Robotics
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Lecture - 16
Robot Kinematics (Contd.)

Now I am just going to discuss once again like how to determine that this particular the T i with respect to i minus 1 that is the transformation matrix of i with respect to i minus 1.

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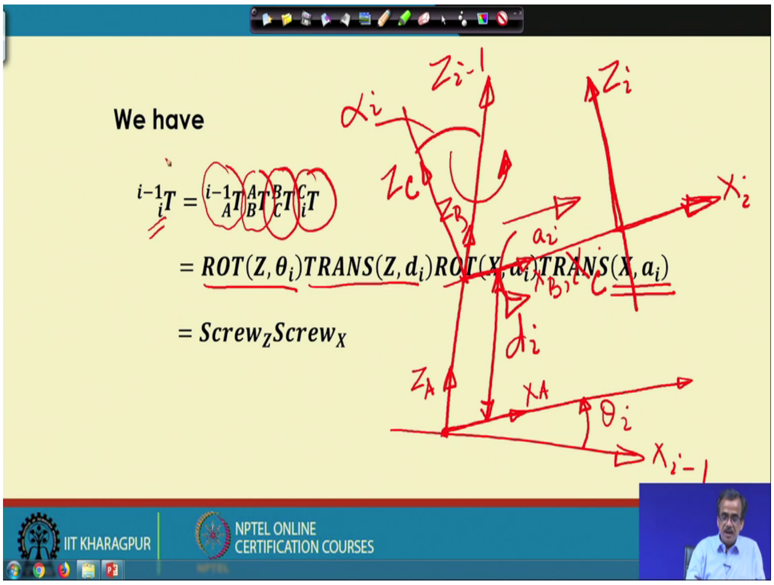

Now if i draw the same thing once again for example, say this is my Z axis, this is my X axis. So, this is your Z i this is xi and supposing that your. So, this is my Z i minus 1 and let me consider this is your X i minus 1.

So, starting from here actually I will have to reach this how to do it. So, what I do is first we take the help of ta with respect to i minus 1. So, what I do? So, we rotate about z. So, this is my Z by angle theta so, that. So, here there is this is this is nothing but the a i. So, I am just going to draw one line parallel to that and this particular angle is nothing but theta i.

So, will start from this Z i minus 1 and X i minus 1 and the first is ta with respect to i minus 1. So, what a do is. So, we define. So, this is nothing but X A and this is nothing but is your Z A and that is nothing but rotation about Z by theta i ok. So, as if I am taking rotation about Z by an angle theta I; so this X i minus 1 will take the position of X A and this particular Z A will remain same along this particular Z i minus 1.

Now, I am just going to do one thing now supposing that; so this T B with respect to a. So, from here actually I will have to reach this particular and supposing that. So, if I can reach here. So, this will become your X B and this will become your Z B and this distance is nothing but the offset that is d i. Now this is nothing but translation along Z by d i translation along Z by d i. So, X B X A will become X B and your Z A will become Z B.

Now I am just going to take this thing like your C with respect to B; so C with respect to B and this particular angle. So, if I draw one parallel here. So, this particular angle is nothing but alpha. So, this is the alpha angle. So, what I will have to do is, I am going to take rotation about X by an angle alpha. So, what will happen to my X C? XC will remain same as X B, but Z C will be different from your Z B. So, this will become equal to Z C and now I am translating along this particular X that is T i with respect to C. So, I am translating along X by i. So, from here I am just going to reach this particular Zi and Xi. This is the way actually I can find out what is T i with respect to i minus 1 ok.

So, this is the way starting from i starting from i minus 1, I can reach this that is T i with respect to i minus 1. Now if you see this particular sequence for example, say the rotation about Z by theta i translation along Z by d i. So, this is one thing then rotation about X by alpha and translation along X by a i.

Now, this rotation about Z followed by translation along Z by d i is nothing but the screw Z. Now this is very simple for example, say if this is the Z axis. So, I am rotating I am a rotating about Z, and if I have got a threaded part here not here. So, the screwed part is going to have some linear displacement along this particular Z direction. So, I am a rotating about Z and there is translation along Z. So, this is nothing but the screw principal and that is why the rotation about Z and translation along Z is nothing but screw Z then a rotation about X translation along X is nothing but the screw X.

So, we will have to follow this particular the screw rule. Now we see I know the expression for rotation about Z by theta i, I know the expression translation along Z by d i then rotation about X by alpha i translation along X by a i.

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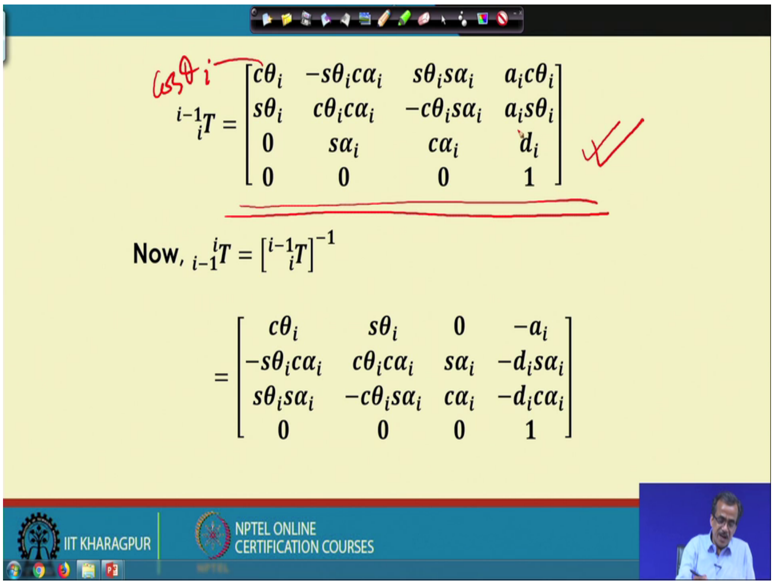

So, I will be getting some matrices and those mattresses like if I multiply then I will be getting. So, these type of 4 cross 4 matrix. So, this is the 4 cross 4 matrix because corresponding to each of these particular thing. So, I can find out the 4 cross 4 matrix. So, there are 4 such matrices each having 4 cross 4 dimension and if I multiply then I will be getting finally,. So, this particular the 4 cross 4 matrix.

Now, here c theta i means cos theta i this is the short form cos theta i minus s theta i is minus sine theta i c alpha i is cos alpha i and so, on and d i is nothing but the offset. So, this is nothing but say T i with respect to i minus 1. So, this is the final matrix which we are going to get. So, I think up to this it is clear to all of you, but here I have got one query there the question that is the rule which you have followed to derive this particular expression, are we not violating the rule for the composite rotation matrix.

The according to the rule for the composite rotation matrix; so whatever we state fast that should go to the end ok, but here we stated first about rotation about Z by theta I, that I have written at the beginning, but not at the end ok. So, my question is are we violating the rule for composite rotation matrix. The answer is no. Now the reason why that particular answer is no is as follows like, if you follow the rule for the composite rotation matrix actually whatever we were doing is nothing but T i minus 1 with respect to i.

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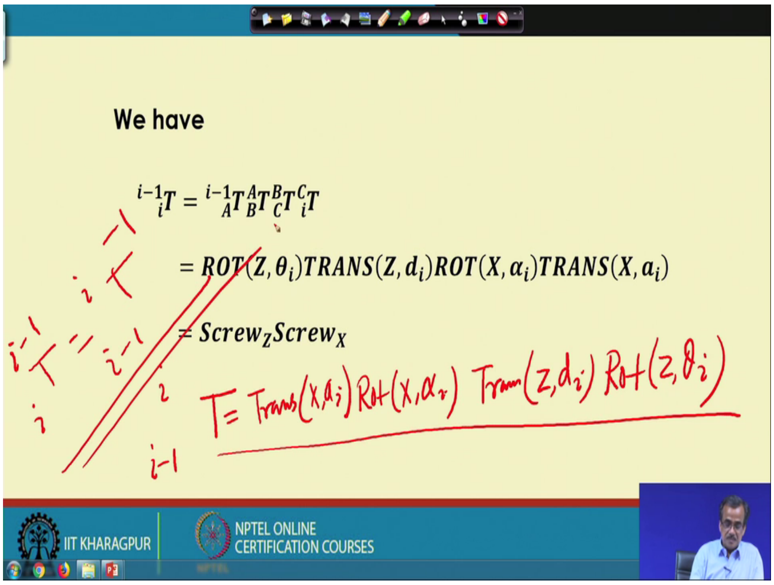

With respect to i and if you want to find out these; so whatever i stated first that I will have to write at the end.

So, rotation about Z by theta I, then comes your translation along this zi, then comes your rotation about X by alpha i then comes your translation along X by a i. So, according to the rule for composite rotation matrix this should be the sequence. Whatever I stated first should go to the end followed by this followed by this followed by this and truly speaking this is nothing but T i minus 1 with respect to i, but what we are trying to find out is just the reverse that is T i with respect to your i minus 1 and all of you know that this particular T i with respect to your i minus 1 is nothing but T i minus 1 with respect to i universe of that ok.

So, we are not violating the rule for the composite rotation matrix and according to that. So, T i minus 1 with respect to i as i told it is the universe of that and if we just try to find out the universe of this particular matrix you will be getting; so this particular thing which is nothing but T i minus 1 with respect to i. So, this is the way actually we can find out the final matrix, using that particular you are the rule for the composite rotation matrix and using the Denavit Hattiesburg notation, we can we can find out the expression for this particular the transformation matrix.

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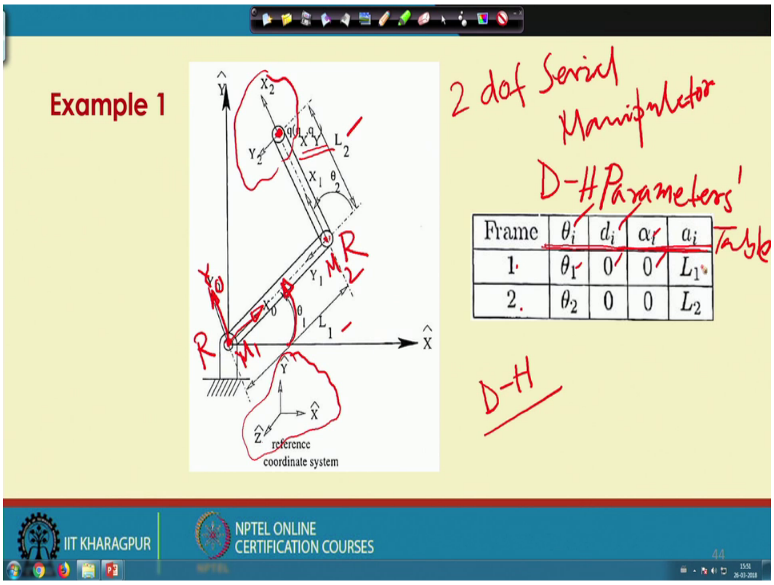

Now I am just going to solve one example one very practical example like using the rules which I have already discussed. So, how to assign the coordinate system and how to carry out the kinematic analysis? Now here for simplicity: so I am just going to consider a very simple problem, a problem of 2 degrees of freedom serial manipulator.

So, 2 degrees of freedom serial manipulator and here we have got 2 joints this is joint 1 this is joint 2 the link one having the length L 1 the link 2 having the length L 2 and this is say the wrist joint or say approximately this is the end effector. And here the joint angles are nothing but is your theta 1 and here with respect to the previous. So, this is nothing but theta 2. So, theta 1 theta 2 are nothing but the joint angles I have already defined the joint angles.

Now, let us try to assign the coordinate system first, according to the D H parameter setting rule or the Denavit Hattiesburg the notations now let us try to concentrate on the first joint. So, as I told that we have got 2 joints here 2 motors here one motor is here another motor is here, here there is no motor. Now here, how to assign the coordinate system to assign the coordinate system? The first thing we will have to see is we will have to see the reference coordinate system. So, this is nothing but my reference coordinate system, we can see that the Z cross s Z cross X is nothing but Y and this is actually the reference coordinate system.

So, with respect to this by following these: I will have to assign the coordinate system here now according to the rule. So, this particular joint is a revolute joint and this is a rotary joint this is also a revolute joint, this is a rotary joint. Now this is in Cartesian X and Y and this particular end effector is having the coordinate qX qY and you forget about Z this is on the 2 dimension.

Now, here actually what we do. So, first thing you will have to find out the Z if I try to find out. So, Z is the axis about which I am taking the rotation. So, Z will be what? Z will be perpendicular to this particular the board. So, Z will be perpendicular to the board and here also the Z will be perpendicular to the board and X is what? The Z here at the first joint and the Z here at the second joint they are parallel.

So, they are mutually perpendicular direction is this direction. So, X should be along that particular the direction. So, Z is perpendicular to the board away from the board and X is in this particular direction and Z cross X. So, this will be my Y not direction and Z is perpendicular to the board which is not shown here and it is coming out of the board.

Similarly, here the Z is perpendicular to the board coming out of the board and this will be my X direction and this will be my Y direction and what we do is whatever coordinate system we assign here, the same thing we copy at the last joint although here there is no motor. So, in place of your X 1 I will have to write X 2 in place of Y 2. So, Y 1 I will have to write Y 2. So, this X 2 and Y 2 is nothing but we are copying this X 1 and Y 1. So, this is an extra coordinate system we are adding at the end although there is no such motor here.

The motor there are 2 motors as I told here I have got motor 1 and here I have got motor 2 just to create that particular the rotary movement. So, this is how to assign the coordinate system, according to the D H parameter setting rule. Now once we have actually assigned this particular coordinate system, now I can prepare the D H parameter table this is called the D H parameters D H parameters table. Now here what we write is your frame 1 frame 2 whenever we write frame 1. So, what you will have to do is, one with respect to 0. Whenever we consider 2 that is 2 with respect to the previous that is 1 and here. So, this particular sequence is very important for example, first we consider theta i next we consider d i if you remember the screw Z rule.

Now, screw Z there is a rotation about Z by an angle theta i, there is translation along Z by an angle d i. So, I am following this particular the screw Z, then I am going to follow the screw X that is your the rotation about X x direction rotation about X then translation along x. So, this is nothing but screw X. So, screw Z screw x.

So, that particular rule will have to follow while writing down this particular your the link and joint parameter in the D H parameters table in some of the textbook they do not follow this particular rule they write in a slightly different fashion like they first write alpha theta then d i ai something like that, but if you write in that particular fashion whenever we are going to write down the forward kinematic equation. So, you will have to make the correction. But if you follow this particular sequence like the screw Z and screw X this particular sequence, you need not make any change and directly you can write down the forward kinematic equation that I am going to show.

Now, how to determine these numerical values or how to find out these variables; now as I told one means what? 1 means 1 with respect to 0 what is theta i? Theta i is this is a rotary joint this is a revolute joint and for this particular revolute joint for example, this type of revolute joint sort of thing. So, this particular angle is the variable ok. So, this theta i has to be variable. So, at this particular joint this is my theta i and that is variable then what is di? By definition if you remember. So, d is the distance between 2 X d is the distance between 2 X measured along Z.

Now, this is X naught direction and this is X 1. So, if I extend X 1 they are going to intersect. So, X naught and X 1 are going to intersect then what is the distance within X naught and X 1 is 0. So, here I have put 0. Now then comes alpha, alpha is the angle between 2 Z if you remember. So, this is actually my Z naught is here perpendicular to the board Z 1 is perpendicular to the board and they are parallel. So, they are included angle is 0.

Now, next is your a i. So, ai is the distance between the 2 z. So, here I have got 1 Z here I have got 1 Z and along X I can find out the mutual perpendicular distance and that is nothing but the length of the link. So, ai is nothing but the length of the link. Next we can find out that frame 2 that is 2 with respect to 1, that is 2 with respect to one. So, once again I have got a rotary joint here revolute joint here. So, the variable is theta i next is the d that is the distance between the 2 x. So, X 1 and X 2 are in the same line. So, the distance between them is 0 the next is your alpha.

So, alpha is your this is the direction of Z 2 and this is hypothetically we have assumed that this is your Z 2 and this is Z 1 they are parallel. So, the angle is 0 then comes your the length of the link. So, this is your Z 1 and Z 2 they are mutually perpendicular. So, this particular L 2 is nothing but the length of the link. So, we can find out all the entries of this particular the D H parameter the table. And once you have found out the entries for that, now very easily can find out what should be the kinematic equation.

So, what I am going to do is I am trying to find out the kinematic equation; the purpose of kinematic equation is to represent the position, and orientation of this particular the position and orientation of this particular say reached with respect to the base code in a frame that I am going to find out.

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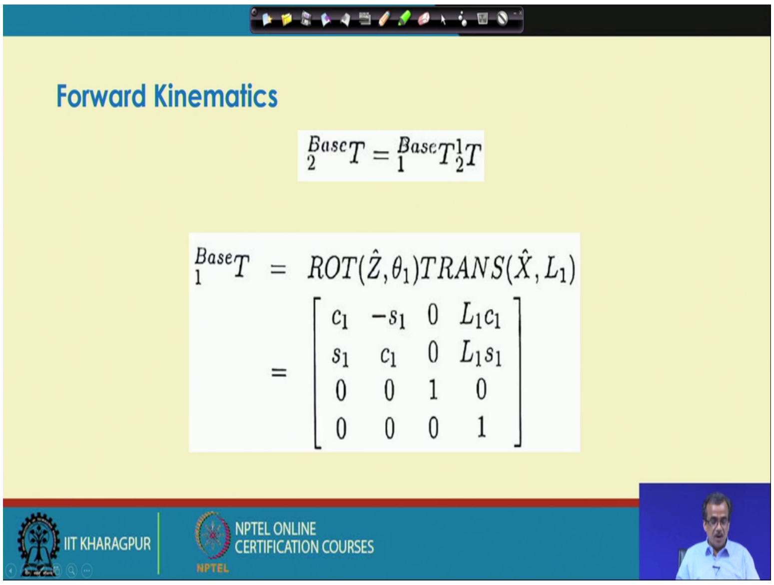

And to get it actually what I will have to do is. So, I will have to take the help of this type of transformation matrix that is T 2 is respect to base T 2 with respect to base. So, this is to with respect to the base. So, T 2 with respect to base is nothing but T 1 with respect to base multiplied by T 2 with respect to 1.

Now, how to find out the T 1 with respect to base, now to write down to find out the T 1 with respect to base you concentrate here.

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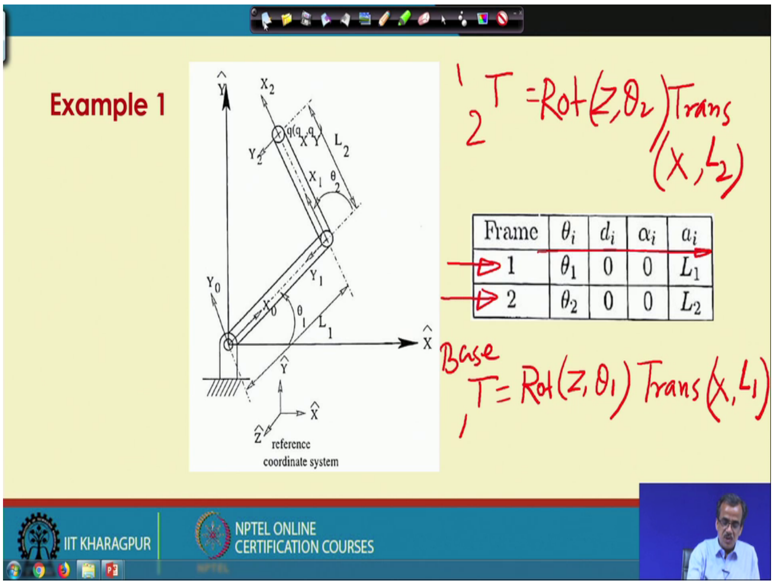

And you just move along this particular direction, without making any change. So, T 1 with respect to base the first is theta 1 that is your rotation about Z by theta 1. Next is 0 0 I am not going to write anything and the in the last one is that translation trans along X by L 1. So, this is what you mean by the T 1 with respect to base.

Similarly, we can also write down that is T 2 with respect to 1. So, I will have to concentrate here and this is nothing but rotation about Z by an angle theta 2, then comes your translation along X by L 2 and all such things actually I am just going to consider next. So, this T 1 with respect to base is nothing but is your rotation about Z by theta 1 translation along X by L 1. So, I know the 4 cross 4 matrix corresponding to this i know the 4 cross 4 matrix corresponding to this and if I just multiply then I will be getting. So, this particular the 4 cross 4 matrix for this T 1 with rest to base.

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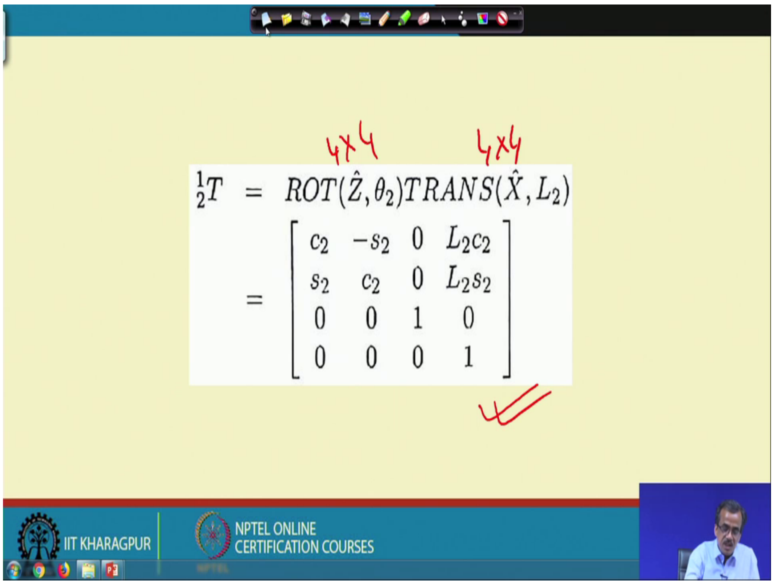

Now, similarly actually what we can do is we can find out this T 2 with respect to one that is your rotation about Z by an angle theta 2 and translation along X by L 2. So, I know the 4 cross 4 matrix here and I know the 4 cross 4 matrix here and I can multiply. So, I will be getting this particular the final matrix.

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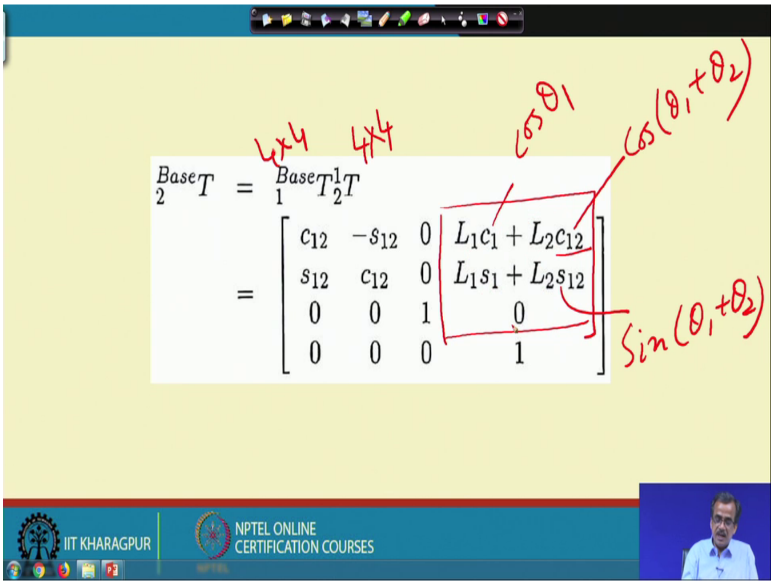

And once we have got this particular the final matrix now, I am in a position to find out what is T 2 with respect to base that is nothing but T 1 with respect to base multiplied by T 2 with respect to base.

Now, if I just multiply. So, this T 1 with respect to base that is nothing but a 4 4 cross 4 matrix, and this T 2 with respect to 1 is nothing but a 4 cross 4 matrix, then I will be getting actually this final 4 cross 4 matrix and here actually this carries information of this particular the position ok.

So, the position information is given by this particular the information and here c 1 means cos theta 1 and c 1 2, 1 2 means cos of theta 1 plus theta 2. Similarly 1 2 is nothing but sine of theta 1 plus theta 2 now if you just compare. So, whatever position information we are getting. So, if I just compare with our general information of technomatic. So, we can find out we can find out that this particular expression is correct.

For example say if you see; so this particular the 2 degrees of freedom serial manipulator. Now if this particular angle is theta 1.

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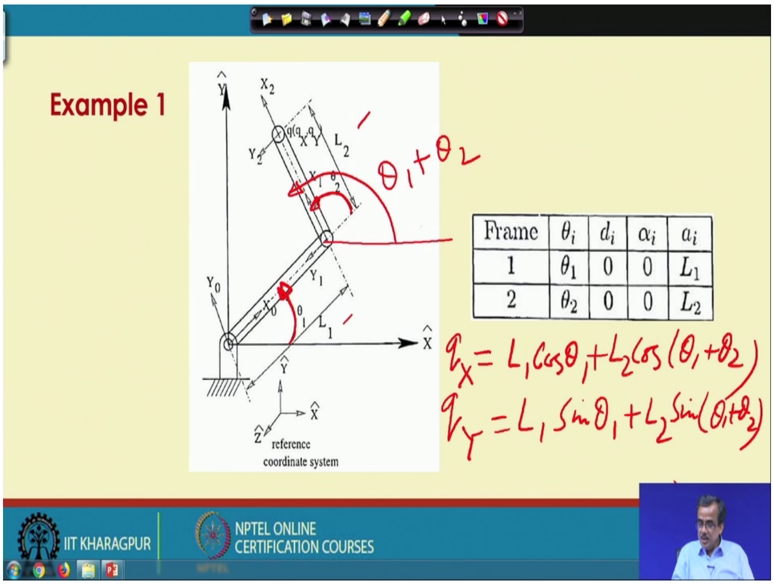

So, this is theta 1 the length of the link is your L 1 and the second link it is L 2 and with respect to these this particular joint angle is theta 2.

Now, with respect to X the total angle with respect to X is nothing but theta 1 last theta 2. So, this is your theta 1 plus theta 2 now very easily you can find out using the principle of technomatic. So, we can find out the general expression for q X is nothing but L 1 cos theta 1. So, this is theta 1 plus L 2 cos of theta 1 plus theta 2. Similarly we can find out this qy is nothing but L one sine theta 1 plus L 2 sine of theta 1 plus theta 2, this we can we can find out using the principle of your techno matric.

Now, the same thing we are getting we are getting after carrying out. So, this particular your that analysis. So, what you can do is. So, we can get this particular the same expression, that is your L 1 cos theta 1 plus L 2 cos theta 2 and that is nothing but qx.

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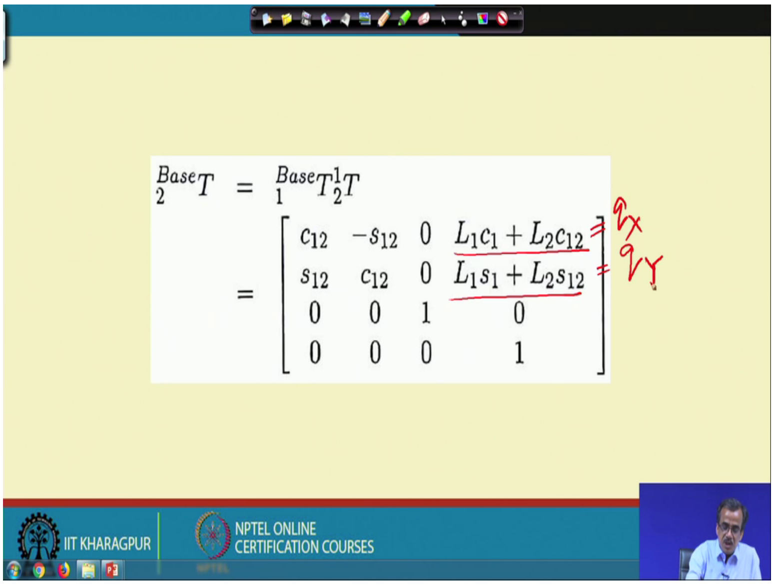

L 1 sine theta 1, L 2 sine theta 2 L 2 sine of theta 1 plus theta 2 is qy and qj is equal to 0.

The same expression we are getting. Now this particular problem is actually known as the forward kinematics problem.

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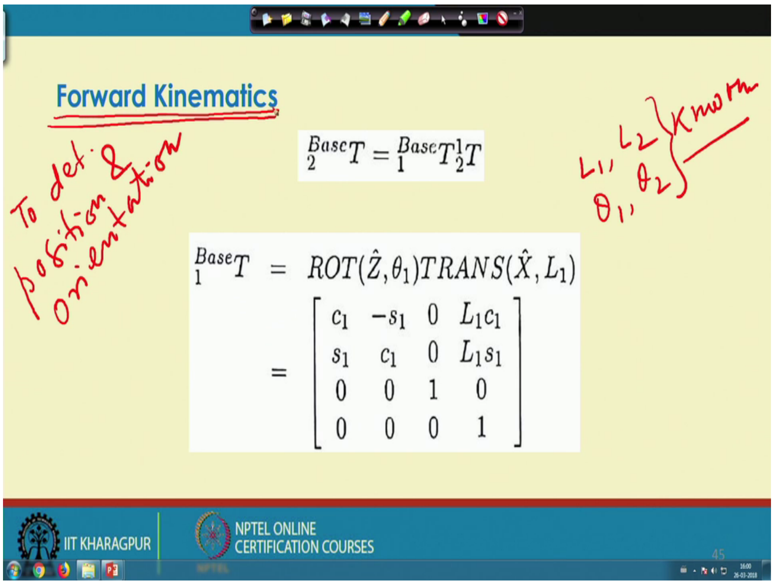

Now, in forward kinematics problem actually what we do is our aim is to determine our aim is to determine your to determine the position and orientation position and orientation of the end effector of the robot, provided the length of the links are known and the joint angles are known. So, that is actually the problem of the forward kinematics.

Once again let me repeat supposing that the length of the links say L 1 and L 2 are known, the joint angles theta 1 theta 2 are known. So, these are known and if these values are known can I not find out the position and orientation of the end effector with respect to the base coordinate system of the robot. If I take the physical example if this is the end effector and this is my base coordinate system, can I not find out the position and different orientation of this particular end effector with respect to the base coordinate system?

So, this particular problem is the problem of your the forward kinematics. So, the forward kinematics problem we can solve very easily using this particular the principle of Denavit Hattiesburg notation and then this frame transformation. So, very easily we are we can find out, we can solve the problem that is your the forward kinematics problem.

Thank you.