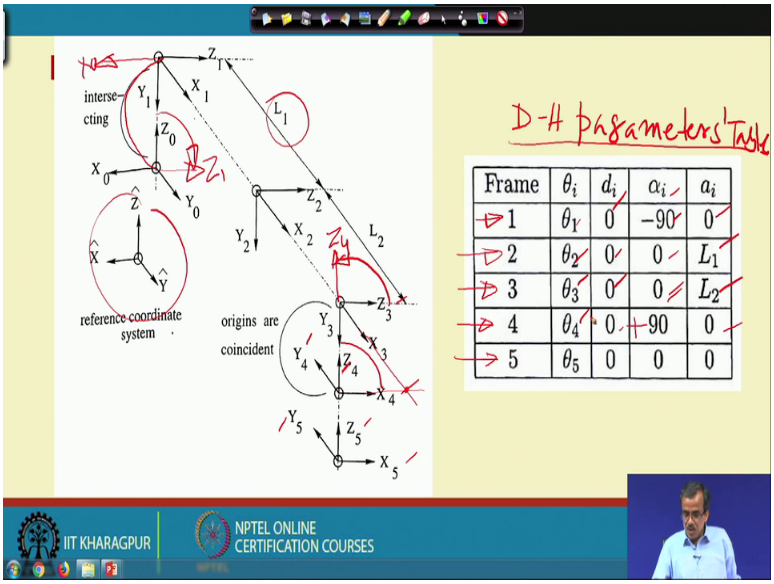

Now, here actually what we do is, we try to assign the coordinate system at the different joints according to the DH parameter setting rule. Now as I told that will have to will have to see the reference coordinate system first. So, if you see the reference coordinate system. So, this is X Y and Z and Z cross X is nothing but Y.

Now if you see the first joint the first joint is nothing but a twisting joint and for these twisting joint. So, Z is what? Z is this particular the base Z. So, this is my Z direction and that is why we have considered. So, this is your Z not and this X naught I have consider along this particular direction in the direction of this particular the reference X reference coordinate X and Y naught is in this particular the direction.

So, this indicates actually the coordinate system at the twisting joint and if you see the second joint is actually a revolute joint and this particular Z and that particular Z. So, they are going to intersect and that is why actually we have written. So, as if this particular and that particular thing they are intersecting and this is the Z about which we take the rotation for this particular the revolute joint.

Now, if this is Z and they are intersecting, now according to the d h parameter setting rule. So, my X could be either this one or it could be this one; that means your Z 1 is selected, now X could be either this direction or that particular direction. So, what we do is, we will have to select X in such a way. So, that we can show the length of the link because the next joint if you see the next joint is once again the next joint is once again a revolute joint and the Z here and the Z here are parallel and if they are parallel, then the mutual perpendicular distance is going to so the direction of X and that is nothing but the length of the link and that is why. So, we have considered. So, this is your Z 1 and this we have selected as X 1.

Now, if this is selected as X 1. So, Z cross X is nothing but Y 1. So, this is your Y 1 ok. The third joint is once again a revolute joint. So, its Z will be parallel to Z 1. So, this is your Z 2 and if you see this particular the manipulator; actually the fourth joint is once again a revolute joint. So, here this particular Z and that particular Z should be parallel and that is why. So, if this Z and that particulars Z are parallel. So, the mutual perpendicular distance should be the X and that is why we have considered this as X 2. So, this is Z 2 and X 2 and Z cross X is nothing but Y 2 and as I discuss. So, here this is once again a revolute joint.

So, I have taken Z along this X 3 along this now Z cross X is nothing but Y 3. So, Z 3 X 3 and Y 3 we can find out now if you see the next joint, the next joint is nothing but a twisting joint and the Z here for the fourth joint and your. So, the Z for this particular fifth joint that is the twisting joint they are going to intersect now if the 2 Z are intersecting. So, I have got both the option. So, Z is selected because this is the twisting joint. So, Z 4 is selected here. So, this is my Z 4.

Now, regarding the X 4; so as this particular Z 3 and Z for they are intersecting. So, X 4 could be either this particular direction or this particular direction. So, anyone can be taken as X 1 X 4, and here I have taken this as X 4 and if this is taken as X 4 then Z cross

X. So, Z cross X will be or Y four. So, this is your Y 4. And as I told that the same coordinate system will be copied at the last that is this is my Z 5 this is my X 5 and this is Y 5.

Now, this completes actually the assignment of these particular the coordinate system. Now here one thing I just want to mention that this is not the only way of showing this type of coordinate system at the different joints, in some of the textbooks they follow a slightly different method that method is something like this. So, at each of the rotary joint there will be some rotation. So, what they want to do is they saw the rotation even a while showing that particular the coordinate system.

For example here there will be some rotation, now if I show that particular rotation. So, this particular X 0 Y 0 Z 0 will be rotated and here once again there is some rotation and if I once again so, that particular rotation then once again it will be rotated.

So, each of this particular coordinate system will be rotated on this particular figure, then it becomes difficult to visualize actually the coordinate system assigned at the different joints. That is why actually this is the better lab representing where we saw the coordinate system at each of these particular joints as if it is going to follow my reference coordinate system. So, this particular reference coordinate system is followed at each of these particular joint and wherever we have some rotation. So, that particular rotation we are going to show while preparing this particular the d h parameter stable.

So, the D H parameter is stable. So, we saw all such rotation now let us see how to fill up; so these particular the table the D H parameters table. Now to fill up actually what we do is first we concentrate on the frame 1 and as I told that while preparing this particular table. So, we will have to follow the screw Z and screw X; that means, I will have to write theta i first then d i then alpha i after that a i.

So, what I do is. So, the first joint that is this particular twisting joint is a rotary joint. So, definitely the variable will be the joint variable and this particular the joint variable I have considered as theta 1. Now here I just want to say that here I can write down theta 1 or if you just draw this particular X naught here. So, if I draw the X naught here let me just draw it X naught here. So, if I draw the X not here. So, the angle between X naught and X 1 is already 90 degree. So, what you can do is in place of theta 1 I can also write down 90 plus theta 1.

Now, if I write 90 plus theta 1 then this particular theta 1 could be the acute angle, but here for simplicity whatever I am doing I am writing the whole thing as a whole as a theta 1; that means, your. So, this particular theta 1 will be more than 90 degree. So, what in place of this 90 plus theta 1 for simplicity, I am writing this particular as theta 1 so, that theta 1 is actually your obtuse angle.

Now, next is your how to find out this particular d; so according to the definition of d. So, d is the offset and that is nothing but the distance between 2 X the distance between 2 X measured along Z. Now these 2 are coinciding they are in fact, intersecting. So, if they are intersecting then the distance between X naught and X 1 is nothing but 0, then we try to find out alpha.

So, by definition alpha is nothing but the angle between 2 Z measured about x. So, if I draw this particular your; so this particular Z 1. So, if this Z 1 if I just draw it here Z 1 if I just draw it here. So, this is my Z 1; so Z naught to Z 1. So, I will have to move in the clockwise direction by 90 degree. So, clockwise we have considered negative. So, this is nothing but negative 90 degree then comes a i. So, a I is the distance between 2 Z if the 2 Z s are parallel. So, we try to find out the mutual perpendicular distance.

Now they are intersecting. So, the distance between Z naught and Z 1 so that is nothing but 0. So, I can fill up actually all the entries corresponding to frame one now we can concentrate on this particular 2. Now 2 means what? 2 means 2 with respect to one now here. So, once again the joint is nothing but a revolute joint. So, the variable is theta 2 and the d is the distance between 2 x. So, X 1 and X 2 they are on the same line. So, the distance is 0 then comes here alpha Z 1 and Z 2 are parallel. So, the angle between them is 0 then comes here a. So, if these 2 Z are parallel Z 1 and Z 2. So, X is along the mutual perpendicular distance and this is actually L one. So, L 1 is nothing but the length of the link. So, this is your L 1 the next is your let me concentrate here.

So, 3 means 3 with respect to 2 now here. So, this particular joint is a revolute joint. So, the variable is theta 3, then comes your d is the distance between 2 X. So, X 2 and X 3 are on the same line. So, this is 0 then comes your alpha is the angle between 2 Z z 2 and Z 3 they are parallel. So, this is 0 now then comes here this a that is Z 2 and Z 3 are parallel and this is the mutual perpendicular distance. So, the length of the link is your L

2 then comes your 4 there is 4 with respect to 3. So, 4 with respect to 3. So, once again this particular joint is a revolute joint. So, the variable is theta 4.

The next is your d, d is the distance between the 2 X. Now here you can see if I extend this particular X 3. So, it is going to intersect the X 4. So, the distance between them is equal to 0. Next is your that the alpha that is the angle between Z 3 and Z 4. Now here if I just draw this particular Z four. So, this is my Z 4 and Z 3 to Z 4 if I want to move. So, I will have to move in the anticlockwise direction by 90 degree. So, this is nothing but positive 90 then comes your A. A is the distance between 2 Z and 2 zs are actually coinciding they are intersecting and coinciding.

So, this particular the distance is nothing but is your 0 and then comes here 5. So, 5 with respect to 4 and this is a twisting joint. So, the joint angle is nothing but is your theta 5 and the other things will be equal to 0. So, this is the way actually we will have to prepare the D H parameter table that is Denavit Hardenberg parameter stable. And once you have got this particular table carrying out the forward kinematics becomes very easy.

Thank you.