Robotics
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Lecture - 14
Robot Kinematics (Contd.)

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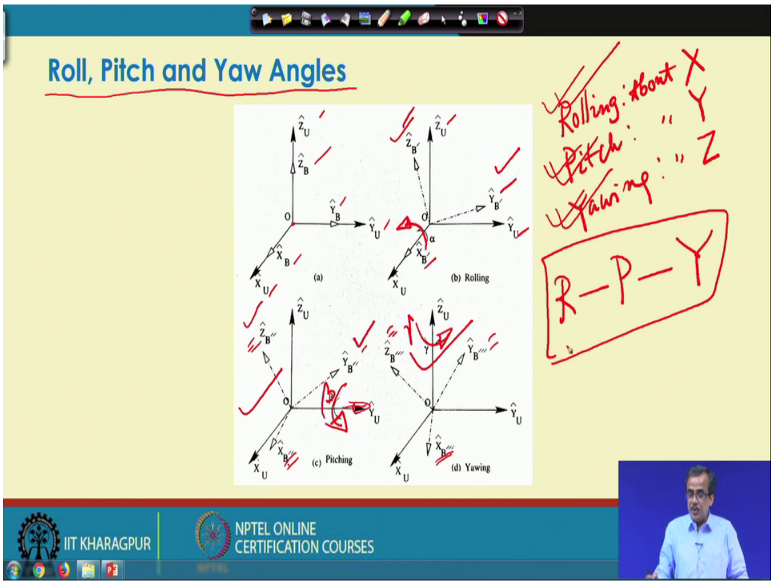

Now, I am going to discuss how to represent the orientation using the Principle of this Roll, Pitch and Yaw angles. Now, the concept of this roll, pitch and yaw actually we have copied from the movement of a ship. Now, let me first define the rolling, pitching and yawing movement of a particular ship and let us see how to copied to represent the orientation with the help of this roll, pitch and yaw.

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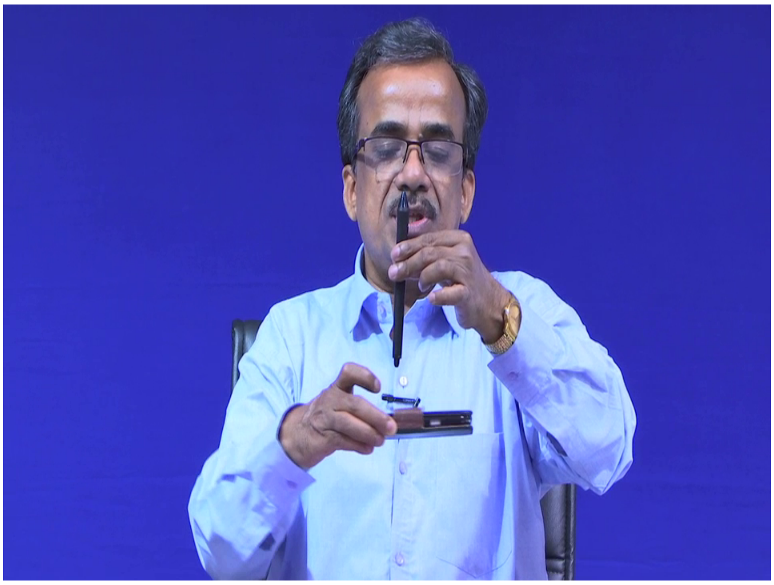

Now, supposing that so, this is nothing but a ship now if I consider that this particular ship now the ship will have rolling movement pitching movement and yawing movement. For example, say if I consider so, this particular the movement so, this particular movement is nothing but the rolling movement. Similarly, if I consider this type of movement of the ship is nothing but the pitching movement and this particular movement of the ship is nothing but the yawing movement; the moment we consider, so, this type of movement of the ship as if this is the axis about which I am taking the rotation.

Similarly, the movement we consider so, this type of movement of this particular ship that is your pitching as if this is the axis about which I am taking the rotation. And the moment we consider like this type of movement of the ship that is called the yawing movement and as if this is nothing but the axis about which I am taking the rotation.

Now, if I call the rotation about X is the rolling movement then the rotation about Y is the pitching movement then rotation about Z is nothing but the yawing movement. So, rolling is nothing but is your the rotation about X then the pitching movement is nothing but about Y, and this particular yawing movement is nothing but the rotation about Z, the same concept we are going to use it here.

Now, let us try to explain the way we can copy here. Now, here so, this particular the universal coordinate system is denoted by once again X U, Y U and Z U and the body coordinate system which is attached to the body the 3D body whose rotation I am just going to represent so, that that particular body coordinate system is B and it has got X B, Y B and Z B and initially they are coinciding and origin is exactly the same that is O for both the coordinate system.

Now, what you do is we take some rotation about the universal coordinate system and we try to rotate that particular B for example, say we first take the rotation about this particular X U. So, this is my X U by an angle alpha in the anticlockwise sense that is plus alpha. Now, if I take rotation about X U and initially X U and X B were coinciding. So, my X B prime will remain same as X U because I have taken rotation about X U. Now, this particular Y B prime will be different from Y U similarly this Z B prime will be different from Z U. So, this is what you mean by the rotation about X.

Now, I am just going to take the rotation about Y U. So, this is nothing but your Y U direction. So, I am taking the rotation about U, Y U by an angle beta in the anticlockwise sense. So, if I take the rotation about Y U by an angle beta so, what will happen to my X B double prime? So, X B double prime will be different from X B prime then Y B double prime will be different from Y B prime and Z B double prime will be different from your Z B prime. And now, I am just going to take the rotation about this particular the Z U. So, if I take the rotation about Z U so, by an angle gamma and in the anticlockwise sense.

So, what will happen to my X B triple prime? So, X B triple prime will be different from X B double prime then Y B triple prime will be different from Y B double prime and Z B triple prime will be different from Z B double prime. So, the final coordinate system the final frame I will be getting X B triple prime, Y B triple prime, Z B triple prime after taking three rotation in a particular the sequence.

Now, here actually what you do is the rotation about this particular X we call it this is nothing but the rolling motion then the rotation about Y we call it this is nothing but the pitching motion and the rotation about Z is nothing but the yawing motion and these particular rotations three rotations are in a particular the sequence. The sequence is nothing but the roll, pitch and yaw the roll, pitch and yaw so, this particular sequence we are going to follow and that is why whenever we are going to write down your the composite rotation matrix.

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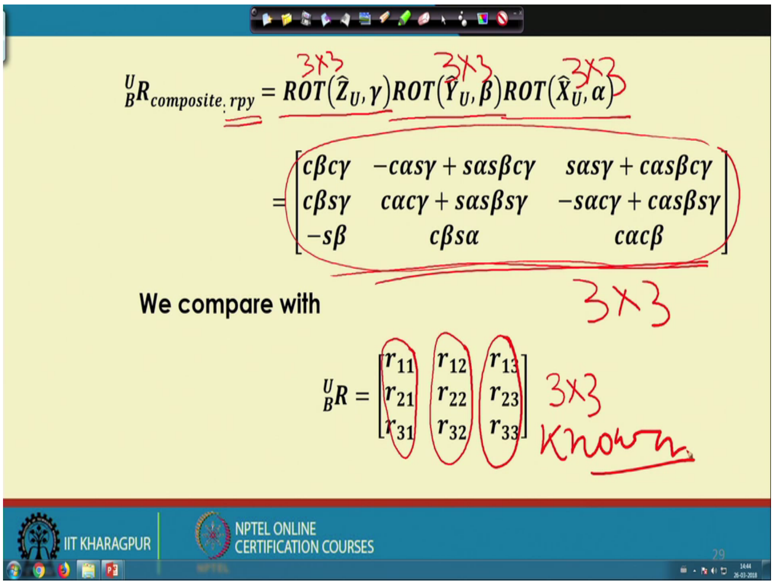

So, we are going to write down the composite rotation matrix as follows like your so, R B with respect to u composite and here I am writing rpy that is roll, pitch and yaw this is very important because if you just change the sequence so, you will be getting all together a different final matrix.

So, this particular the rpy that is the roll, pitch and yaw. So, this particular sequence will I have to write. So, rotation of B with respect to U, that means, I am just going to find out what is the orientation of that particular body with respect to the universal coordinate system. So, we will have to find out this R B with respect U composite comma rpy that is roll pitch yaw and once again the same rule. So, whatever I stated first that will go to the end. So, that is rotation about X U by an angle alpha followed by the rotation about Y U by an angle beta, followed by the rotation about Z U by an angle gamma.

Now, each of these rotation matrices are nothing but 3 cross 3 matrix. So, this is a 3 cross 3 matrix. We know the expression rotation about Z by an angle gamma similarly the rotation about Y U by an angle beta is 3 cross 3 and this is rotation about X U by an angle alpha is once again 3 cross 3 and if you multiply then ultimately I will be getting this as the final matrix and this is once again a 3 cross 3 matrix,. So, using the concept of the roll, pitch and yaw so, I will be getting that this is nothing but the final form of the rotation matrix .

Now, in Cartesian coordinate system which I have already discus the same rotation can be represented with the help of your 3 cross 3 matrix and this is nothing but r 1 1, r 1 2, 1 3 these are the elements for the first row then r 2 1, 2 2, 2 3 then 3 1, 3 2, 3 3. Now, in vector form if you see so, this is corresponding to if you remember this is the normal, vector this is corresponding to the sliding vector and this is corresponding to the approach vector.

So, if this is known and this is the final expression for the rotation matrix using the concept of the roll, pitch and yaw and element wise if I just compare; so, I will be able to find out corresponding to this known rotation if it is known what should be the angle for this rolling, pitching and yawing so, that we can determine very easily.

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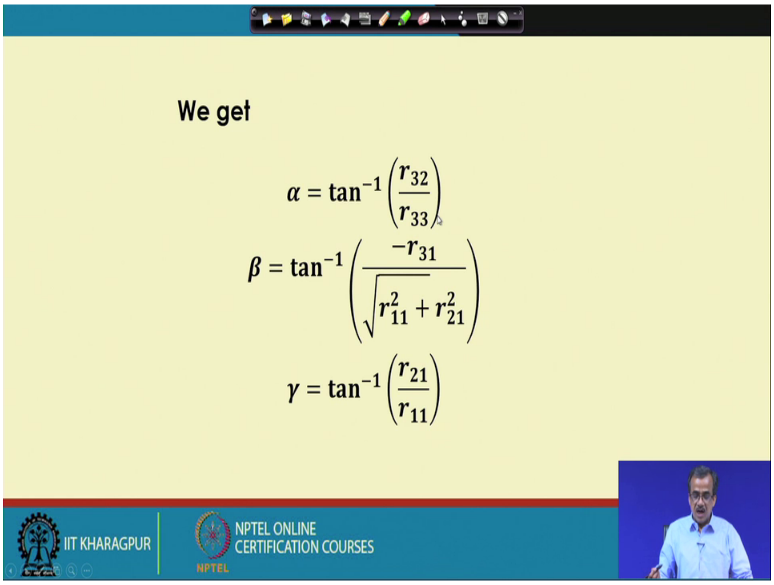

So, what we will have to do is element wise we will have to compare and then we will have to find out. For example, the angle of roll or the rolling alpha is nothing but tan inverse r 3 2 divided by r 3 3. So, if I see r 3 2 divided by 3 3, r 3 2 divided by 3 3. So, if I see this particular thing. So, r 3 2 is this r 3 3 is this and if I compare r 3 2 is nothing but this and r 3 3 is nothing but this.

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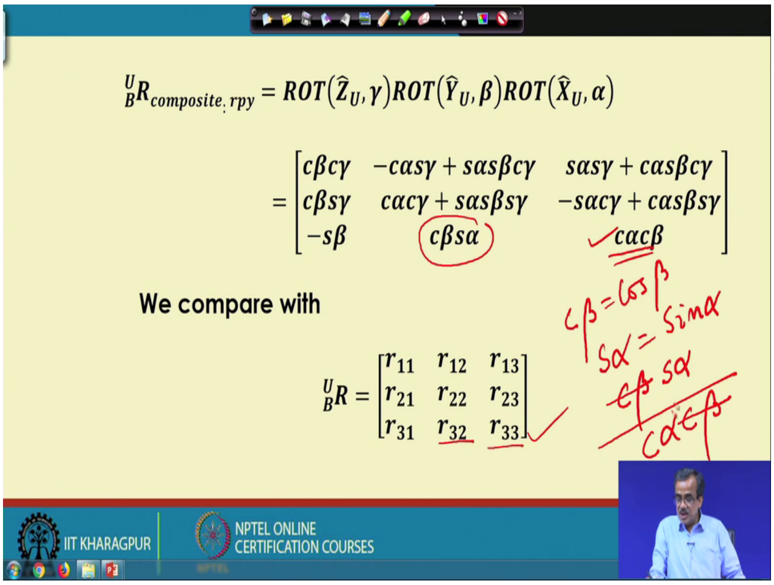

So, 3 2 divided by 3 3 that is c beta means cos beta. So, I just forgot to tell you that in sort we are writing. So, this is nothing but your cos beta, s alpha is nothing but is your the sine alpha. So, c beta s alpha c beta cos beta sine alpha divided by your cos alpha then comes your cos beta so, this particular thing. Now, if I just see so, c beta that is cos beta cos beta gets cancelled. So, I am getting 10 alpha sine alpha divided by the cos alpha. So, this is nothing but is your 10 alpha now if it is 10 alpha. So, very easily I can find out that alpha is nothing but tan inverse your 3 r 3 2 by r 3 3.

Now, following the same principle I can also find out the angle for this particular pitching. So, beta is nothing but tan inverse minus r 3 1 divided by square root of r 1 1 square plus r 2 1 square it is up to this, ok. So, we can find out this particular the beta. Similarly, the angle gamma that is nothing but the angle of yaw so, that can be determine as tan inverse r 2 1 divided by r 1 1. So, if I know the orientation in Cartesian coordinate system to achieve the same orientation I can also find out what should be the corresponding values for the angle of the rolling, pitching and yawing. And that is why the orientation of the robot can also be actually expressed in the using the principle of the roll, pitch and yaw.

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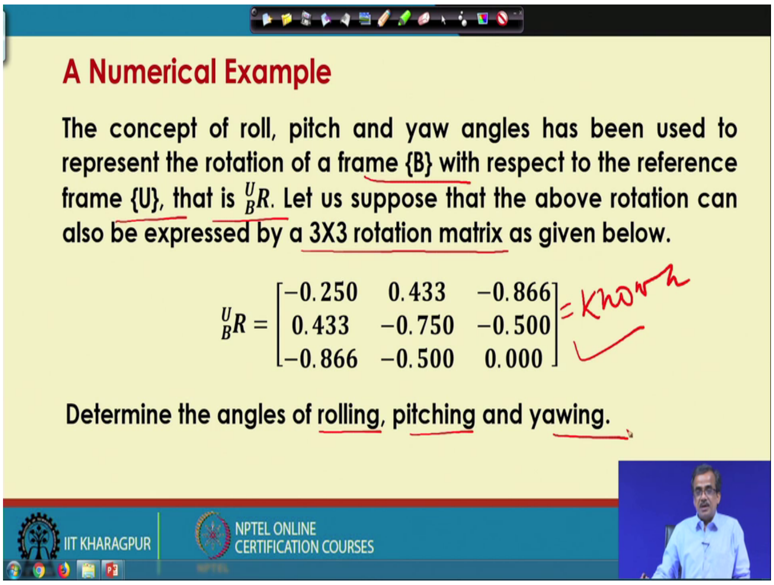

Now, here just to explain it further I have I am just going to take the help of one numerical example. So, if I take the help of this numerical example will understand that very easily we can find out the angle for the rolling, pitching and yawing. The statement of the problem is as follows the concept of roll, pitch and yaw angels has been used to represent the rotation of B the rotation of B with respect to the reference frame U that is your R B with respect to U.

Let us suppose that above rotation can also be expressed by a 3 cross 3 rotation matrix as given bellow. So, this particular rotation matrix is known. So, this is given. So, determine the angles of rolling pitching and yawing it is very simple.

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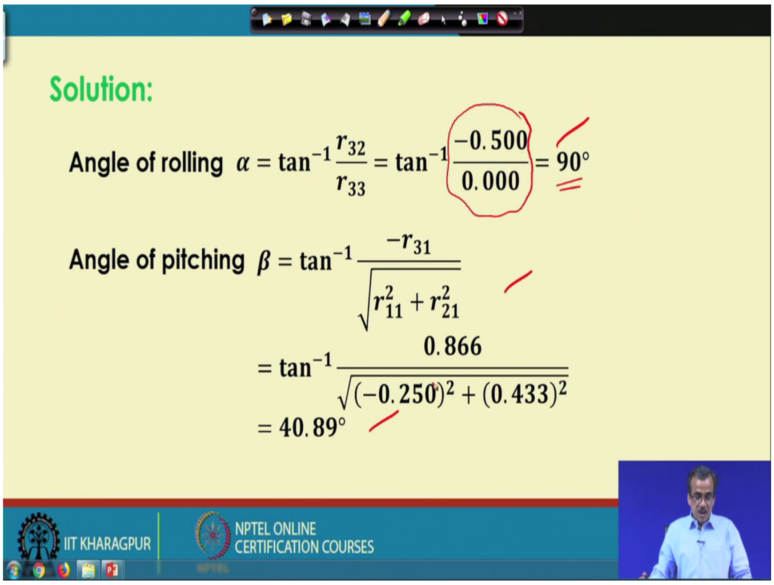

So, what I am we are going to do is so, very easily we are going to use those expression that is angle of rolling alpha is nothing but tan inverse r 3 2 by r 3 3. So, this is coming as tan inverse minus 0 point 0.5 divided by is your 0.000. So, this is nothing but actually the infinity infinite and that is why your this alpha is equal to 90 degree. Same, the angle for pitching beta is tan inverse your minus r 3 1 divided by this and if you just put the numerical value and calculate you will be getting 40.89 degree.

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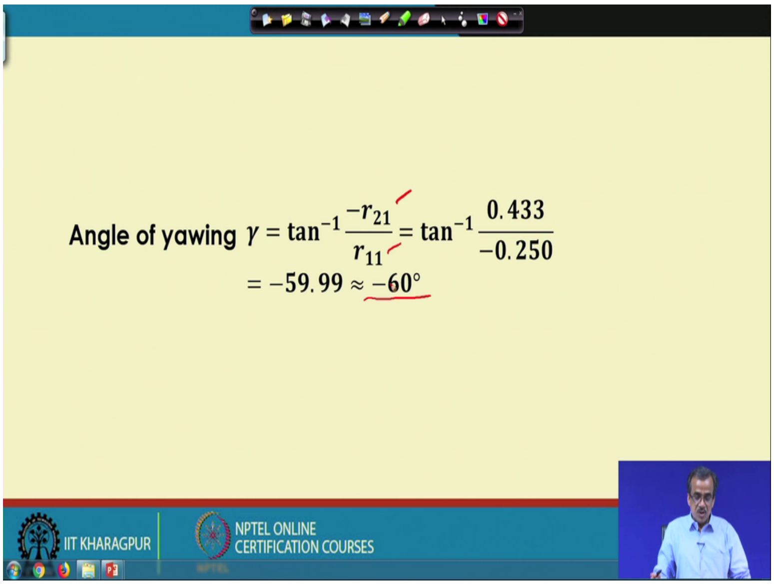

Now, similarly we can also find out actually the angle for the yawing. So, this alpha this gamma is nothing but tan inverse minus r 2 1 divided by r 1 1 and if you put the numerical value. So, we will be getting this is equal to more or less approximately equal to minus 60 degree, ok. So, if we consider that positive is clockwise and your positive is anticlockwise and your negative is clockwise. So, this is nothing but the clockwise. So, so this is the way actually we can find out this the angles for the rolling, pitching and yawing.

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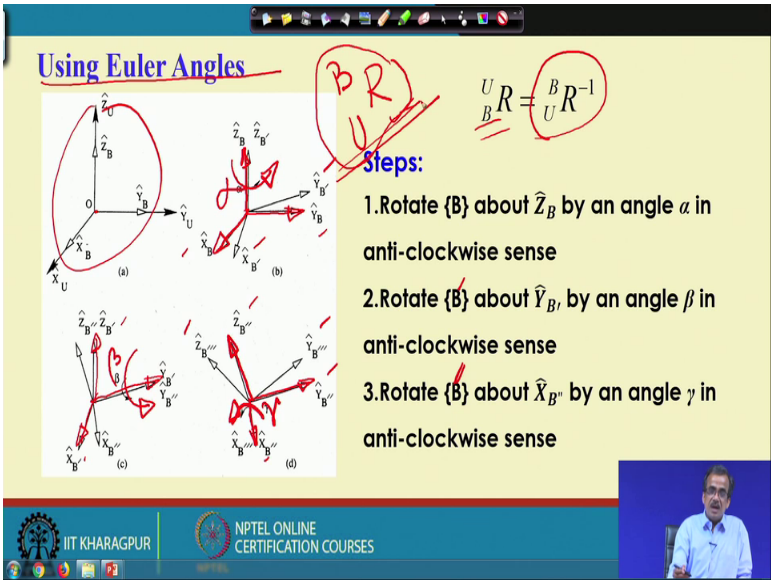

Now, I am just going to discuss another method which is also very frequently used to represent the orientation of this particular object. Now, supposing that the same that the initial coordinate systems are the same for example say. So, we are discussing this Euler angles we have got this X U, Y U, Z U it is the universal coordinate system X B, Y B and Z B is a body coordinate system and the origin is nothing but O.

Now, here actually what I do is initially they are coinciding, but after that we are going to leave this particular universal coordinate system and we are going to rotate only the B coordinate system with respect to the rotated coordinate system, but with not respect to the universal coordinate system. But, what is our aim? Our aim is to determine; what is R B with respect to U that is rotation of B with respect to U that we are trying to find out ok, but that will find out by following some sort of indirect method.

So, what we will do is we will try to find out first what is R U with respect to B and we will try to find out the inverse of that and that is nothing but R B with respect to U let me repeat. So, we are trying to find out first what is R U with respect to B and as I told that in this particular method we are taking rotation of B with respect to B itself and we will be taking rotation with respect to the rotated frame at itself.

Now, let us try to explain; so initially so, you forget about U coordinate system. So, initially so, this is actually the origin this is my X B this is my Y B and this is my Z B. So, what I do is we rotate B about Z B by an angle alpha in the anticlockwise sense. So, we are rotating with respect to this Z B by an angle alpha. Now, if I do that what will happen to my Z B prime Z B prime will remain same as Z B because we took the rotation about Z B, but what will happened to my X B prime.

So, this X B prime will be different from X B, then Y B prime will be different from your Y B and now, I am just going to take rotation. Rotate B prime it should be B prime about this Y B prime by an angle beta in the anticlockwise sense. So, this is nothing but Y B prime. So, whatever Y B prime we have got so, you draw this particular Y B prime similarly you draw this particular Z B prime and you draw this X B prime.

Now, I am just going to rotate about this Y B prime by an angle beta in the anticlockwise sense. So, what will happen to my Y B double prime? Y B double prime will remain same as Y B prime, but what will happen to X B double prime? X B double prime will be different from X B prime. Similarly, your Z B double prime will be different from your the Z B prime and after that actually what we do is we rotate B double prime B double prime with respect to X B double prime. Now, here let me draw this X B double prime. So, this is nothing but X B double prime this is nothing but is your Y B double prime and this is nothing but the Z B double prime.

Now, we are going to rotate this B double prime with respect to X B double prime. So, this is my X B double prime. So, I am just going to rotate by the angle gamma in the anticlockwise sense. So, what will happen to my X B triple prime? X B triple prime will remain same as X B double prime, but Y B triple prime will be different from Y B double prime and Z B triple prime will be different from Z B double prime and till now all the rotations we have taken with respect to the B coordinate itself and I have not yet involved I have not yet use this particular the universal coordinate system.

Now, let us try to understand one thing. So, initially so, this particular U coordinate system and the B coordinate system they were coinciding and after that we took rotation of B with respect to B itself, that means I am rotating the B coordinating system, but I am not doing anything with U. Now, can I not consider a similar situation that here initially they are coinciding and U is kept constant and B is rotating can I not find out one equivalent situation that as a B is kept constant and U is rotated by the same angle in the opposite direction? So, let me repeat initially the U coordinate system and B coordinate system were coinciding. Now, B is rotated by some angle say alpha in the anticlockwise direction now can I not say that this is equivalent to the situation that B is kept constant and U is rotated by the same angle alpha in the opposite direction, that means, just like the velocity and relative velocity that particular the concept sort of thing.

Now, the reason why I am going for this type of thing because my aim is to determine actually the final aim is to determine R B with respect to U, but before that I will have to find out what is R U with respect to B. Now, if I want to find out what is R U with respect to B and if I do not include U I cannot find out this R U with respect to B. Just to include U with B I am taking in the help of. So, that particular concept that means using that particular concept if I write down the composite rotation matrix. So, this is nothing but R U this is nothing but R U with respect to B and Euler angles will have to write down Euler angles here.

So, that is nothing but whatever I consider first, but with the negative sign because we rotated B keeping U fixed. Now, we are considering as if we are rotating U keeping B fixed in the opposite direction. So, we are considering the rotation about Z B by an angle minus alpha followed by the rotation about Y B by an angle minus beta, the rotation about X B by an angle minus gamma. Now, we know the expression of each of this particular the rotation matrix.

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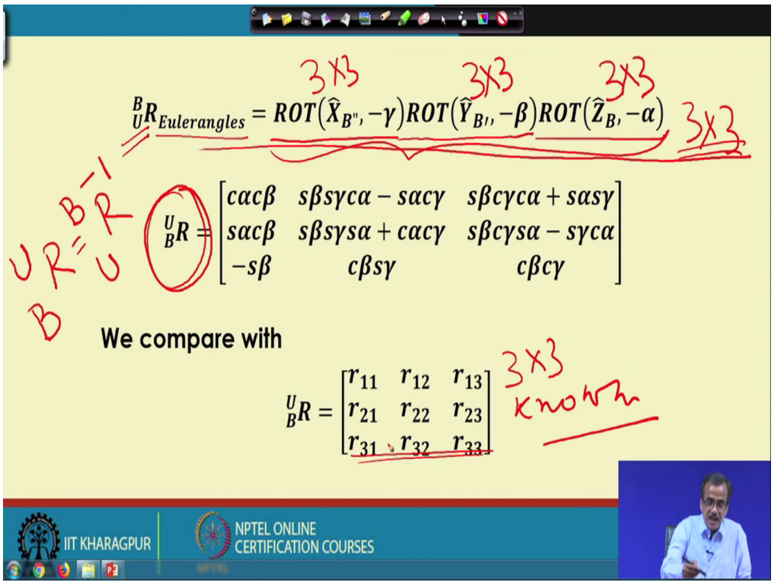

For example, rotation about X minus gamma so, I can write down the 3 cross 3 matrix. Similarly, rotation about Y B by an angle beta, so, I can write down this particular 3 cross 3 matrix then comes your rotation about Z B by an angle alpha. So, I can write down the 3 cross 3 matrix and these three 3 cross 3 matrices I can multiply. And finally, I will be getting one 3 cross 3 matrix and that is nothing but is your R U with respect to B, but what we need is just the reverse. So, what I need is R B with respect to U. So, this R B that is the rotation of B that is the body coordinate system with respect to the universal coordinate system and that is nothing but see your R U with respect to B inverse of that.

Now, if I can find out so, this R U with respect to B so, this is nothing but a 3 cross 3 matrix and this is a pure rotation matrix and as it is a pure rotation matrix so, very easily we can find out that the inverse because for a pure rotation matrix the inverse is nothing but the transpose. So, that means, the row will column and column will be row. So, whatever matrix we are getting here the 3 cross 3 matrix you try to find out the transpose of that and that is nothing but the inverse. So, you will be getting the rotation of B with respect to U. So, this particular thing we can find out. So, using this Euler angles we can represent the orientation with respect to the universal coordination system.

Now, supposing that that in Cartesian coordination system so, this particular R B with respect to U, so, this particular 3 cross 3 matrix is known to us supposing that this is known to us. Now, if it is known then element wise if I just compare so, this particular matrix is equal to this then element wise if I compare. So, very easily we can find out the numerical values for this particular the alpha beta and gamma that is the Euler angles. So, let us see how to find out the Euler angles values. So, alpha is nothing but tan inverse r 2 1 divided by r 1 1. So, r 2 1 divided by r 1 1. So, r 2 1 r 2 1 is nothing but this and r 1 1 is nothing but this. So, r 2 1 is nothing but your sine alpha cos beta.

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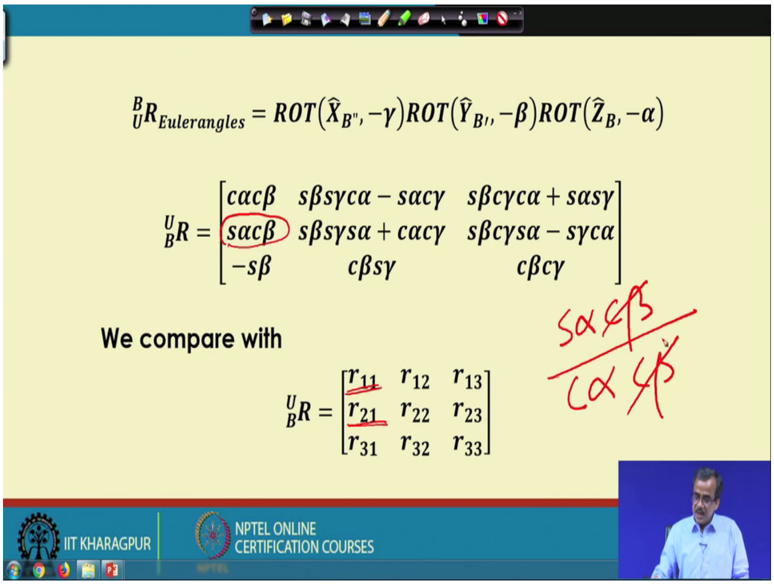

So, sine alpha cos beta s alpha c beta that is sign alpha cos beta and r 1 1 is nothing but is your cos alpha cos beta so, c alpha c beta. So, cos beta, cos beta gets cancelled and I will be getting tan alpha and if I get tan alpha. So, very easily I can find out alpha is tan inverse this. Similarly, I can find out your alpha is this, beta is nothing but this. So, minus r 3 1 divided by r 1 1 square plus r 2 1 square it should come up to this, ok. Then gamma is nothing but tan inverse r 3 2 divide by your r 3 3.

So, in this way actually we can find out we can represent both the position as well as orientation in different coordinate system. And if we can represent the position and orientation in different coordinate system the same robot you can control in different coordinate system and that is why if you see the remote controller for the robot. For example, I mention about that that is the teach-pendant. So, while discussing the robot teaching methods I discussed about the teach-pendant. So, using the teach-pendant so, in different coordinate system I can control the manipulator or that particular the robot.

So, once again let me repeat that the position can be expressed either in Cartesian coordinate system or in cylindrical coordinate system or in spherical coordinate system. Similarly, the orientation or the rotation can be represented either in Cartesian coordinate system or in roll, pitch and yaw or in Euler angles.

Thank you.