Robotics
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Lecture - 15
Robot Kinematics (Contd.)

Let us take the example of my own hand in the form of a serial manipulator. Say this is the serial manipulator this is the end effector ok. Now if I consider if I want to find out the position and orientation of this particular end effector, with respect to the fixed the coordinate system. Then what I will have to do is I will have to assign the coordinate system at the different joints, and then we will have to find out the frame and then we will have to talk about the frame transformation.

Now, to make it possible actually the first thing we will have to do is, at each of this particular joint. So, we will have to assign the coordinate system. Now to assign the coordinate system we will have to follow certain rules, and those rules actually are nothing but the Denavit Hartenberg notation rules, and this particular concept that was proposed in the year 1955 by Denavit and Hartenberg.

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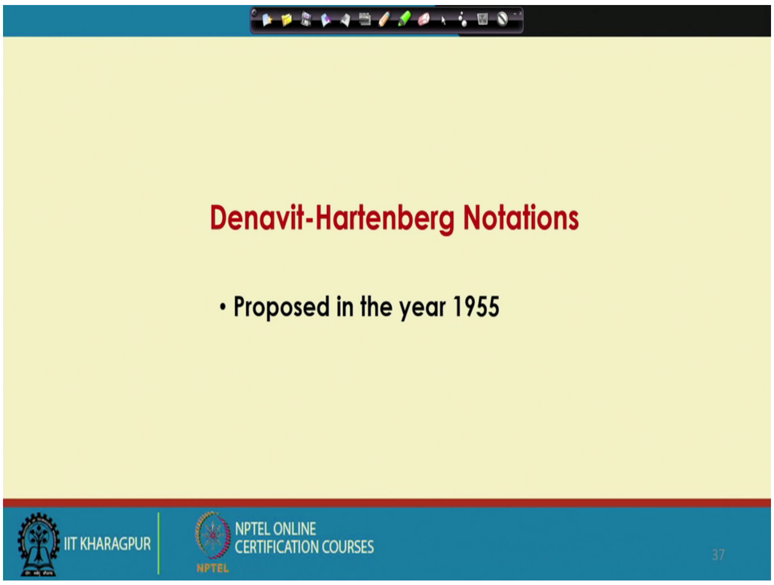

Now, according to this Denavit and Hartenberg rules actually we can assign coordinate system at the different joints. Then very easily we will be able to find out the position and orientation of this particular end effector with respect to the base coordinate frame and vice versa. So, now, we are going to discuss the Denavit Hartenberg notation and Denavit Hartenberg rules to assign the coordinate system at the different joints.

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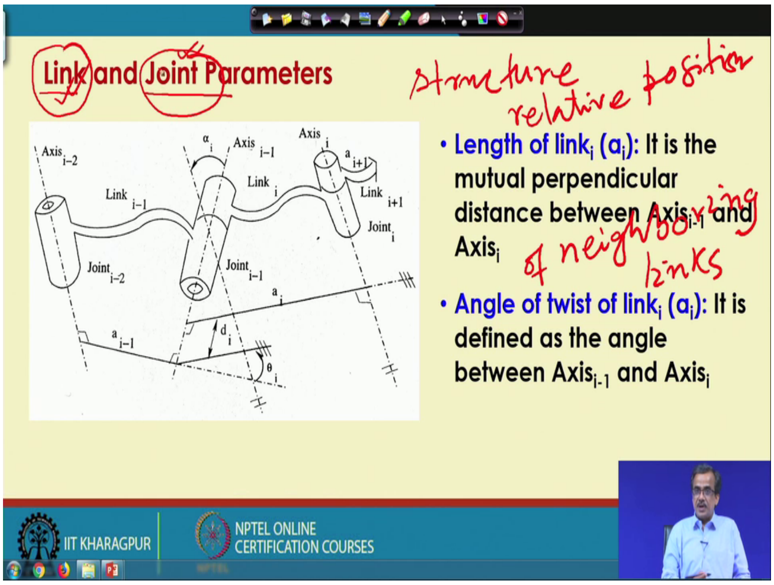

Now, before I go for that. So, what we will have to do is, we will have to define a few parameters. For example, say we will have to define 2 link parameters and there are 2 joint parameters. Now before I am just going for deriving or explaining the meaning of this link parameter and the joint parameter, let me tell you the purpose of using the link parameters and the joint parameters. Now the link parameters are used to represent the structure of a link.

So, if I want to represent the structure of a link. So, I will have to use the link parameter. Similarly the joint parameters are used to represent relative position relative position of neighboring links of neighboring of neighboring links. So, the relative position of the neighboring links if you want to find out we take the help of your the joint parameter ok.

Now, I am just going to define; so this particular the link parameter and the joint parameter. Now here the first thing I am just going to define is the link parameter that is the length of a link. The length of a link that is denoted by a i, and by definition it is the mutual perpendicular distance between the a x is i minus 1 and axis i. Now before that let me tell you that in this particular the sketch supposing that this is nothing but the joint i. So, this is the joint i similarly this is joint i minus 1 this is joint i minus 2. So, for this particular joint I; so this is the axis i the joint i minus 1 the axis is axis i minus 1 and for joint i minus 2 the axis is denoted by I minus 2.

Now here, if you see this particular axis i and axis i minus 1. So, this particularly actually this part this upper part is actually in 3 d. So, if I consider this axis i and axis i minus 1, they may lie on the same plane they may not lie. For example, say if I consider say this is one axis say axis i and this is another axis the axis i minus 1. So, they could be like this they could be like this they could be like this. So, they may not lie on the same plane or there is a possibility that both of them are lying on the same plane same 2 d plane.

Now, supposing that they are not lying on the same plane, they are lying on the two different plane. So, this is this is axis i this is axis i minus 1 and they are lying on two different planes, if they are lying on the two different planes like this or these or this. So, I can find out the mutual perpendicular distance, and that particular mutual perpendicular distance is nothing but the length of link i that is denoted by a i.

Now, similarly if they are lying on the same 2 d plane the plane of the board now here. So, the mutual perpendicular distance will be this. So, this will be the mutual perpendicular distance and they are parallel. So, if they are parallel and lying on the same 2 d plane. So, I can find out the mutual perpendicular distance. So, once again let me repeat that particular definition, the length of link that is a i it is the mutual perpendicular distance between axis i minus 1 and axis i.

So, this is nothing but axis i minus this axis i this is nothing but axis i minus 1 and here. So, this particular is nothing but is your a i that is the length of the link and this particular angle is 90 degree this particular angle is 90 degree. And, this axis i and axis i a according to this figure might be they are lying on two different planes now here. So, this particular angle is 90 degree and this is also 90 degree, and this is the mutual perpendicular distance and this a i is nothing but the length of the link i.

Now, let me let me let me take one very special case supposing that; so axis i and axis i minus 1. So, this is my axis i minus 1 and this is your axis i they are going to intersect at this particular point, and supposing that they are lying on the same 2 d plane. And if they are going to intersect at this particular point then what will be the value for this particular a i? A i will become equal to 0.

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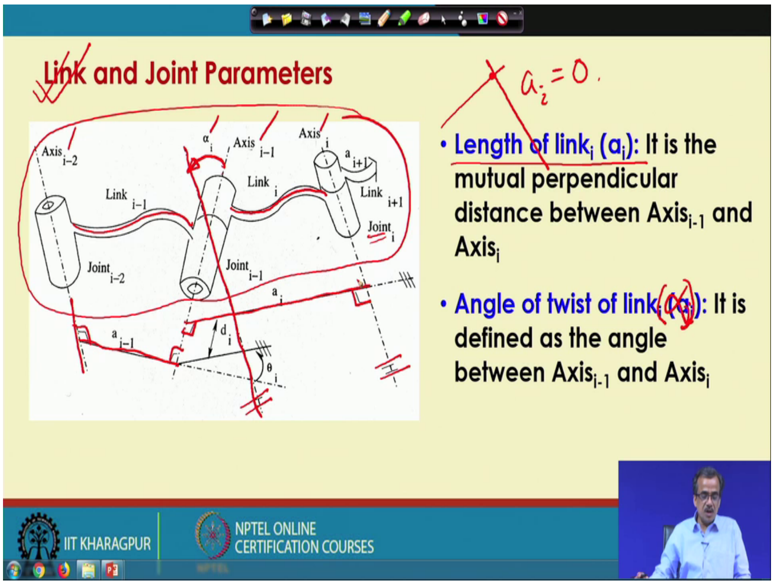

For this type of case actually a i becomes equal to 0.

So, this particular the length of the link, so that could be 0 and it could be nonzero. So, this is what you mean by the length of link i. The next is your the angle of twist of link i think here there is a mistake. So, this particular symbol we generally use alpha. So, this is alpha i. So, in place of a i this is actually alpha i this is the alpha i ok. So, angle of twist of link that is denoted by alpha i it is defined as the angle between the axis i minus 1 and axis i ok.

Now, this is the axis i this is my axis i minus 1. So, what you do is, this particular axis i we draw it here now if I just draw it here; so this particular axis i. So, this particular line is parallel to this ok. Now here I will be getting one angle, the angle between axis i minus 1 and axis i measured from axis i minus 1. So, this angle is nothing but alpha that is your the angle of twist alpha. So, this is actually the angle between the axis i minus 1 and x is i.

Now, remember. So, this particular angle alpha that is the angle of twist, it could be either positive or negative or sometimes it may also become 0. If the 2 axis are parallel then this particular angle will become equal to 0 here. So, this axis this angle is measured from axis i minus 1 to i and here it is anticlockwise. So, this is your positive alpha similarly if it is found to be clockwise. So, alpha could be negative also ok. So, these two things are actually nothing but the link parameters and the link parameters are used to represent the structure of a particular the link. For example, this is the link this is a link i, this is your link i minus 1. So, this is a i similarly this is your a i minus one. So, this is the mutual perpendicular distance between; so axis i minus 2 and axis i minus 1. So, this particular is a i minus 1. So, this is a i minus 1 and this is your a i.

So, till now I have defined only the 2 link parameters, now I am just going to define the joint parameters.

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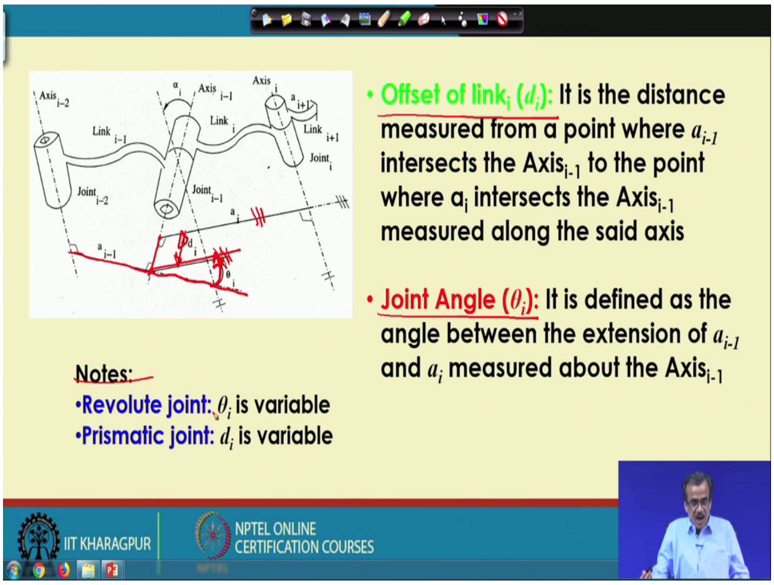

The purpose of the joint parameters I have already told that just to represent the relative position of the neighboring links. Now here there are 2 the joint the joint parameters, one is called the offset of link that is denoted by d i the link offset. And this particular link offset is nothing but the distance between the 2 points the first point is a point where a i minus 1 intersects the axis i minus 1 and another point where a i intersect the axis i minus 1.

So, this is the distance measured from a point where a i minus 1 intersects the axis i minus 1 to the point, where a i intersect the axis i minus 1. So, this particular distance is actually nothing but the link offset and that is denoted by d i ok. So, this is actually the d i. Now this particular d i this link offset it could be 0, sometimes the link offset becomes equal to 0 and it could be the positive value also and in some special case due to the coordinate system, it may take some negative value also.

Now, then comes the joint angle that is denoted by theta i, now this particular joint angle is defined as the angle between the extension of a i minus 1 and a i measured about axis i minus 1. So, if I extend. So, this particular a i minus 1. So, I will be getting this particular line and so, this line is actually parallel to this particular a i, now this angle between the extension of a i minus 1 and a i measured from a i minus 1 is nothing but the joint angle that is your theta i and theta i is actually measured from a extension of a i minus 1 to a i. Now this theta i it could be once again 0 it could be positive or it could be negative.

Now, here I have put actually 2 notes for a revolute joint theta i is the variable it is very obvious for example, if I take the example of a revolute joint like this. So, this particular joint is a revolute joint. So, if I take this is the axis about which I am taking the rotation and if you can sent it on this particular angle. So, this particular angle is the joint angle and that is actually a variable ok.

So, for a revolute joint; so theta i is the variable and what about the other three parameters. Other three parameters like a i, alpha i, d i those are kept constant similarly for a prismatic joint d i there is a link offset is the variable and the other three remaining parameters for example, say a i alpha i and theta i those things are kept constant. So, for revolute joint theta i is the variable, for prismatic joint d i is the variable.

Now, till now actually we have defined the 2 link parameters and 2 joint parameters and with the help of these four terms. So, I am just going to now state the rules to find out how to get that particular the coordinate system at a particular the joint how to assign the coordinate system at a particular the joint.

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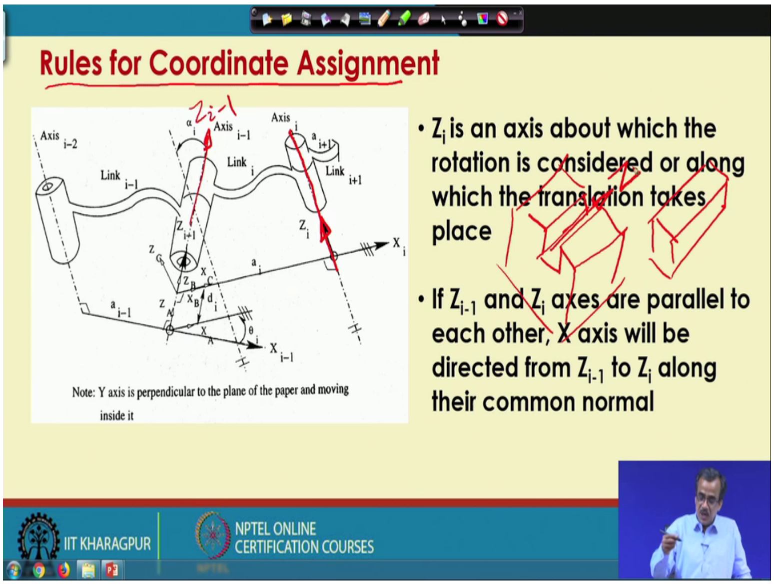

The rules for the coordinate assignment; now here we will have to find out now remember one thing, the first thing we will have to do is we will have to assign the Z coordinate system, next we try to find out the x coordinate system and after that we go for the y coordinate system.

Now, let us see how to represent or how to find out that Z i axis first. The rule for determining the Z i axis is as follows; Z i is an axis about which the rotation is considered and along which the translation takes place, now let us try to understand. So, Z i is an axis about which the rotation is considered now here according to this so, for example, for this particular joint. So, they see the axis about which I am taking the rotation. So, this is nothing but is your Z i.

Similarly here, you can see that. So, this is the axis about which I am taking the rotation in this particular joint. So, this particular is actually directs this is the axis that is Z i minus 1. So, this is Z i and this is your Z i minus 1 now let me repeat. Now let me take the example of the same example of the revolute joint. Now this is the axis about which I am taking the rotation. So, this particular thing is going to represent my Z axis and if it is a linear joint if there is a translation, I have already taken example of the linear joint like say q and qa for example, let me take a very simple sketch once again.

So, for example, say if I take this type of sketch for this linear joint, which I have already discussed for example, this type of linear joint if I take, and here if I just insert one key sort of thing. So, this particular key I am just going to insert here ok. So, this will be your Z direction. So, this is the direction of Z; so actually for the linear joint. So, Z is the axis about which; so this particular the translation takes place and if it is a rotary joint. So, j is the axis about which that a rotation takes place ok.

Now, I am just going to consider one case if Z i minus 1 and Z i axes are parallel to each other ok, then X axis will be directed from Z i minus 1 to Z i along their common normal. Now as I mentioned that these particular things your this axis i that is Z i and this particular Z i minus 1 say they may belong to two different planes, but they could be parallel or they are lying on the same plane they could be parallel. So, if they are found to be parallel. So, they are the mutually perpendicular direction, that is your the way we define this particular a i. So, a i is actually the direction of your x.

So, X will be along the length of the link ok; so x. So, once again let me repeat if the 2 Z axes are parallel. So, X will be along their common normal and; that means, you are here Z i minus 1 and Z i are parallel. So, this is a i direction. So, this is nothing but my X direction. So, this is your X i direction. So, I will already got this particular your Z i and X i see similarly. So, this particular is your Z i minus 1 and this is your X i minus 1, this is X i minus 1 and this is Z i minus 1 ok.

Now, let us see now there could be some other cases ok. So, I am just going to discuss those special cases.

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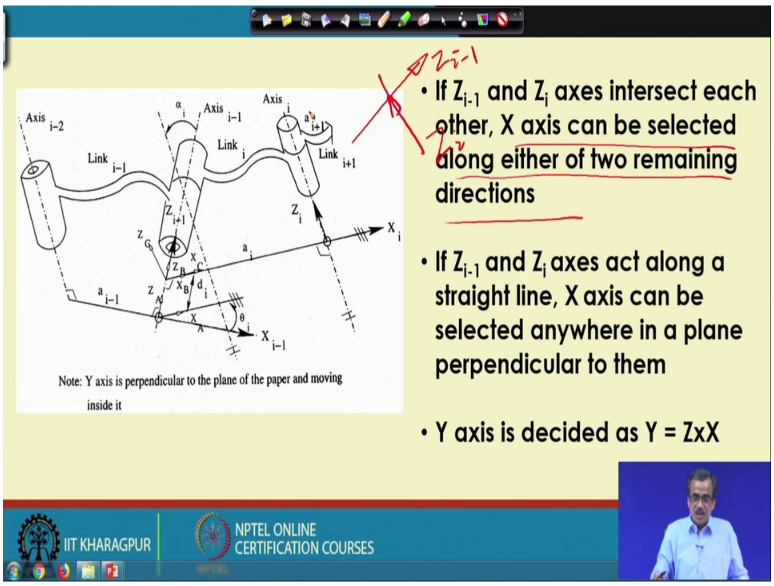

For example say if Z i minus 1 and Z i axes intersect each other. So, x axis can be selected either of the 2 remaining direction. Now if Z i and Z i axes intersect each other. So, this I have already discussed little bit supposing that this is my Z i minus 1 and this is my. So, this is Z i minus 1 and this is the Z i axes.

So, this Z i minus 1 and Z i they are intersecting. If they are intersecting then what will happen to the value of the length of the link a i that will become equal to 0. And if they are intersecting then x can be selected the rule is x can be selected along either of the 2 remaining direction; so at a particular joint. So, Z has been selected. So, I have got 2 remaining direction. So, out of the 2 remaining direction anything can be selected anyone can be selected as X. So, this is one very special case.

Now, similarly there could be some other very special case like if Z i minus 1 and Z i axes act along a straight line; that means, they are collinear. So, Z i minus 1 and Z i they are collinear, then x axis can be selected any higher in a plane perpendicular to them. So, if they are found to be collinear it is a very special case for example, say might be this is my Z i minus 1 and this is my Z i.

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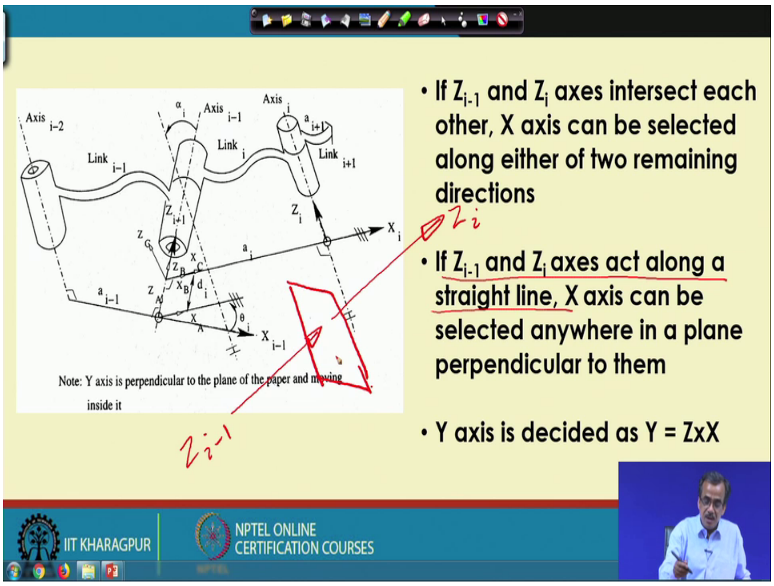

So, this is Z i and this is Z i minus 1 it is a very special case. So, what I will have to do is I will have to consider a plane, which is perpendicular to them.

Now a plane perpendicular to them could be of this type for example, if I just draw it here roughly one plane perpendicular to them could be something like this, which is perpendicular to this particular line. Now x may lie on this particular plane ok. So, this is actually one possibility similarly there could be another possibility like. So, this is Z i minus 1 and this is Z i. Now I am just going to define a plane which is perpendicular to both now that particular plane could be something like this also. So, my x can lie here also ok. So, both the possibilities are there. So, what you can do is; so these for these special cases. So, we will have to find out very carefully that particular the x direction.

So, till now we have discussed actually how to determine the Z axis and then your x axis and this particular sequence has to be maintained; that means, first we will have to find out the Z axis, then we go for your X axis and after that we try to find out the Y axis. Now Y axis is nothing but the Z cross X. Now Z X these are all unit vector. So, I can find out that the cross product of this particular Z and X and Y will be nothing but Z cross X.

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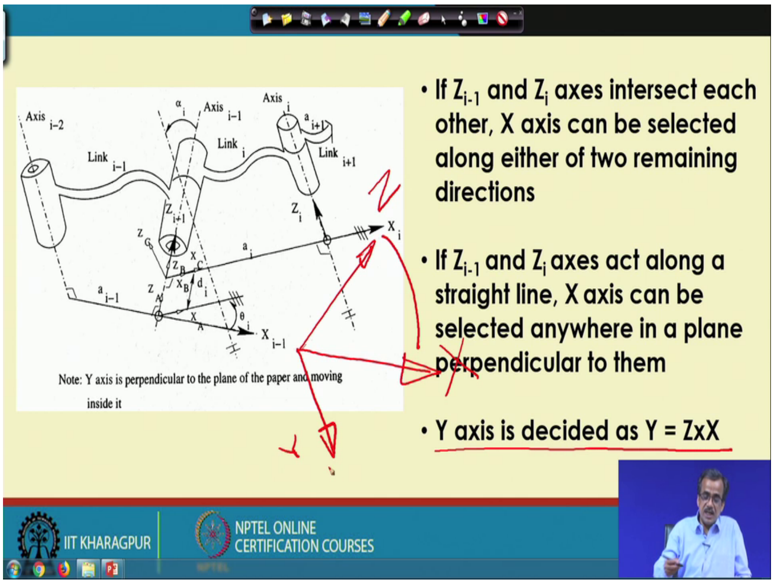

Now, let me take a very simple example supposing that say this is my Z. So, I have already defined and say this is my X. So, that also I have got, now I will have to find out your y direction now the Z cross X. So, according to the rule of cross product; so Z cross X will be something like this. So, this will be the direction of this particular the Y. So, Z cross X will be Y are you getting a fine. So, this is the way actually we will have to find out the Y direction.

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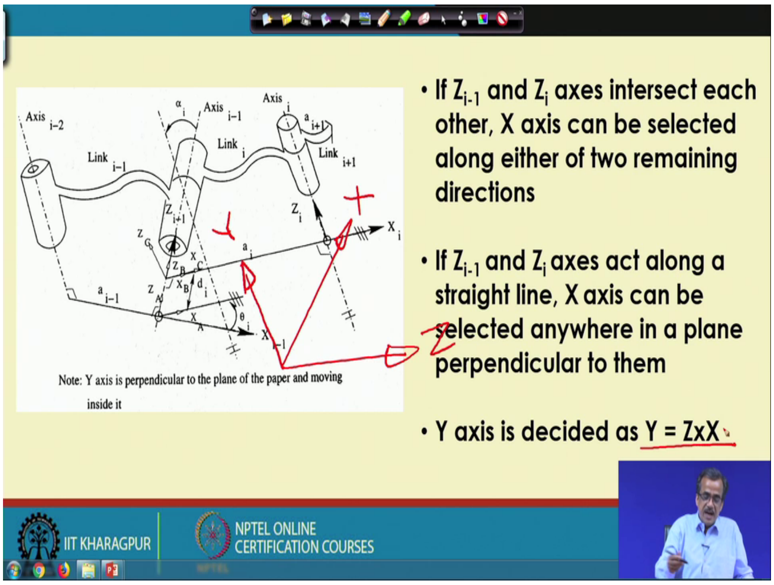

Now, if I just take the reveres the reverse problem for example, say this is my Z and this is my X ok.

Now, Z cross X. So, Z cross x will be something like this. So, this will be Y. So, Z cross x will be something like Y. So, we will have to be very careful while determining, so this particular your Y direction. So, Y is nothing but is your Z cross X and by following this particular rule at each of the joint. So, we can actually define the coordinate system like your XY and Z. And once you have got this particular thing now what you will have to do is.

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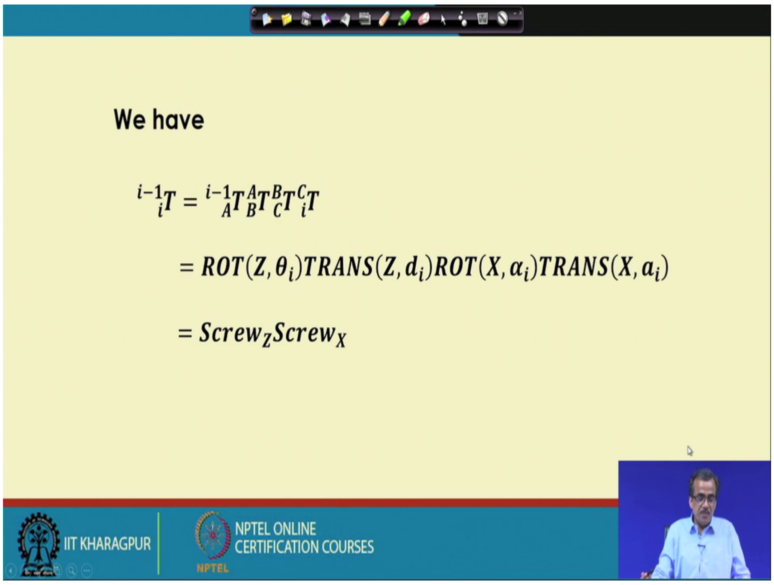

So, we will have to find out what is this transformation matrix that is T i with respect to i minus 1.

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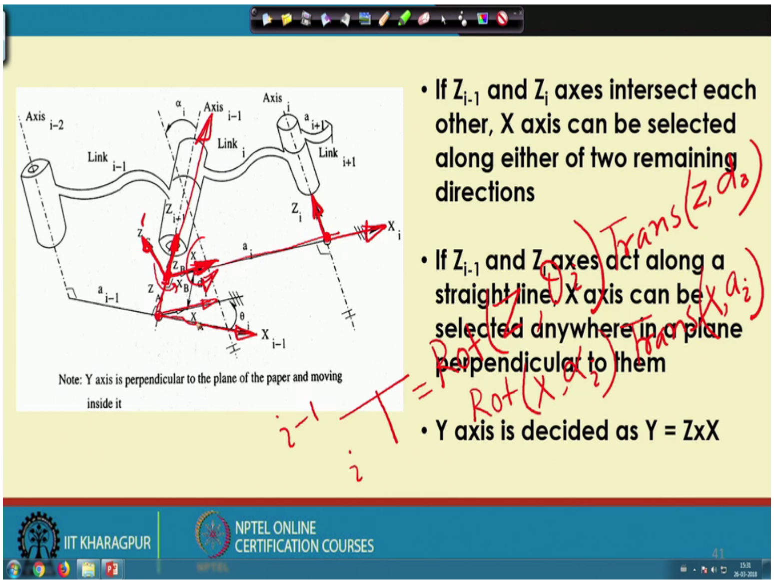

So, to determine this T i with respect to i minus 1; so this is nothing but i and this is nothing but is i minus 1. So, my aim is to determine my aim is to determine that is T i with respect you are i minus 1 so that I will have to find out how to find out? So, my i minus 1 frame is here and i-th frame is here. So, from here, so I will have to reach this particular how to do it? Now to do that actually what we do is, now I am just going to follow one sequence of rotation and translation let us try to follow.

So, let me try from x i minus 1 and Z i minus 1. As I told that this is nothing but is your x i minus 1 and this is nothing but is your Z i minus 1. So, what I will have to do is. So, I will have to take some rotation by an angle theta i about this particular your Z axis ok, then only I will be able to move here ok. So, I will be getting this is nothing but is your X A and I am taking rotation about Z A, so Z Z. So, Z A will remain same as Z i minus 1 ok. So, first I will have to take some the rotation sort of thing that is nothing but I am just going to write down. So, rotation about Z by angle theta i, the next from here I will have to reach this particular point how to reach? I will have to translate along z. So, I am just going to translate trans along Z by d i. So, I will be getting actually your this particular thing that is X C and this is your Z zc sorry X B and this is your Z B.

And after that actually I am just going to take some rotation about this particular x axis by an angle alpha. So, I am just going to take rotation about X by an angle alpha and then I will be getting these as nothing but X C because I have taken a rotation about X C and Z B will take the position like Z C. So, Z C will be something like this and ones we have got this particular X C and Z C now I can translate along x direction.

So, trans along this particular x direction by this particular amount a i. So, I will be able to reach particular this point and I will be getting this Z i and X i. So, with the help of these translation and rotation actually starting from here; so I am just going to reach this particular the point. Now this particular thing, in fact I am just going to repeat this explanation I am just going to repeat it.

Thank you.